Unconditioned Zone Model

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Philip Niles
Larry Palmiter
Bruce Wilcox
Ken Nittler

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1. Introduction

1.1 What it is designed to model:

The unconditioned zone model (UZM) performs an annual thermal simulation of two unconditioned zones (UZ's): an attic and a crawl space. The program can thermally model typical roofs, ceilings with knee walls, end walls, floors, trusses, duct systems, and ground components.

Convection and radiation inside the UZ are treated separately to model radiant barrier and ventilation affects better than is afforded by the simple combined coefficient method. The program accounts for infiltration from the conditioned zones and natural ventilation through roof and crawl space vents. Wind, solar and sky radiation effects are included.

Each of the unconditioned zones may be thermally connected to up to two conditioned zones (CZ's) via ceilings, knee walls, floors, and infiltration. The conditioned zones are assumed to be modeled by an alternate program, the conditioned zone model (CZM), that determines the conditioned zone hourly conditions. Micropas was used to model the conditioned zones for the development work on the UZM program.

Each conditioned zone has an air-handler associated with it, and each air handler can have supply and/or return ducts in the UZ, and in the conditioned zone itself--see Figure 1.1. Both air handlers' can operate independently in either a heating, cooling, or off mode. The CZM program is assumed to determine whether an air handler should be in a heating or cooling mode, or floating, and if heating or cooling, the magnitude of the load that must be met by the air handler to keep the CZ at its current set point. In the off mode case the UZ is modeled during the hour without duct or air handler effects.

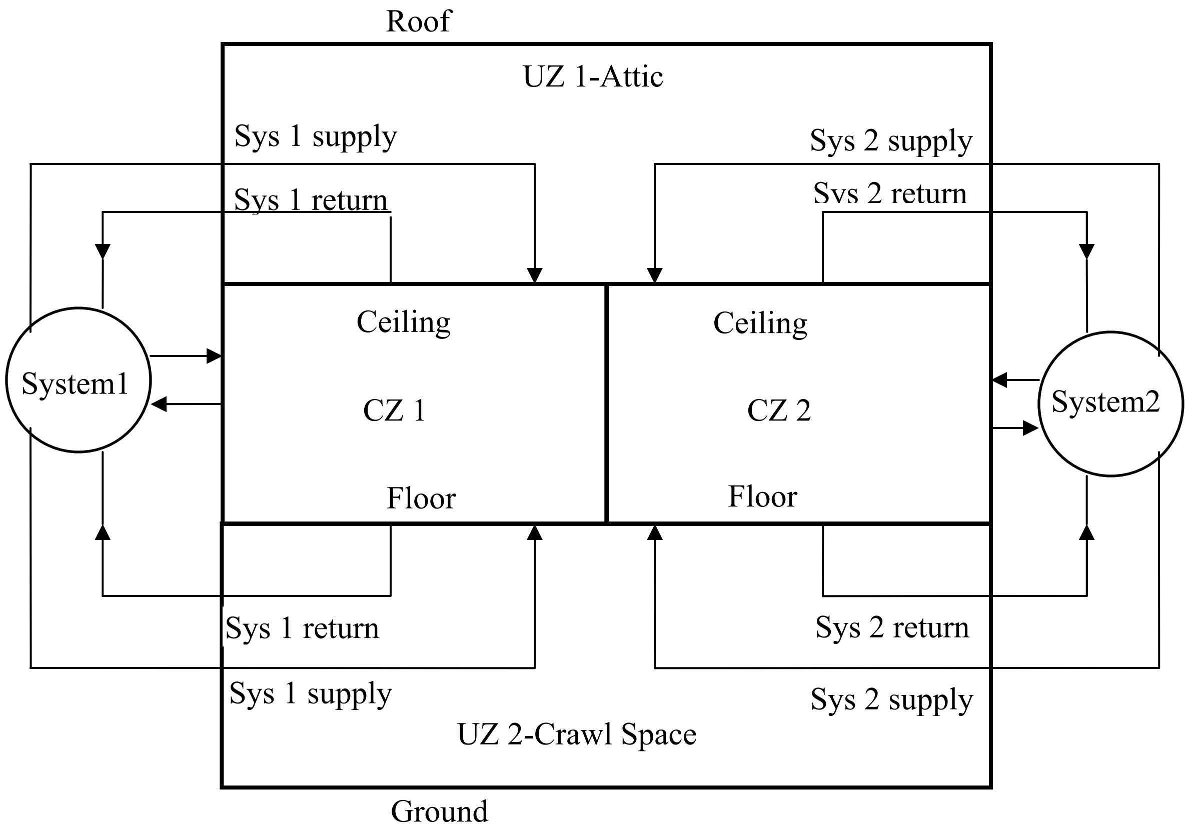

Figure 1.1. Schematic of zones and air handler systems. All ducts are assumed to be in either the conditioned or unconditioned zones but for clarity are shown partly outside of the envelope.

The UZM determines duct losses, their effect on the conditions of the UZ's and CZ's, and their effect on the heating or cooling delivery of the air handler system.

The duct system model allows unequal return and supply duct areas, with optional insulation thickness. The ducts can have unequal supply and return leakages, and the influence of unbalanced duct leakage on the UZ and CZ natural infiltration is taken into account.

1.2 How the program works

The unconditioned zone model is designed as a stand alone simulation to be implemented essentially as a subroutine of an hourly conditioned zone model, the CZM. After input of all unconditioned zone structural data and air handler system flow rates, the CZM supplies hourly data to the UZM consisting of each CZ's:

- operating mode (heating, cooling, or off)
- air handler nominal capacities
- heating or cooling load
- the zone air temperature
- the natural infiltration rate
- wind, insolation, long-wave sky radiation, and ambient temperature

Given the hourly ambient and CZ conditions, the UZM models the UZ's for the same hour, but on a sub-hourly time step to better model envelope heat capacity effects. UZ envelope mass temperatures are updated each sub-hour time step using a finite difference routine. An interior energy balance on all of the thermally coupled interior heat sources and sinks is used each sub-hour time step to determine the UZ air and mean radiant temperatures.

Based on the air handler hourly nominal capacities supplied, the UZM determines the duct losses each sub-hour time step for each CZ. If Qdel, the net heat delivered to a CZ, is greater than the CZ's load, the CZ's air handler is set to run a fraction of the sub-hour time step, so that Qdel matchs the load. On the other hand if Qdel at full nominal capacity is less than the CZ's load, the CZ's capacity is increased until the load is met, overriding the nominal capacity value. That is, the system is always assumed to be able to handle the load, regardless of its nominal capacity. If does so by increasing or decreasing the temperature change through the air handler without changing it's flow rate. This strategy is in accord with the California ACM (see ref: Residential Alternative Calculation Method) procedures with respect to system capacity specifications.

Heat transfer networks:
All heat transfer rates determined each sub-hour time step to update mass temperatures, duct losses, infiltration loads, and zone temperatures, are based on the temperatures of all nodes at the beginning of the sub-hour time step. This is consistent with the forward-difference (Euler) method of solving for mass node temperatures (see Sebald (1985). If the temperature or heat flow is only available hourly (such as ambient temperature and insolation) then the value at the beginning of the hour is used, with no interpolation scheme employed.

The UZ's are modeled with separate convection and long-wave radiation networks to handle the heat exchange between interior surfaces and the UZ air. The convection network uses fixed or buoyancy sensitive convection coefficients to connect all interior surfaces to a UZ air temperature node representing a fully mixed air in the zone. The long-wave radiation network uses a mean-radiant-temperature node as a clearinghouse for radiant interchange between the UZ surfaces. Each interior surface is connected to the MRT node with a conductance calculated from the surface emissivities and view factors (see Section 3).

Mass updates:
Massive envelope elements are represented by parallel path lumped resistance and lumped heat capacity network elements (see Section 2.1).

The mass nodes are updated each sub-hour time step based on conditions at the start of the sub-hour time step using a forward difference solution of the mass node differential equation. After the masses are updated, an energy balance on the whole UZ network determines the air and mrt temperatures. All massless temperature nodes are dissolved from the network. To keep the forward difference method stable and accurate, the sub-hour time step used to solve the UZM is set below the mass node time-constant expected for typical constructions (see Section 7).

Ducts:
The duct model builds on the procedure given by Palmiter (see Francisco and Palmiter, 2003), that uses a steady state heat exchanger effectiveness approach to get analytical expressions for instantaneous duct loss and system efficiencies. The UZM model, developed for this program by Palmiter, makes use of many of the same fundamental steady state equations and approach, but given the considerable complexity of the multiple duct systems, does not do a simultaneous solution of all the equations which a generalized Francisco and Palmiter scheme may imply. Instead the approach takes advantage of the small time steps used in the code, and in effect decouples the systems from each other and the UZ by basing all losses and other heat transfers occurring during the sub-hour time step on the driving conditions of Tair and Tmrt known at the beginning of the time step, similar to how heat transfers are determined during mass temperature updates (Section 7).

Other assumptions made in the UZM duct program: mass and thermal siphon effects in the duct system are ignored; off-cycle losses due to infiltration through holes in the duct work are not modeled; only sensible loads are addressed. The duct system model is discussed in more detail in Section 5.

Interaction:
The UZM program is structured such that the CZ and UZ programs run sequentially each hour with no iteration between programs. The main thermal interaction between the CZ's and UZ's is the supply and return duct air flows that the air handler delivers to satisfy the CZ loads. Since the CZ loads are always satisfied by the effectively unlimited air handler capacities as discussed above, there is no need to iterate hourly between the UZM and CZM models, as there would be if the capacity were limited--a limited capacity would not always allow the CZ to stay at its current hour's setpoint and iteration between the CZ and UZM would be required to determine the CZ temperature for the hour. Although iteration is not needed in this case, the following three thermal/flow interactions between the UZ's and CZ's require approximate algorithms in order to avoid interation.

1. Ceiling and Floor Conduction

   The heat conducted through the ceiling and floor due to the temperature differences between the CZ's and the UZ's is determined by the UZM model, which calculates this heat transfer every sub-hourly time step, and sums them to get the hourly total heat transfer to each CZ. Since the load sent by the CZM should have included this heat transfer, iteration could be used to correct the load. To avoid iteration, these hourly values are returned to the CZM at the end of the hour and the CZM uses these values as part of the next hours CZ load's sent to the UZM. The time lag is considered to have only a minor impact on the accuracy of the results.

2. Ceiling and Floor Infiltration.

The ceiling and floor infiltrations are set as fixed fractions (possibly different for the ceiling and floor) of the CZ natural infiltration rate. The CZ natural ventilation rates and the fractions are hourly inputs from the CZM to the UZM. The affect of duct unbalanced leakage on the CZ's natural infiltration rate used in this calculation is ignored in determining the ceiling/floor infiltration.

The direction of the infiltration flow is determined by the relative magnitudes of the CZ and ambient temperatures. If it is hotter in the CZ than outdoors then the floor infiltration flow goes from the crawl space to the CZ and the ceiling infiltration goes from the CZ to the attic. These flows reverse if it is cooler in the CZ than outdoors. If the crawl space doesn't exist, no floor infiltration is assumed to occur.

When the direction of infiltration flow is such that the infiltration flow from a UZ goes into the conditioned zone it may change the CZ load since infiltration from the UZ's will likely be at a different temperature than ambient. Instead of correcting the load through iteration, the correction to this load made in the UZM. The form of the correction is based on the requirement that the CZM has calculated it's natural infiltration load to be all coming from air at the ambient temperature. The correction is made by adjusting the load sent by the CZM, essentially by adding the part of the CZ load from the attic or crawl space infiltration (ceiling and floor infiltration into the CZ can't occur simultaneously), and subtracting the overcounted part of the load from the ambient temperture. These corrections are made based conditions at the beginning of the simulation hour. See Section 5.2.1).

3. CZ Infiltration of Outdoor Air due to Unbalanced Duct Leakage.

Operation of the air handler system with an unbalance duct leakage will pressurize or depressurize the conditioned zone, increasing or or decreasing the infiltration into the CZ from outdoors compared to the natural infiltration value. This will change the CZ infiltration rate and thus the infiltration part of the CZ load. To avoid iterating to correct the load, it is accounted for in the UZM by adjusting the heat delivered by the air handler system each sub-hour time step so that the heat delivered is what would be delivered if that change in infiltration load had been accounted for--see Section 5.7 and 6.3.1.

The latter two iteration avoidance approximations, cause errors in situations like: the CZM tells the UZM there no load this hour, so the air handler is not operated, although it would have operated at least a fraction of the hour if the correct load had been known by the CZM. And other similar scenarios.

2. <u>Modeling of Components:</u>

2.1 <u>Frame Construction Model:</u>

Frame walls with different specs have different 'constructions'. The following constructions are available.

attic frame constructions:

Construction Number (ncon) | Construction Description

1	roof
2	ceiling for cz1
3	ceiling for cz2
4	knee wall for cz1
5	knee wall for cz2
6	end wall

crawl space frame constructions:

Construction Number (ncon)	Construction Description
1	floor for cz1
2	floor for cz2
3	end wall

As shown in Figure 2.1, constructions can have more than one sub-surface called a 'component'. Each frame wall construction is broken into two components, corresponding to two parallel heat flow paths, the part through the joists, and the part between the joists. The inside surfaces of components are connected to the Tair and Tmrt nodes of the adjacent UZ. The outside surface nodes are connected to Tamb if they are roof or end wall surfaces, or to the adjacent CZ temperature if they are ceiling or floor surfaces. The lines between dots represent conductances calculated from the frame construction inputs. Roof and end wall outside surfaces can also receive solar and long-wave sky radiation (see Appendix B).

Each massive layer of a component is represented by a lumped capacitance at its midline, with half of the lumped resistance connecting each side of the capacitance to the adjacent layer: a "tee" circuit.

The number of constructions and components that can be used to model a UZ is fixed for the attic and crawl space UZ's as shown in the following two tables. For example, the program input for construction number 3 in the attic is always used to define the parallel path components 5 and 6 for the ceiling of conditioned zone 2.

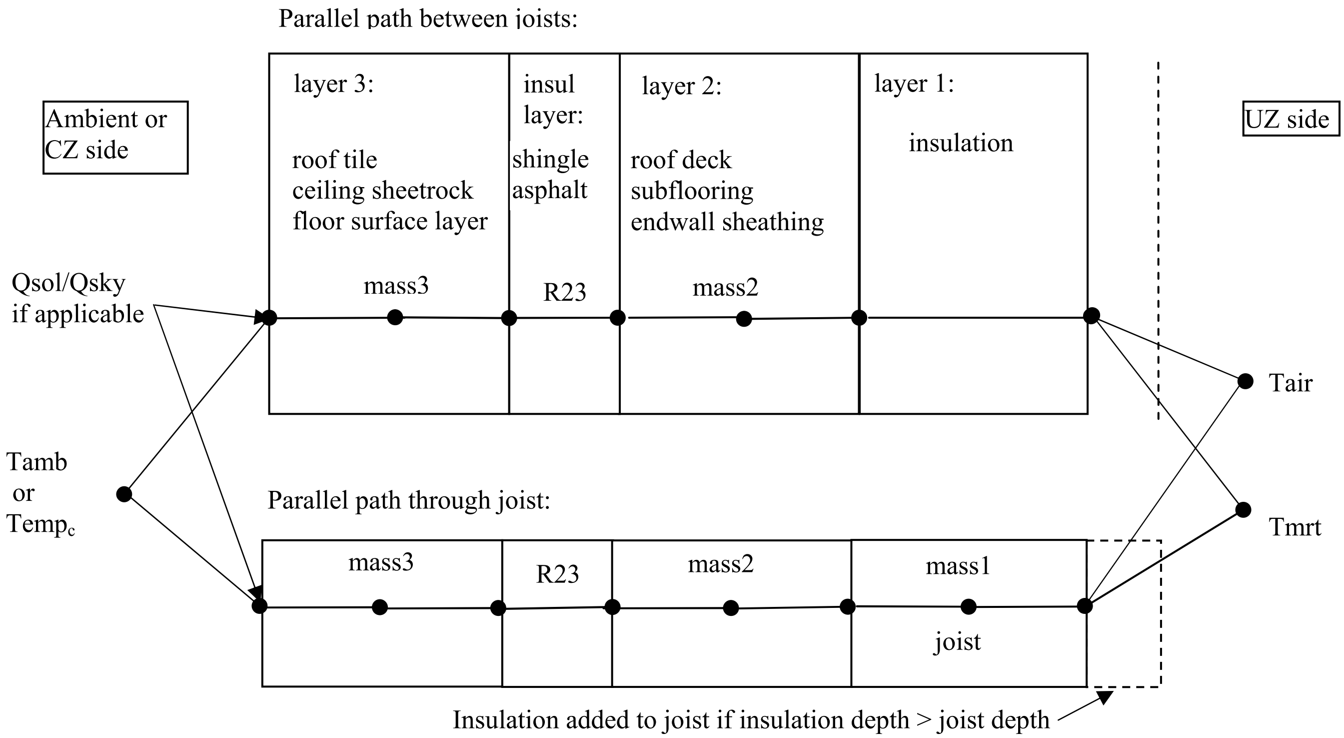

Figure 2-1. Network showing how a construction specifies two components; between-joists and through-joists.

Table 2-1. Attic Components		
Description of component	Component number	Corresponding construction number
roof btwn joists	1	1
roof thru joists	2	1
ceiling btwn joists, CZ 1	3	2
ceiling thru joists , CZ 1	4	2
ceiling btwn joists, CZ 2	5	3
ceiling thru joists, CZ 2	6	3
knewall btwn joists, CZ 1	7	4
knewall thru joists, CZ 1	8	4
knewall btwn joists, CZ 2	9	5
knewall thru joists, CZ 2	10	5
endwall btwn joists	11	6
endwall thru joists	12	6
trusswall	13	
supply duct of CZ 1	14	
supply duct of CZ 2	15	
return duct CZ 1	16	
return duct CZ 2	17	
UZ air	18	

Table 2-2. Crawl Space Components		
Description of component	Component number (ncom)	Corresponding construction number (ncon)
floor btwn joists, for CZ 1	1	1
floor thru joists, for CZ 1	2	1
floor btwn joists, for CZ 2	3	2
floor thru joists, for CZ 2	4	2
endwall btwn joists	5	3
endwall thru joists	6	3
trusswall	7	
ground	8	
supply duct for CZ 1	9	
supply duct for CZ 2	10	
return duct for CZ 1	11	
return duct for CZ 2	12	
air	13	

Input Data for Each Construction:

Each construction number (ncon) in the above table is specified by the following input variables. Example values in brackets represent an asphalt-shingle/felt/strand-board roof construction, without insulation:

ff framing fraction for framed surface construction [0.07].
epsbf emissivity of surface between-joists inside UZ [0.9].
dj depth of joists [3.5-inches].
d1 thickness of insulation layer, UZ side, between joists [0-inches].
k1n nominal conductivity of insulation layer [0.025 Btu/hr-ft-F].
d2 thickness of inner mass layer (subfloor or roof deck) [15/32-inches].
k2 conductivity of inner mass layer [0.052 Btu/hr-ft-F].
vc2 volumetric heat capacity of inner mass layer [10.7 Btu/ft^3-F].
R23 R value of insulation between mass layers [0.06 hr-ft^2-F/Btu].
d3 thickness of outer mass layers [0.38-inches].
k3 conductivity of outer mass layer [0.072 Btu/hr-ft-F].
vc3 volumetric heat capacity of outer mass layer [21 Btu/ft^3-F].
epso emissivity of outside surf of construction facing ambient or an adjacent CZ [0.9].
alfao solar absorptivity of outside surface if facing outdoors [0.95].

The program sets the density of all joists to 34 lb/ft^3, joist conductivity to 0.08 Btu/hr-ft-F, and joist volumetric heat capacity to13 Btu/ft^3-F. Exposed joist surfaces are set to have an emissivity of 0.9.

Frame constructions can be modeled with varying depths of insulation. Insulation of depth d1 is represented by layer 1 in the between-joists path of Figure 2.1. The k1n values are corrected for temperatures above and below 70 F by the correlation used in EnergyGuage USA (Parker, et al, 1999) and based on Wilkes (1981) data:

$$k1 = (k1n)\cdot\left(1+0.00418\cdot\left(\frac{Tsurf+Tm2}{2} - 70\right)\right); \text{ temperatures in } ^{o}F.$$

where k1 and k1n are in the units of Btu/hr-ft-R.

If the component has no insulation, d1 is set to zero and epsbf is set to the emissivity of the underside of the decking. For radiant barriers epsbf is set to the appropriate low emissivity. If a construction has no insulation layer then the joist would protrude into the UZ air and act as a heat transfer fin. For the nominal properties of fir joists (of dj > 3.5"), the fin effectiveness (defined as $q_{with-fin}/q_{from-base-area}$) is less than 1, so that the fin retards heat flow compared the case where the joist is completely removed and heat is transferred only from its footprint area on mass layer 2, the underside of the roof decking for example. However, in this case the decking also is acting as a fin, conducting heat laterally from the between-joist part of the decking to the joist base. Two dimensional finite-element studies show that these fin effects tend to cancel each other and that the correct heat transfer is close to that obtained by the parallel path analysis for the configuration with the joist completely removed. The mass of the removed joist material is accounted for by adding it to a fictitious internal UZ wood wall, the trusswall, designed to model the dynamics of the trusses--see Section 2.2.

Similarly, if the insulation depth d1 is finite but less than dj, the part of the joist extending beyond the insulation, by an amount (d1 - dj), is removed to the trusswall, leaving the rest of the joist, whose surface is now flush with the joist end, to be modeled as part of the through-joist parallel path.

If the insulation is thicker than the joists, i.e., d1 > dj, a layer of insulation of thickness d1 - dj is applied to the UZ end of the joist (not shown in Figure 2.1).

The surface area of the ceiling and knee wall constructions (2, 3, 4 & 5) are required inputs. These areas are split into the areas of the two parallel path components of these constructions using the constructions framing fractions.

The roof area is determined from the total ceiling area and roof pitch, required inputs. The roof can be either a single hip roof, which is set as square, or a gable roof with a plan-view aspect ratio different than 1. The gable end wall areas are calculated by the program. The crawl space end wall areas are calculated from a crawl space height input.

Flat roofs can be modeled by setting the roof pitch to zero, but the attic height is required as an additional input in this case. Sealed attics (cathedralized ceilings) can be modeled assigning the appropriate insulation to the roof and ceiling.

The amount of total horizontal insolation impinging on an area equal to the total ceiling area is distributed uniformly on the roof. If there are attic end walls, the average of the vertical surface insolation from the four cardinal directions is assumed to be incident on the end walls.

2.2 Trusswall Model:

The purpose of the 'trusswall' is to consolidate the trusses and other miscellaneous masses that are not part of the envelope framing into one mass wall. The weight of the trusses in the attic is an input to the code. The attic truss weight is defined to include the weight of the roof and ceiling joists, but not the knee wall or end wall joists.

The weight of the parts of the roof and ceiling joists that are kept as part of the joist path thermal network (see Section 2.1) is subtracted from the truss weight. That is, their mass is accounted for in the joist path thermal network and doesn't need to be modeled as part of trusswall. The weights of the end wall and knee wall joists that are removed from the joist path thermal network are added to the trusswall. That is, the end and knee wall joists weren't originally part of the truss weight so any part removed from the joist path thermal network must be added to the trusswall mass.

The crawl space does not have trusses, but a crawl space trusswall is still needed to model any parts of the crawl space floor and end wall joists that are removed from the joist path thermal network. These parts are added to constitute the crawl space trusswall.

The trusswalls in the attic and crawl space are each represented in the model by a single interior wall. The trusswall is defined to have the same thermal and density properties as input for joists. It's thickness is set at 3/4-inches (half the thickness of 2-by-x framing), and an area determined by the trusswall weight. It is adiabatic on one side, with the exposed side coupled to the attic air node with a convection coefficient of 0.5 Btu/hr-ft^2-F, and to the mrt node by a radiation coefficient h_r based on 70-F. The trusswall is modeled with one mass node, in the same tee circuit form as other mass layers.

2.3 Ground Model:

The crawl space floor is at ground level. The ground is modeled with two mass layers, with the top of the top layer connected to the crawl space air and mrt temperature nodes through convection (see Section 4.3.1) and radiation coefficients and the bottom of the bottom layer connected through a conductance to the annual average ambient temperature, representing the deep ground temperature. The soil conductivity is set at k = 0.8 Btu/hr-ft-F, and its volumetric heat capacity at vcsoil = 24 Btu/ft^3-F. The ground surface emissivity is set at 0.9. The layers are nominally fixed at 2-inches for the surface layer and 4-inches for the deeper layer, values chosen to give reasonable dynamic response over short time periods on the order of hours. The conductance from the bottom of the bottom layer to the deep ground node is chosen to yield a overall crawl space to outdoor R value of 12 hr-ft^2-F/Btu for a soil conductivity of 0.8. If the soil conductivity is changed in the initialization, the R value is scaled there accordingly as R = (12 hr-ft-F/Btu)((0.8 Btu/hr-ft-F)/ksoil). Long term transient behavior of the ground is not modeled because of the difficulty of the accompanying long warm-up period requirement.

3. Radiant Heat Transfer Network

The radiant model described below was developed by Joe Carroll (Carroll 1980a, 1908b, Carroll & Clinton 1982). It was chosen because of its relative simplicity, its freedom from heat balance errors, and it's relatively good accuracy regarding heat flux distribution for typical building applications (see Carroll 1981). It does

The unconditioned zone radiation model uses a mean radiant temperature node, Tmrt, to act as a clearinghouse for the radiation heat exchange between surfaces, much as does the air node $Tair_u$ for convective exchange. A three surface example of the radiation network is shown in Figure 3.1. Using this method at the beginning of the simulation, the UZ model determines the radiant conductances ($h_b \cdot A_1 \cdot F_1'$,

$h_b \cdot A_2 \cdot F_2'$, etc.) shown in Figure 3.1. Figure 3.2 shows the radiation network combined with the convective heat transfer network. The model also accounts for the absorption of radiation in the air itself, so that the air and mrt nodes are thermally coupled to each other as well as to the interior surfaces.

The principle inputs are the interior surface areas in the unconditioned zone, the emissivities of these surfaces, and the overall volume to surface area ratio of the zone. All of the interior surfaces, including ducts, are assumed to exchange heat between each other as diffusely radiating gray body surfaces. The mrt model essentially maps the real surfaces onto the surface a sphere and the view factors between the real surfaces is based on the view factors between their corresponding spherical surfaces. The view factors between surfaces on inside of sphere only depend on the surface areas, independent of their relative positions on the sphere. Thus the view factors found in the UZ model only depend on surfaces areas, avoiding the need for any three dimensional unconditioned zone geometry data.

3.1 MRT View Factors:

For an enclosure of black body surfaces, the net radiant long-wave heat transfer from surface i is given by:

$$Q_i = \sigma A_i \sum_j F_{ij} \cdot (T_i^4 - T_j^4) \qquad (3\text{-}1)$$

where σ is the Stefan-Boltzmann constant, A_i is the area of surface i, and F_{ij} is the conventional view factor from surface i to j. Linearized, this equation can be written in the useful form:

$$Q_i = h_b \cdot A_i \sum_j F_{ij} \cdot (T_i - T_j) \qquad (3\text{-}2)$$

where h_b [Btu/hr-ft^2-R^3] is the net black body radiation transfer per unit area and per unit temperature difference:

$$h_b = 4 \cdot \sigma \cdot Tbar^3, \qquad (3\text{-}3)$$

where Tbar [°R] is the average temperature of the surfaces.

To simplify the radiation network Carroll (1980a) developed the 'MRT view factor method' for handling the radiant interchange. In this method a radiant temperature node, T_{mrt}, is coupled with all of the surfaces. Assuming that an appropriate form of the conductance hr_i can be determined, the net radiant long-wave heat transfer from any surface is:

$$Q_i = A_i \cdot hr_i \cdot (T_i - T_{mrt}). \qquad (3\text{-}4)$$

If there were only three surfaces total, it would be possible to do an exact Δ-Y transformation such that the hr_i values would be uniquely determined by the A_iF_{ij} values. However, for more than three surfaces a Δ-Y transformation (generalized to more than three legs) is not possible in general, so one cannot in general determine hr_i values for any arbitrary room geometry that will give the same heat transfer as equation (3-1). However there is one class of geometries for which a generalized Δ-Y transformation is possible and this is the geometry represented by curved surfaces that completely tile the inside surface of a sphere. In this case, the view factor from surface i to surface j is equal to the fraction that area a_j is of the total surface in the enclosure. That is:

$$F_{ij} = \frac{a_j}{\sum_{\text{all } j} a_j} = \frac{a_j}{a_S}, \qquad (3\text{-}5)$$

where a_S is the surface area of the sphere. This view factor is exact for the view from any area curved a_i on the inside surface of a sphere to any other area a_j also on the inside surface of the sphere. This is because the length of a chord between any two points on a sphere is proportional to the cosine of the angle of incidence at each end of the chord, thus the $1/r^2$ distance effects exactly cancel out the cosine effect which occurs at each end of the chord. Carroll's method essentially maps the flat surfaces, A_i, of an actual enclosure, to corresponding curved surfaces, a_i, on the inside of a sphere of total area $\sum a_i$.

Applied to spherical surface a_i, equation (3-2) can be written:

$$Q_i = h_b \cdot a_i \sum_j F_{ij} \cdot (T_i - T_j)$$

Substituting (3-5) into this, the net heat flow from surface 1 is given by:

$$Q_i = h_b \cdot a_i \sum_{j=1 \to n} \frac{a_j}{a_S} \cdot (T_i - T_j),$$

This can be written:

$$Q_i = h_b \cdot a_i \sum_{j=1 \to n} \frac{a_j}{a_S} \cdot T_i - h_b \cdot a_i \sum_{j=1 \to n} \frac{a_j}{a_S} \cdot T_j$$

$$= h_b \cdot a_i \cdot (T_i - T_{mrt}) \qquad (3\text{-}6)$$

where T_{mrt} is the area-weighted mean radiant temperature for this black body situation:

$$T_{mrt} = \sum_{j=1 \to n} \frac{a_j}{a_S} \cdot T_j$$

Defining the ratio of the spherical area to the corresponding flat surface area as Carrroll's "mrt view factors", F_i:

$$F_i = \frac{a_i}{A_i} \qquad (3\text{-}7)$$

Substituting (3-7) into (3-6) gives the net heat transfer from the flat surface a_i:

$$Q_i = h_b \cdot A_i \cdot F_i (T_i - T_{mrt})$$

Thus the value of the conductances in equation (3-4) are:

$$hr_i = h_b \cdot F_i \tag{3-8}$$

From geometry, the ratio of the area of circular disc A_i generated by slicing a sphere with a plane, to the area of the spherical cap a_i above and below the circular disc is equal to 1 minus the ratio of the spherical cap to the total surface area of the sphere. That is,

$$\frac{A_i}{a_i} = 1 - \frac{a_i}{a_S}$$

It is this formula that Carroll uses to map flat enclosure surfaces A_i to their spherical counterparts a_i. That is, this formula is used in eq. (3-7) to determine the view factor F for any shape surface i as follows:

$$F_i = \frac{a_i}{A_i} = \frac{1}{1 - \frac{a_i}{a_S}}$$

Since $a_i = A_i F_i$, and $a_S = \sum_{i=1 \to n} A_i \cdot F_i$, this equation becomes:

$$F_i = \frac{1}{1 - \frac{A_i \cdot F_i}{\sum_{i=1 \to n} A_i \cdot F_i}} \tag{3-9}$$

Suppose one of the interior surfaces of total area A_i is composed of N_i identical flat sub surfaces, each at the same temperature, and similar views to each other, like the facets of a geodesic dome. The F's would be the same if each facet is treated as a separate surface. To avoid redundant solutions to eq. (3-9), it is easy to show that A_i can be treated as one surface in eq. (3-9) if N_i is introduced into eq. (3-9) as follows:

$$F_i = \frac{1}{1 - \frac{A_i \cdot F_i / N_i}{\Sigma(A_i \cdot F_i)}} \tag{3-10}$$

In the code, N_i is set to unity for each component i, except for the trusswall and endwalls. Sinces the trusswall is a consolidation of the may truss surfaces, it's N is set to 10. Since the two endwalls of a gabled roof are modeled as one, the end wall N is set to 2. The crawl space end wall N is set to 4. Results are relatively insensitive to the N value. The use of N_i to additional advantage is discussed by Carroll (1980b).

There are 18 possible attic components (see Table 2-1). Equation (3-10) constitutes a set of 18 nonlinear equations to be solved for 18 unknowns, F_i. This set of equations is solved numerically in the code by successive substitution starting with all F_i terms on the right hand side of eq. (3-10) set to unity. This solution method apparently always converges for realistic (constructable) zone geometries.

3.2 Application to Thermally Grey Surfaces:

The above methodology is extended to gray surfaces (i.e., surfaces with long-wave hemispherical emissivity $\varepsilon_i < 1$) by adding the Oppenheim surface conductance $\varepsilon_i/(1 - \varepsilon_i)$ to the conductance corresponding to $h_b A_i F_i$, as shown in Figure 3-1. The surface conductance correctly accounts for surface absorption and reflection.

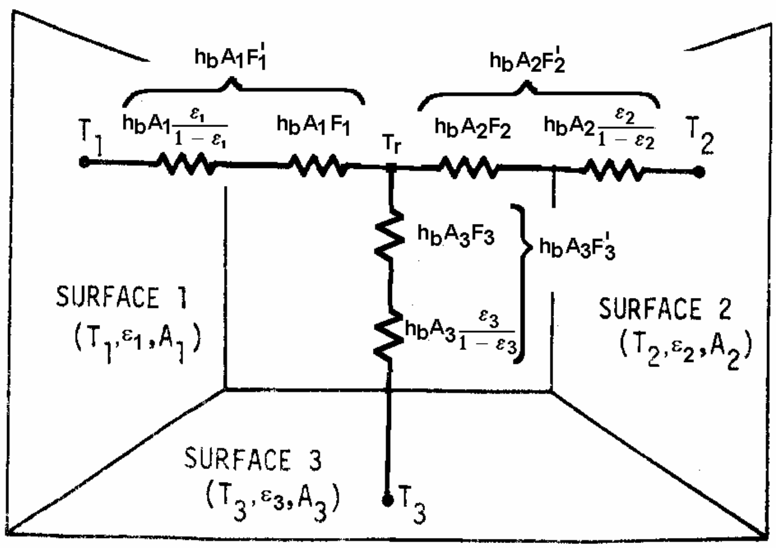

Figure 3-1. Linearized radiation mrt network representing gray surfaces.

Thus for gray surfaces, hr_i of equation (3-8) is replaced by:

$$hr_i = h_b \cdot F_i' = \frac{h_b}{\frac{1}{F_i} + \frac{1-\varepsilon_i}{\varepsilon_i}} \tag{3-11}$$

Thus, the final expression for the net radiant heat transfer from surface i is:

$$Q_i = h_b \cdot A_i \cdot F_i'(T_i - T_{mrt}) \tag{3-12}$$

Figure 3-2 shows the radiative heat transfer circuit combined with a convective heat transfer circuit.

3.3 Air Absorption:

The air and radiant nodes are coupled somewhat by the opacity of the air to long-wave radiation. Carroll (1980a) gives an air emissivity by the following empirical dimensional equation that is based on Hottel data:

$$\varepsilon_a = 0.08 \cdot \varepsilon_s \cdot \ln\left(1 + \frac{4}{\varepsilon_s} \cdot \frac{V}{A} \cdot rh \cdot Patm \cdot \exp\left(\frac{T_a}{17}\right)\right) \tag{3-13}$$

where ln is the natural logarithm, and,

ε_s = the area-weighted average long-wave emissivity for room surfaces.

V/A = the room volume to surface area ratio. For multi-room zones the appropriate value for a typical room is the total zone volume divided by the total zone surface area, in meters.

rh = the relative humidity in the zone. ($0 \le rh \le 1$).

Patm = atmospheric pressure in atmospheres.

Ta = zone air temperature, in °C.

The values of Patm and rh used in the code are annual average values.

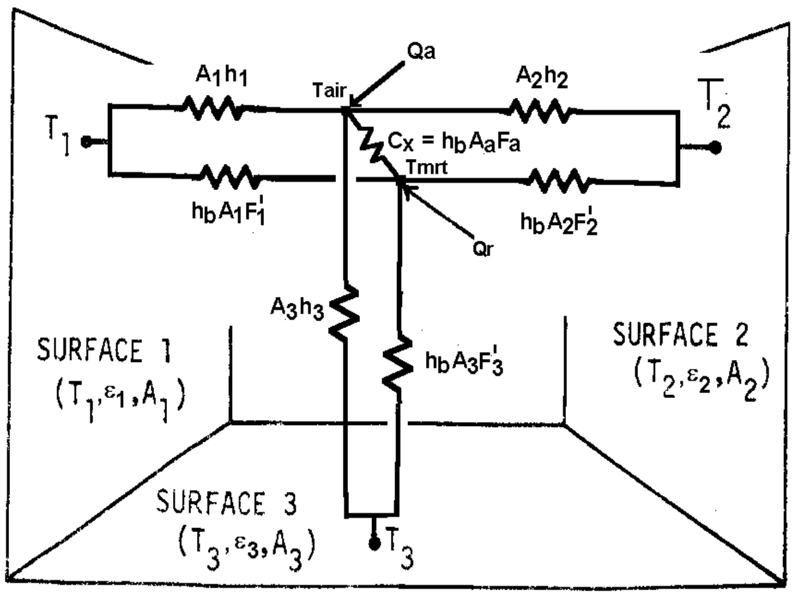

Figure 3-2. Convection and radiation networks combined.

Following a heuristic argument Carroll assigns an effective area A_a to the air that is the product of ε_a and the sum of all of the zone surface areas, as if the absorbing part of the air were consolidated into a surface of area A_a.

$$A_a = \varepsilon_a \cdot \Sigma A_i. \tag{3-14}$$

Using this area, the value of F_a for this 'surface' can be calculated along with the other F_i by equation (3-10). The value of the conductance between the air and radiant nodes in Figure 3-2 is given by:

$$Cx = h_b \cdot A_a \cdot F_a \tag{3-15}$$

Cx is discussed in Sections 7 and 8.

4. Convective Heat Transfer

The convection network in the UZ interior is shown schematically in Figure 3-2. The model assumes well mixed air inside the UZ at a single temperature Tair. Convection from surfaces inside the UZ is assumed to be by natural convection only, without any allowance made for air movement due to infiltration or other effects.

4.1 Fixed Convection Coefficients:

The convection coefficients for the following surfaces are fixed in the UZ program initialization at the value hcVert = 0.5 Btu/hr-ft^2-F:

- UZ side of attic knee walls.
- UZ side of attic and crawl space end walls.
- attic and crawl space trusswalls.
- outside of ducts. (The ACM assumes a combined coefficient of R=0.7: For $h_r \approx 0.9$, hc~=(1/0.7)-0.9 = 0.53 btu/hr-ft^2-F).

4.2 Fixed combined coefficients:

Combined convection & radiation coefficients are assigned to the CZ side of the ceilings, knee walls, and floors:

- The CZ side of the CZ ceilings and knee walls: hc = 1.5 Btu/hr-ft^2-F.
- The CZ side of the CZ floor: hc = 1.3 Btu/hr-ft^2-F.

4.3 Variable convection coefficients on surfaces inside UZ:

4.3.1 Coefficients for Top of CZ ceiling, underside of CZ floor, and ground surface:

For a hot ceiling facing a cooler attic air, a cold floor facing a hotter crawlspace air, or for the ground hotter than the crawl space, Mill's equations 4.95 and 4.96 (due to McAdams) for horizontal plates are applied:

$$Nu_L = 0.54 \cdot Ra_L^{1/4} \quad \text{for } 10^5 < Ra_L < 2x10^7 \qquad (4\text{-}1)$$

$$Nu_L = 0.14 \cdot Ra_L^{1/3} \quad \text{for } 2x10^7 < Ra_L < 3x10^{10} \qquad (4\text{-}2)$$

Stratification is assumed to dominate and the convection coefficient is set to zero for a cold CZ ceiling facing a hotter attic air, a hot CZ floor (i.e. crawlspace ceiling) facing a cold crawl space air, or the ground colder than the crawl space.

4.3.2 Coefficients for underside of hot roof over cold attic air:

The correlation for tilted heated and cooled plates from Mills (1992), equations 4.85 and 4.86 (due to Churchill and Chu), are used.

$$Nu_L = 0.68 + 0.67 \cdot (Ra_L \cdot \psi)^{1/4} \quad \text{for } Ra_L < 10^9 \qquad (4\text{-}3)$$

$$Nu_L = 0.68 + 0.67 \cdot (Ra_L \cdot \psi)^{1/4}(1 + 1.6x10^{-8} \cdot R_L \cdot \psi)^{1/12} \quad \text{for } 10^9 < R_L < 10^{12} \qquad (4\text{-}4)$$

$$\psi = (1 + (0.492/Pr)^{9/16})^{-16/9}$$

The Rayleigh number is defined with gravity g replaced by $g \cdot \cos(\theta)$, where θ is the angle of the roof plane from the vertical, and L is the length of the plane in the direction of boundary layer flow. Characteristic length L is taken as 10-ft in the code. This equation is applicable to suspended plates with $\theta < 88^o$ for heated plates facing downward, and $\theta < 60^o$ for cooled plates facing downward.

For the case of a hot roof over a colder attic, the correlation is not strictly applicable to horizontal ($\theta = 90^o$) roofs. Nevertheless, at $\theta = 90^o$ this correlation predicts $Nu_L = 0.68$ which corresponds to a convection coefficient close to zero. Since this in effect mimics the stratification that would occur under a horizontal hot roof, the correlation is used in the code for all roof pitches including horizontal.

4.3.3 Coefficients for underside of cold roof over hot attic:

The above equations (4-3 & 4-4) are applicable to this situation, but limited to roofs with $\theta < 60^o$, making them inapplicable to common roof pitches. Using horizontal plate correlations in this case will exaggerate the heat flow for higher pitches. Since equations (4-1) and (4-2) are applicable for horizontal surfaces at 90^o tilt and (4-3) and (4-4) at 60^o tilt, the correlation used in the code interpolates linearly between the two results for angles between 60 and 90^o. For angles smaller than 60^o (pitches higher than 6.9/12) equations (4-3) and (4-4) are used.

4.4 Variable convection coefficients on outside of UZ envelope:

The hourly wind velocity used in the following convection coefficient correlations are modified to account for the local terrain and building height as described in following Sections. In addition, when the weather tape gives a zero wind velocity for the hour, then the zero is replaced by a fixed finite value: VwindZero. This is used because weather tape wind velocities are measured with instruments that report zero below a certain threshold wind velocity. VwindZero is meant to represent an average velocity for winds lower than the threshold.

4.4.1 Akbari Correlation:

The Akbari correlation (from private communication) is currently used for determining the convection coefficient on the outside of the end walls of both the attic and crawl space. It can be optionally chosen to determine the roof convection coefficient. The wind velocity used in applying the model is the weather tape wind velocity multiplied by a shelter factor, Sw, and an additional wind multiplier, Wsm, to adjust wind speed for roof height.

$$hc[W/m^2\text{-}C] = 6.7757 + 1.2157\ V[m/s]$$

$$hc[Btu/hr\text{-}ft^2\text{-}F] = 1.1939 + 0.0958V[mph]$$

4.4.2 Clear, Gartland and Winkelmann Correlation:

The correlation by Clear, et al (2001) is sensitive to the direction of heat flow, surface roughness, and combines both free and forced convection.

4.4.2.1 For roof hotter than ambient:

Clear's equation (11-a) is used for this situation, which gives the convection coefficient averaged over roof surface area and wind direction.

For surface temperature greater than ambient,

$$hc = 0.15 \cdot \eta \cdot \frac{k}{L_n} \cdot Ra_{Ln}^{1/3} + 0.037 \cdot \frac{k}{L_{eff}} \cdot R_f \cdot Re_{Leff}^{4/5} Pr^{1/3}$$

Velocity is free-stream wind speed at roof level. The wind velocity used in applying the model is the weather tape wind velocity multiplied by a shelter factor, Sw, and an additional wind multiplier, Wsm, to adjust wind speed for roof height. For a rectangular area A (coded as total ceiling area) and perimeter P (perimeter of ceiling), the following parameters are needed:

L_n = A/P, the characteristic length for natural convection.
$L_{eff} \approx (0.899-0.032*L_1)*L_2$, the effective forced convection length.
$L_1 = 4(\sqrt{A})/P$
$L_2 = 4*A/P$
$\eta = \dfrac{1}{1 + \dfrac{1}{\ln(1+Gr_{Leff}/Re_{Leff}^2)}}$, the weighting factor for natural convection.

Film temperature in the code is taken as the average of ambient air temperature and that of the outer most mass node of the roof construction. The roughness multiplier R_f is obtained from Clear's Table 7 as follows:

ASHRAE roughness number	Example surfaces with this roughness number	Forced convection multiplier, R_f
6	Glass, paint on pine	1.00
5	Smooth plaster	1.11
4	Clear pine	1.13
3	Concrete	1.52
2	Brick, rough plaster	1.67
1	Stucco	2.10

4.4.2.1 For roof colder than ambient:

The Clear correlation uses critical length xc:

$$xc = 5 \cdot 10^5 \cdot \frac{Leff}{Re_{Leff}}$$

The effective critical length, xceff, is equal to the smaller of xc and the smallest roof width. If xceff is larger than the largest roof length then:

$$hco = \eta \cdot \frac{kair}{Ln} \cdot 0.27 \cdot Ra_{Ln}^{1/4} + \frac{kair}{Leff} \cdot Rf \cdot 0.664 \cdot Re_{Leff}^{1/2} \cdot Pr^{1/3}$$

Otherwise,

$$hco = \eta \cdot \frac{kair}{Ln} \cdot 0.27 \cdot Ra_{Ln}^{1/4} + \frac{kair}{Leff} \cdot Rf \cdot \left(0.037 \cdot (Re_{Leff}^{0.8} - Re_{xceff}^{4/5}) + 0.664 \cdot Re_{xceff}^{1/2}\right) \cdot Pr^{1/3}$$

4.5 Ceiling Bypass Model

A simple model was implemented to simulate ceiling bypass heat transfer, the heat that is transported from the CZ's to the attic via miscellaneous inter-wall cavities in the CZ that may be partially open to the attic, as for example around a fireplace unit. Natural convection in the cavity when the conditioned zone is hotter than the attic is assumed to be the main mechanism for the bypass heat transfer. The conductance, when $Temp_c > Tair_u$, is given by:

$$qbp_u = U_1(Temp_1 - Tair_u) + U_2(Temp_2 - Tair_u)$$

where, the conductances follow a simple power law dependence on the temperature difference:

$$U_1 = Ubp_1(Temp_1 - Tair_u)^{nbp}$$

$$U_2 = Ubp_2(Temp_2 - Tair_u)^{nbp}$$

Ubp_1 and Ubp_2 are coefficients depending on the cavity geometry. Although an exponent of nbp on the order of 1/4 can be assumed for laminar convection, there is no current empirical basis for determining the coefficients Ubp_c. If the ACM rule of $U_c = 0.02Aceil_c$ were implemented, then nbp would be chosen as zero.

5. Duct System Model

5.1 Description of model:

The duct system performance is analyzed at every sub-hour time step. The duct air temperatures are calculated each sub-hour time step assuming they are operating at steady state, in equilibrium with the thermal conditions at the beginning of the sub-hour time step in the UZ in which they transit. Heat capacity effects of the ducts are ignored.

During each sub-hour time step, the following steps are taken to find the duct system operating conditions--the air temperatures in each duct, the losses, etc.

Initially, for each sub-hour time step, the duct systems performance is determined when operating at full capacity, independent of the hours load. The procedure starts at the return registers in each CZ, where the duct air temperatures are the current hours CZ air temperatures. The CZ air entering the return register heats or cools, or both, as it traversed through each component of the duct system: the return duct, the return plenum, the heating/cooling device, and the supply ducts. That is, the duct air temperature rises or drops immediately downstream of the return register (where returns leaks are assigned to occur) due to mixing of leakage air at the UZ temperature with the return air from the CZ. It may also increase or decrease in temperature as it mixed with the air from the return duct in another UZ (if any) in the return plenum. After being heated or cooled by the air handler at its nominal heating/cooling capacity, it then is additionally heated or cooled by supply duct conductive gains/loses to the UZ interior.

Summing all the gains and loses in temperature of the duct air as it travels through the system gives the supply temperature for each supply duct for each system, allowing the heat delivered at full capacity, Qdel, to be determined.

If the above useful heat delivered at full capacity is more than required by the load, then the equipment capacity is reduced to meet the load by assuming the system is only running the fraction Qload/Qdelivered, of the sub-hour time step. The needed capacity, Qneed, a program output, is this fraction of the nominal capacity. The duct losses for the time step are also reduced by this fraction.

On the other hand, if the heat delivered at full nominal capacity is smaller than the load, then the capacity is temporarily raised this sub-hour time step to allow the heat delivered to exactly meet the load. The needed capacity, Qneed, in this case is allowed to exceed the nominal capacity (see Sections 1.2 and 5.9). Duct losses are recalculated based on the changed supply temperature corresponding to the increased capacity.

The above calculations are done each sub-hour time step and the average Qneed summarized in the hourly output.

The above steps are presented in detail in the following sections, in the same sequence as described above, and in essentially the same sequence as in the source code.

UZM accounts for the fact that the duct losses and system performance is indirectly affected by the ducts heat losses/gains which affect the UZ interior air and mrt temperatures, which effects the ceiling heat transfer to the conditioned zones. In addition, unbalanced duct leakage can affect the ventilation rate in the UZ, affecting the UZ temperature. Unbalanced leakage also affects the CZ load itself by pressuring and depressurizing the conditioned zone thereby affecting its infiltration rate (see Section 6.3) .

5.2 Duct System Inputs:

In most cases in this section, the subscripted variables stand for arrays, and have a counterpart in the code that is an array with the same indices, but possible a modified name. In some cases variables not subscripted in the code and subscripted here for clarity.

nomenclature:

The subscript u stands for UZ number u = 1 for the attic, and u = 2 for the crawl space.
The subscript m stands for the mode of air handler operation: 0 off, 1 heating, 2 cooling.

The subscript c stands for CZ number and its associated air handler system. c is limited to 1 or 2.

The UZM can model the systems in the full complexity of Fig 1.1. However, if modeling a one system building, that system must be system #1 (i.e., c =1). Also, the UZM can simultaneously model both an attic and a crawl space of a building, but it can only model a building with a crawl space if it also has an attic. The attic need not have ducts in this case.

Per Annual Run Inputs:

The following data is obtained from the CZ to model the duct/air-handler system for hours of the simulation:

Duct areas:

$Asd_{c,u}$ = supply duct inside area for air-handler c in UZ u.
$Ard_{c,u}$ = return duct inside area for air-handler c in UZ u.

Duct Insulation R values:

$Rsd_{c,u}$ = supply duct R for air-handler c in UZ u
$Rrd_{c,u}$ = return duct R for air-handler c in UZ u.

These areas and R-values can represent branching ducts. The values input for $Asd_{1,1}$ and $Rsd_{1,1}$, for example, might represent the single values input for the area and R value of a single constant diameter supply duct of system c in the attic UZ. This is correct, but $Asd_{1,1}$ and $Rsd_{1,1}$ are also intended to also be able to represent the supply duct even if it has branches in the attic.

To do this, consider the same example of a system #1 supply duct in attic. The ratio $Asd_{1,1}/Rsd_{1,1}$ is the 'UA' of the supply duct. Based on the method suggested by Larry Palmiter (see Palmiter and Kruse, 2003), the simplifying assumption is made that the UA of the supply system is the sum of the UA's of it's individual branches. Thus, if A_i is the area of branch i, and R_i is its insulation R-value, then

$$UA = \frac{(Asd_{1,1})}{(Rsd_{1,1})} = \sum \frac{A_i}{R_i}$$

Assuming that

$$Asd_{1,1} = \sum A_i, \qquad (5.1)$$

then,

$$Rsd_{1,1} = \frac{\sum A_i}{\sum \frac{A_i}{R_i}} \qquad (5.2)$$

Equations (5.1) and (5.2) are used to get the effective area and R value of a branching supply for system #1 in UZ 1. The composite areas and R-values of the other ducts of the systems are found similarly. These latter equations must be implemented in the CZM to handle branching duct systems, and are not part of the UZM code.

In addition to representing branching duct systems, $Rsd_{c,u}$ can also represent systems with buried or partially buried ducts by using effective duct U values given by the ACM manual. It is assumed that these ACM values represent the part of the duct loss that goes to the UZ, and not the part conducted directly through the insulation and ceiling into the CZ. The loss from the ducts directly to the CZ need not be explicitly accounted for since if they are not delivered by conduction to the CZ the corresponding energy is retained in the supply air and the losses are in effect delivered by the supply air. This has the effect of slightly exaggerating duct losses to the attic since the duct air temperatures will be slightly in error.

Emissivities:

$epss_{c,u}$ = supply duct emissivity for air-handler c in UZ u
$epsr_{c,u}$ = return duct emissivity for air-handler c in UZ u.

Duct Leakage:

$Ls_{c,u}$ = the fraction of the flow through the system c air handler fan that is leaked from the supply duct in unconditioned zone u. The leak is assigned to occur near the supply register so that the leakage air is at the supply register temperature.

$Lr_{c,u}$ = the fraction of the flow through the system c air handler fan that is leaked into the return duct in unconditioned zone u. The leak is assigned to occur at the return register. The air leaking into the duct is at the UZ temperature.

System Flow:

$Flow_{m,c}$ = the flow rate in cfm (at standard conditions) through the air handler for the cooling and heating modes, for of each system.

Flow Distribution:
How much of the air handler flow of system c goes through each of its return and supply ducts is given by the per run input flow fractions:

$Fmr_{c,u}$ = fraction of flow of system c in the return duct located in unconditioned zone u.
$Fms_{c,u}$ = fraction of flow of system c in the supply duct located in unconditioned zone u.
$Fmrc_c$ = fraction of flow of system c in the return duct located in conditioned zone c.
$Fmsc_c$ = fraction of flow of system c in the supply duct located in conditioned zone c.

For a given system c, the sum of the return duct fractions must add to one: $Fmr_{c,1} + Fmr_{c,2} + Fmrc_c = 1$. Similarly for the supply duct fractions.

Hourly Inputs:

Each hour the UZ model receives the following information from the CZM:

Cap_c = the nominal system capacity for this hour, positive for heating and negative for cooling, in Btu/hr, for each system.

Qld_c = the load for this hour for each conditioned zone, in Btu/hr, positive for heating and negative for cooling.

5.2.1 Load Adjustment for Ceiling/Floor Infiltration:

As discussed in Section 1.2, the load received from the CZM is corrected in the UZM code to account for some of the CZ natural infiltration arriving from the attic or crawl space rather than all from outdoors. Since this part of the infiltration is at a different temperature than ambient, the CZ load must be adjusted. The ceiling and floor infiltration rates are given by equations (6-1) and (6-2). Using only the negative (i.e., into the CZ) values of ceilInf and flrInf, the load is adjusted as follows:

$$Qld_c = Qld_c[\text{from CZM}] + ceilInf_c \cdot (Tair_1 - Tamb) + flrInf_c \cdot (Tair_2 - Tamb)$$

5.3 Return Duct Air Temperatures:

Following the procedure indicated in Section 5.1, the return duct air temperatures are determined first. Utilizing the heat exchanger effectiveness approach (see Mills(1992, and Appendix A), the temperature of the system c return duct air entering the return plenum from a return duct located in UZ number u is given by:

$$Tout_{c,u} = Er_{m,c,u} \cdot Teqr_{c,u} + (1 - Er_{m,c,u}) \cdot Tmix_{c,u}$$

where $Er_{m,c,u}$ is the effectiveness of the return duct of system c in unconditioned zone u when operating in mode m:

$$Er_{m,c,u} = 1 - EXP\left(\frac{-(Urtot_{c,u})}{(Mcpr_{m,c,u})}\right)$$

where $Urtot_{c,u}$ is the total radiant conductance between the duct air and the equivalent surroundings temperature $Teqr_{c,u}$:

$$Teqr_{c,u} = (Frda_{c,u} \cdot Tair_u + Frdr_{c,u} \cdot Tmrt_u),$$

$Frda_{c,u}$ is the fraction of return duct (dissolved surface node) heat transfer that goes to the $Tair_u$ node.

$$Frda_{c,u} = \frac{(Urc_{c,u})}{(Urc_{c,u}+Urr_{c,u})}$$

$Frdr_{c,u}$ is the fraction of the heat transfer from the c,u return duct air that goes to the $Tmrt_u$ node.

$$Frdr_{c,u} = \frac{(Urr_{c,u})}{(Urc_{c,u}+Urr_{c,u})}$$

The U terms are the conductances from the duct air to the mrt and air nodes, determined as described in Appendix A.

$Urr_{c,u}$ = conductance from return duct air to Tmrt.
$Urc_{c,u}$ = conductance from return duct air to Tair.
$Urtot_{c,u} = Urc_{c,u} + Urr_{c,u}$

These conductance values, and the similar supply duct values in Section 5.6, are also used in Section 8.

The term $Mcpr_{m,c,u}$ is the flow conductance (see above) for the return duct flow:

$$Mcpr_{m,c,u} = Mcp_{m,c} \cdot Fmr_{c,u}$$

The total system flow, $Mcp_{m,c}$ is in the "flow conductance" form with the units Btu/hr-F:

$$Mcp_{m,c} = Flow_{m,c} \cdot c_p$$

where c_p is the volumetric heat capacity, which is taken as 1.08 Btu/hr-F-cfm for dry air at the ASHRAE standard conditions of density = 0.075 lb_m/ft^3 and c_p = 0.24 Btu/lb_m-F.

The term $Tmix_{c,u}$ is the mixed air just downstream of the return duct leakage given by:

$$Tmix_{c,u} = Lr_{c,u} \cdot Tair_u + (1 - Lr_{c,u}) \cdot Temp_c$$

where $Temp_c$ is the temperature of conditioned zone c's air, assumed to be well-mixed.

5.4 Return plenum temperature and return duct conductive heat losses:

The heat loss rate from the return duct via convection and radiation, needed in the UZ energy balance, is thus:

$$qlr_{c,u} = Mcpr_{m,c,u} \cdot (Tmix_{c,u} - Tout_{c,u})$$

The final return plenum temperature of system c is found by summing the contributions to its plenum temperature from the return ducts in each UZ and the return ducts located in the conditioned zone. That is,

$$Trp_c = Fmrc_c \cdot Temp_c + \sum_{\text{over } u} Fmr_{c,u} \cdot Tout_{c,u}$$

5.5 Temperature rise through air handler heating or cooling equipment:

If the mode is heating or cooling, the temperature rise through the air handler heating or cooling equipment of system c at nominal capacity Cap_c is given by:

$$dte_c = \frac{Cap_c}{(Mcp_{m,c})}.$$

The program considers no heat losses or gains from the air-handler components other than from the ducts.

5.6 Supply plenum and supply register temperatures:

The supply plenum temperature is given by:

$$Tsp_c = Trp_c + dte_c$$

The supply register temperature for the supply duct of system c in unconditioned space u is:

$$Tsr_{c,u} = Teqs_{c,u} + (1- Es_{m,c,u}) \cdot (Tsp_c - Teqs_{c,u}) \qquad (5\text{-}3)$$

where $Es_{m,c,u}$ is the effectiveness of the supply duct of system c in UZ u when operating in mode m:

$$Es_{m,c,u} = 1 - EXP\left(\frac{-(Ustot_{c,u})}{(Mcps_{m,c,u})}\right)$$

Substituting the Tsp_c equation above into this and rearranging gives:

$$Tsr_{c,u} = (1 - Es_{m,c,u}) \cdot dte_c + Tsrhx_{m,c,u} \qquad (5\text{-}4)$$

where

$$Tsrhx_{m,c,u} = (1-Es_{m,c,u}) \cdot Trp_c + Es_{m,c,u} \cdot Teqs_{c,u}$$

Tsrhx is the temperature that would be delivered to the supply register with the current modes flow rate but with zero capacity such that $dte_c = 0$. The duct system is then acting as a heat exchanger between the connected CZ's and UZ's.

The term $Teqs_{c,u}$, similar to $Teqr_{c,u}$ of Section 5.3, is an equivalent environmental temperature defined by

$$Teqs_{c,u} = (Fsda_{c,u} \cdot Tair_u + Fsdr_{c,u} \cdot Tmrt_u),$$

where

$$Fsda_{c,u} = \frac{(Usc_{c,u})}{(Usc_{c,u}+Usr_{c,u})}$$

$$Fsdr_{c,u} = \frac{(Usr_{c,u})}{(Usc_{c,u}+Usr_{c,u})}$$

$Usr_{c,u}$ = conductance from supply duct air to Tmrt.
$Usc_{c,u}$ = conductance from supply duct air to Tair.
$Ustot_{c,u} = Usc_{c,u} + Usr_{c,u}$

The supply duct flow rate is:

$$Mcps_{m,c,u} = Mcp_{m,c} \cdot Fms_{c,u}$$

5.7 Heating/Cooling Delivered and Supply Duct Conductive Heat Loss:

Given $Tsr_{c,u}$, from above, the heat delivered to the conditioned zones by way of the supply ducts located in one or both of the unconditioned zones is given by summing the sensible heat delivered via each unconditioned zones:

$$Q \text{ delivered from ducts in UZ's } = \sum_{\text{over } u}[Mcpsr_{m,c,u} \cdot (Tsr_{c,u} - Temp_c)] \qquad (5\text{-}5)$$

where $Mcpsr_{m,c,u}$, the flow out the supply register after the supply leakage is removed, is given by:

$$Mcpsr_{m,c,u} = (1 - Ls_{c,u}) \cdot Mcps_{m,c,u}$$

The heat delivered to the conditioned zones by way of ducts in the conditioned zone, which are assumed to have no losses or unbalanced leakage, is given by:

$$Q \text{ delivered directly to conditioned zone} = Fmsc_c \cdot Cap_c \qquad (5\text{-}6)$$

In addition, if there is an unbalanced duct leakage in one or both UZ's, then the conditioned zone is pressurized or depressurized, changing its infiltration rate to a value different than used by the CZM to determine the load it sent to the UZM. The CZM program cannot account for this change in the conditioned zone load without UZM/CZM iteration. To avoid iteration, it is accounted for in the UZ model as a change in heat delivered. The magnitude of the heating or cooling that is delivered is increased by the amount that the magnitude of the load should have been increased due to Laddcc (see Sections 1.2 and 6.3.1).

$$Q \text{ delivered to compensate for changed CZ infiltration} = Laddc_c \cdot (Temp_c - Tamb) \qquad (5\text{-}7)$$

Adding the Q's of equations (5-5, 5-6, and 5-7) together gives the net heating (+), or cooling (-), delivered by the system c as:

$$Qdel_c = Fmsc_c \cdot Cap_c + Laddc_c \cdot (Temp_c - Tamb) + \sum_{\text{over } u}[Mcpsr_{m,c,u} \cdot (Tsr_{c,u} - Temp_c)]$$

Substituting the expression for $Tsr_{c,u}$ from equation (5-4) into this, $Qdel_c$ can be put in the form:

$$Qdel_c = Qdel1_c + Qdel2_c,$$

where $Qdel1_c$ is the part of Qdel that is independent of air handler capacity:

$$Qdel1_c = Laddc_c \cdot (Temp_c - Tamb) + \sum_{\text{over } u}[Mcpsr_{m,c,u} \cdot (Tsrhx_{m,c,u} - Temp_c)].$$

$Qdel2_c$ is the part of Qdel that is linearly dependent on the air handler capacity:

$$Qdel2_c = Fmsc_c \cdot Cap_c + \sum_{\text{over } u}[Mcpsr_{m,c,u} \cdot (1 - Es_{m,c,u}) \cdot dte_c]$$

The rate of supply duct conduction losses this sub-hour time step is given by:

$$qls_{c,u} = Mcps_{m,c,u} \cdot (Tsp_c - Tsr_{c,u})$$

5.8 Duct System Performance when the Load is less than the Heat Delivered at Full Capacity:

If Qld_c is smaller than the capacity $Qdel_c$, then the system runs only part of the sub-hour time step. In this case the run time fraction is:

$$Frun_c = \frac{Qld_c}{Qdel_c}$$

The capacity required to meet the load is defined as $Qneed_c$:

$$Qneed_c = Frun_c \cdot Cap_c$$

The duct conductive and leakage losses are also reduced by the same $Frun_c$ fraction--see Section 8.1.

5.9 Duct System Performance when the Load is greater than the Heat Delivered at Full Capacity:

In this case, the system is set to run for the full sub-hour time step, but in contrast to a real system with limited capacity, the UZ model assumes that the load is always met. This strategy is in accord with the California ACM procedures regarding system capacity specifications, and is convenient since iteration between the CZ model and UZ simulation models is not needed to correct the CZ temperatures were the load not met.

From the Qdel1 and Qdel2 equations it can be seen that the capacity needed in this case is:

$$Qneed_c = \frac{Qld_c - Qdel1_c}{Qdel2_c} \cdot Cap_c$$

Thus, the temperature rise through the air handler needs to be:

$$dte_c = \frac{Qneed_c}{(Mcp_{m,c})}$$

The supply plenum temperature becomes:

$$Tsp_c = Trp_c + dte_c.$$

The supply register temperatures is determined reusing equation (5-3):

$$Tsr_{c,u} = Teqs_u + (1- Es_{m,c,u}) \cdot (Tsp_c - Teqs_{c,u})$$

The supply duct losses now become:

$$qls_{c,u} = Mcps_{m,c,u} \cdot (Tsp_c - Tsr_{c,u})$$

The $Qneed_c$'s from each of the sub-hour time steps during the hour are averaged over the hour and reported in the output as $Qneed_c$. The supply and return duct conduction loss terms $qls_{c,u}$ and $qlr_{c,u}$ are used in the energy balance of the UZ each sub-hour time step (see $G6_u$ in Section 8.1) .

6. Infiltration Model

6.1 Natural Infiltration Model

The UZ natural ventilation rates are determined using a modified version of the attic model of Walker, et al (1995), which determines the attic ventilation rate as a result of stack and wind effects. Attic leakage may be a combination of soffit vents, roof ridge vents, gable end vents, or vents located on the roof deck.

A number of assumptions were made in applying the Walker model. The pressure exponent n, used in the pressure-flow relationship, was chosen as 0.5. The interaction coefficient B_1 was chosen as zero, resulting in the stack and wind flows being added in quadrature. The Walker algorithm requires the wind velocity at the eave height, modified by a shelter factor S_w. The wind velocity used in applying the Walker model is the weather tape wind velocity multiplied by a shelter factor, Sw, and an additional wind multiplier, Wsm, to account for eave height adjustment to wind speed.

Deviating from the Walker model, the CZ ceiling and floor infiltration are handled in the UZM code separately from the Walker algorithm (see Section 6.2 below), so that C_{floor} in the Walker algorithm is set to zero. For the crawl space infiltration, the Sherman Grimsrud leakage distribution parameters used in the Walker model, R_w and X_w, are set to zero. The output of the Walker algorithm is the UZ natural infiltration rates $Linfu_u$, in flow conductance units of Btu/hr-F.

6.2 Ceiling and Floor Infiltration:

Infiltration to or from the UZ's through the CZ's ceilings and floors is modeled as discussed in Section 1.2.

Each hour the CZM sends the UZM the natural infiltration rate, $Linfc_c$ (in the flow-conductance form, Btu/hr-F--see Section 5.2), for each CZ. Also received hourly for each CZ are the fraction, $fceilInf_c$, of the natural ventilation flow that is going up through the ceiling from the CZ into the attic, or down through the ceiling from the attic into the CZ. And similarly the fraction, $fflrInf_c$, of $Linfc_c$, that goes through the floors between the CZ's and the crawl space.

The direction of the infiltration flow is determined by the relative magnitudes of the CZ and ambient temperatures. If it is hotter in the CZ than outdoors then the floor infiltration flow goes from the crawl space to the CZ and the ceiling infiltration goes from the CZ to the attic. These flows reverse if it is cooler in the CZ than outdoors. If the crawl space doesn't exist, no floor infiltration is assumed to occur. Thus,

$$ceilInf_c = fceilInf_c \cdot Linfc_c \cdot SGN(Temp_c - Tamb), \quad (6\text{-}1)$$

where the SGN function sets the sign of ceilInf to be that of the SGN arguement. Similarly for the floor infiltration:

$$flrInf_c = fflrInf_c \cdot Linfc_c \cdot SGN(Tamb - Temp_c). \quad (6\text{-}2)$$

These infiltration rates are used:

- to adjust the UZ infltration rates (Section 6.3.2) and loads (see $G2_u$ in the Section 8.1).
- to adjust the CZ infiltration rates (Section 6.3.1) and loads (Sections 5.2.1, 5.7).

6.3 Infiltration and Unbalanced Flow.

6.3.1 Conditioned Space Infiltration Adjustment

The effect of the unbalanced duct leakage of either of the air handler systems on the natural infiltration of the conditioned zone it serves is determined by first adding up the unbalanced duct leakages of it's ducts in the unconditioned zones. Depending on the magnitude and sign of the sum, this unbalanced leakage will pressurize or depressurize the conditioned zone and thus retard or augment the natural infiltration into the conditioned zone. The ceiling and floor infiltration rates discussed in Section 6.2 will not effect the CZ pressurization since these are already included in the conditioned zones natural infiltration rate.

The interaction between the unbalanced leakage and the natural infiltration is determined using the model developed by Palmiter and Bond (1991a, 1991b, 1992). It can be summarized with the following algorithm, known as the '1/2 rule'.

> If L represents the leakage, or other fixed flow from a constant flow source, into a zone (+ into, - out) and N represents the natural infiltration to the zone without leakage, then accounting for the interaction, the correct infiltration to the zone can be estimated by:
>
> If |L| < 2N then:
>
> $$\text{Ncorrected} = \text{N} - \frac{\text{L}}{2}$$
>
> If L > 2N then:
>
> $$\text{Ncorrected} = 0$$
>
> If L < 2N then:
>
> $$\text{Ncorrected} = -\text{L}$$

This can be translated as follows. Assume as above that the net unbalanced duct leakage flow is defined as positive if it pressurizes the CZ--it is actually defined as negative in the source code. If it's magnitude is less than twice the CZ natural infiltration, and it is positive, then the CZ pressurization is assumed to decrease the natural ventilation rate of the CZ by half the magnitude of the leakage; if negative, depressurizing the CZ, the CZ natural infiltration is increased by half the magnitude of the leakage. If the leakage is over twice the natural infiltration, and positive, then the natural infiltration is assumed to be totally suppressed; if negative, the total CZ infiltration is taken to be equal in magnitude to the leakage rate.

In the code, $Laddc_c$ is the variable name for the infiltration rate that must be added to the uninfluenced natural infiltration rate to arrive at the correct rate.

Since the infiltration rate of the conditioned zones is subject to correction by the amount $Laddc_c$, the load reported by the CZM to the UZM must have been overestimated by the amount $Laddc_c \cdot (Tamb - Temp_c)$. Instead of correcting the load, this rate of heat transfer is added to the amount delivered by the duct system. See Section 5.7.

6.3.2 Unconditioned Space Infiltration Adjustment:

The procedure here is similar to the above, but deals with unbalanced leakage from both the duct system and from infiltration through the ceiling or floor, and their effect on the UZ infiltration rate.

The total infiltration to a UZ from the CZ's ceilings and floors is first determined from the algebraic sum of the ceiling and floor infiltrations given by equations 6-1 and 6-2:

The total ceiling infltration into the attic is:

$$c2uInf_1 = ceilInf_1 + ceilInf_2$$

The infiltration into the crawl space is:

$$c2uInf_2 = flrInf_1 + flrInf_2$$

This infiltration (+ to UZ) is then added to the unbalanced duct leakage from all ducts from both systems in the UZ (+ to UZ). This gives the total leakage into the UZ that interacts to change its natural infiltration rate. Using the above one-half- rule, the correction to be added to the natural infiltration rate for each UZ is determined. It is called $Laddu_u$ in the code. The one-half-rule is used again in the same way, except without the unbalanced duct leakage included with the ceiling and floor infiltrations, to determine $Ladduoff_u$ when the system is not operating.

$Laddu_u$ and $Ladduoff_u$ are used in determining the UZ air temperature each sub-hour time step--see the Section 8-1.

7. Mass Temperature Updates

There are up to 49 mass nodes in the frame wall, trusswall, and ground models. The temperatures of these masses are updated every sub-hour time step using the Euler forward-difference numerical integration method, whereby the change in temperature of the mass during the sub-hour time step is based on the driving conditions at the beginning of the sub-hour time step. The driving conditions are the temperatures of adjacent mass nodes, the temperatures of the conditioned or unconditioned zones if adjacent to the mass, and any other heat flow rates directly applied to the mass, such as insolation.

The general procedure in updating a particular mass is to first dissolve out all massless nodes in the network that connect the mass node to the driving temperatures. Then apply the forward difference algorithm to determine the change in mass temperature over the sub-hour time step that results from the driving boundary conditions.

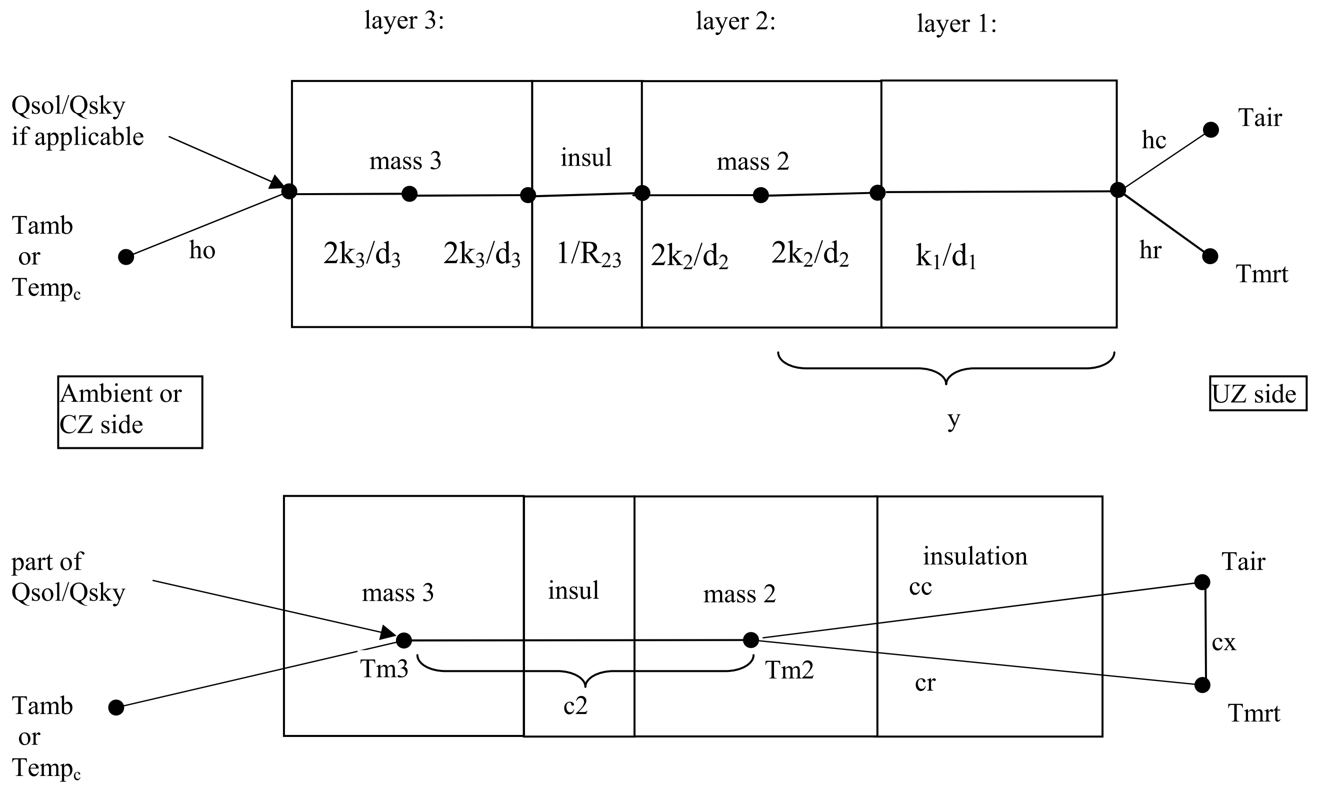

Fig. 7-1. Network for heat flow path between joists. Before and after dissolving massless nodes.

To illustrate the procedure, the update equations for the mass 2 node in the between-joists path of Figure 2-1 is shown as follows. The overall conductance between mass 2 and mass 3 in Fig 7-1 is given by:

$$c2 = \frac{2k_2 \cdot k_3 \cdot Acom}{d_2 \cdot k_3 + d_3 \cdot k_2 + 2k_2 \cdot k_3 \cdot R_{23}}$$

where Acom is the area of the between-joists part of the framing construction. The overall conductance between mass 2 and Tair is given by a Y-Δ transformation to be:

$$cc = A \cdot \frac{y \cdot h_c}{y + h_c + h_r}$$

where

$$y = \frac{2k_1 \cdot k_2}{k_1 \cdot d_2 + 2k_2 \cdot d_1}$$

Similarly,

$$cr = A \cdot \frac{y \cdot h_r}{y + h_c + h_r}$$

The Y-Δ transformation also necessitates a coupling between the air and mrt nodes:

$$cx = A \cdot \frac{h_c \cdot h_r}{y + h_c + h_r}$$

A coupling conductance similar to cx results from each of the mass nodes connected to the air and mrt nodes, and in addition due to the absorption of some of the radiation by the air. The air absorption cross coupling is found in Section 3, and is the only cross coupling shown in Fig. 3-2 since Y-Δ transformations were not involved in that example. These cross coupling do not effect the mass update procedure outlined here, but are needed to determine Tair and Tmrt as shown in Section 8.

The conductivities and thickness in these expressions are inputs for the construction type considered. They are identified with the subscripts (u,ncon) in other sections of this documentation. The area A represents the components area $A_{u,ncom}$. The conductances cc, cr, and cx represent $cc_{u,ncom}$, $cr_{u,ncom}$, and $cx_{u,ncom}$ in the source code and are determined therein for all components. The convection and radiation coefficients hc and hr can vary hourly and are described in Sections 3 and 4.

Equating the heat transfer into mass 2 to its rate of change in internal energy gives the following differential equation for mass temperature Tm_2

$$\frac{dTm_2}{dt} = \frac{Ts_2 - Tm_2}{\tau_2}$$

where Tm_2 is the mass temperature, and Ts_2 is the temperature Tm_2 would have if steady state were reached:

$$Ts_2 = \frac{cc \cdot T_{air} + cr \cdot T_{mrt} + c_2 \cdot Tm_3}{cc + cr + c_2}$$

Tm_3 is the temperature of mass node 3.

τ_2 is the time constant of mass node 2:

$$\tau_2 = \frac{v_2 \cdot d_2 \cdot A}{cc + cr + c2}$$

where v_2 is the volumetric heat capacity of the mass node (Btu/ft^{3}·F).

To integrate the differential equation over a sub-hour time step, the forward difference procedure assumes that the right hand side of the equation remains constant over the time step at its value at the beginning of the time step. In this case the mass temperature at the end of the time step becomes:

$$Tm_2(t+\Delta t) = Tm_2(t)\left(1 - \frac{\Delta t}{\tau_2}\right) + Ts_2\left(\frac{\Delta t}{\tau_2}\right),$$

where Δt is the sub-hour time step. This solution is unstable when $\tau_2 < \Delta t$. Although the code is designed to run with any time step, it is set at a low value to avoid instability and other inaccuracies associated with large time steps. The development version of the code gives warnings whenever a mass node update is performed when $\tau_2 < \Delta t$. In the program development process it has been found that a 6-minute time step conservatively avoids instability problems for realistic constructions. Although the program does not stop execution when $\tau_2 < \Delta t$, severe instability can sometimes lead to a program abort, usually because some temperature becomes so big a mathematic function in the code fails.

All of the mass nodes are updated in an analogous fashion each sub-hour time step, including those for the ground and trusswall networks described in Sections 2.3 and 2.4. The mass updating sequence is irrelevant because they are updated based only on the known conditions at the beginning of the time step. In updating mass nodes with solar and sky radiation "current' sources, such as mass #3 in the bottom of figure 7-1, the fraction of the source Q (top of fig. 7-1) that is applied directly to the mass node (bottom of fig. 7-1) is obtained by superposition.

8. Energy Balance

Once all of the mass node temperatures are updated--i.e., predicted for the end of the sub-hour time step, an energy balance between all interior surfaces and heating or cooling sources is used to determine the Tair and Tmrt at the end of the sub-hour time step.

Figure 8-3 shows the same example as Figure 3-2, but with the surface temperatures T_1, T_2, etc, replaced by the mass node temperatures $Tm_{u,1}$, $Tm_{u,2}$, etc., of the mass nodes adjacent to the UZ interior. The conductance between the mass nodes and Tair and Tmrt are the convective and radiant conductance $cc_{u,ncom}$ and $cr_{u,ncom}$ similar to the cc's and cr's determined in Section 7. The Tair to Tmrt cross coupling cx in this case represents the sum of all $cx_{u,ncom}$'s resulting from the Y-delta transformations, and from radiation absorbed by the air.

Figure 8-3 shows a number of network branches (shown in figure 3-2 as Qa and Qr) that represent sources other than the mass nodes that affect the air and mrt temperatures and must be accounted for in the energy balance.

8.1 Air Node Balance:

For each UZ, summing all instantaneous rates of heat transfer to the $Tair_u$ node to zero gives:

$$Tair_u = \left(\frac{G1_u + G2_u + G3_u + G4_u + G5_u + G6_u}{H1_u + H2_u + H3_u + H4_u + H5_u}\right) \tag{8-1}$$

where $G1_u$ is a conductance term to account for infiltration of air at ambient temperature into the UZ (see Section 6.3.2).

$$G1_u = [Linfu_u + (1 - Frunmax)\cdot Ladduoff_u + Frunmax\cdot Laddu_u]\cdot Tamb$$

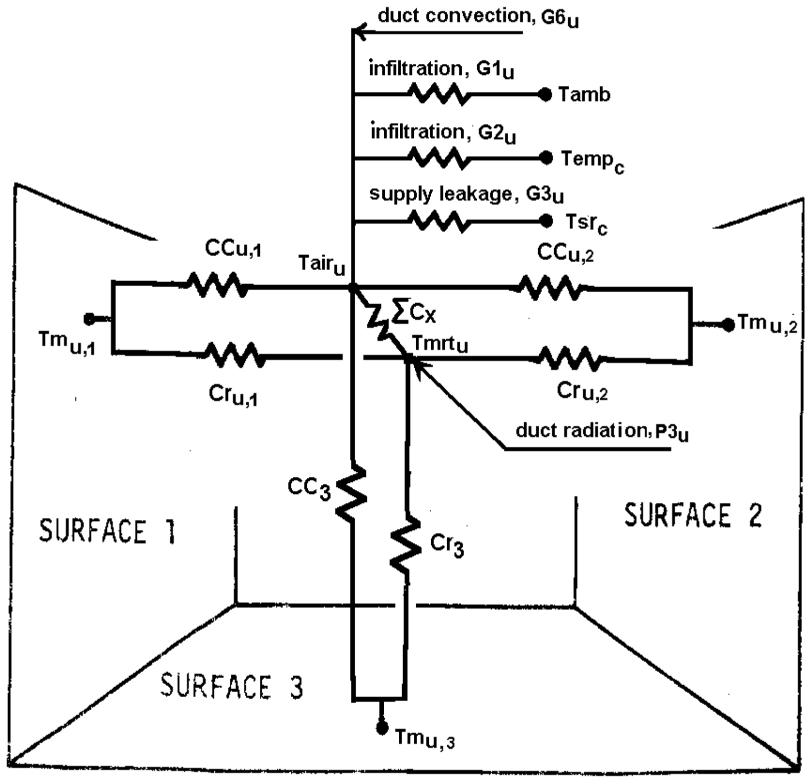

Figure 8-3. Final Radiation and Convection Networks

The corresponding term in the denominator is:

$$H1_u = Linfu_u + (1 - Frunmax) \cdot Ladduoff_u + Frunmax \cdot Laddu_u$$

The term $Linfu_u$ is the natural infiltration rate into UZ u this hour found as described in Section 6.1. $Laddu_u$ is the increase or decrease in $Linfu_u$ caused by unbalanced duct leakage and ceiling or floor infiltration, both happening during the same time period. Ladduoff is that due to ceiling or floor infiltration alone. See Section 6.3.2.

The correction $Laddu_u$ is assumed to apply for the fraction Frunmax of the sub-hour time step and the correction $Ladduoff_u$ during the rest of the sub-hour time step, when the system is off and has no flow.

If only one system is operating during the sub-hour time step Frunmax = Frun for that system. If both are operating, Frunmax is taken as the larger of the two run time fractions for the two systems. This is a simplification that exaggerates the effect of the induced infiltration when one system's run time is short relative to the other but the resulting loss of accuracy is likely small.

$G2_u$ is a conductance term to account for infiltration of air from the CZ's into the unconditioned zone u at temperature $Temp_c$.

If the attic (u = 1) is being modeled, the contributions of infiltration air through the ceiling is:

$$G2_u = \sum_c (\text{ceilInf}_c \cdot \text{Temp}_c)^+$$

where the superscript + means only positive terms are summed--i.e., only air entering the attic.

If the crawl space (u = 2) is being modeled:

$$G2_u = \sum_c (\text{flrInf}_c \cdot \text{Temp}_c)^+$$

The corresponding denominator terms for u = 1 and 2 are:

$$H2_u = \sum_c (\text{ceilInf}_c)^+$$

$$H2_u = \sum_c (\text{flrInf}_c)^+$$

$G3_u$ is a conductance term to account for supply leakage flow into the UZ from both systems. It applies only during the system run time Frun_c:

$$G3_u = \sum_c ((\text{Frun}_c \cdot Ls_{c,u} \cdot \text{Mcps}_{m,c,u}) \cdot (\text{Tsr}_{c,u}))$$

The corresponding term in the denominator is:

$$H3_u = \sum_c (\text{Frun}_c \cdot Ls_{c,u} \cdot \text{Mcps}_{m,c,u})$$

The term $G4_u$ represents the convective heat transfer to Tair from all of the mass nodes adjacent to the interior of the UZ being modeled. See Section 7 for the basis of the cc, cr, and cx conductances.

$$G4_u = \sum_{\text{adj mass nodes } i} (cc_{u,i}) \cdot (Tm_{u,i})$$

The corresponding term in the denominator is:

$$H4_u = \sum_{\text{adj mass nodes } i} (cc_{u,i})$$

The term $G5_u$ partly represents the direct conductance between Tair and Tmrt due the air's absorption of radiation, and partly due to the conduction introduced by the Y-Δ transformations. The sum is over all interior surfaces, including the trusswall and ducts.

$$G5_u = \sum_{\text{interior nodes } j} (cx_{u,j}) \cdot \text{Tmrt}_u$$

The corresponding term in the denominator is:

$$H5_u = \sum_{\text{interior nodes } j} (cx_{u,j})$$

The $G6_u$ term is the rate of convective heat transfer to the UZ air from the supply and return ducts:

$$G6_u = \sum_c \left((Frun_{c,u})\cdot(Frda_c\cdot qlr_{c,u} + Fsda_{c,u}\cdot qls_{c,u})\right)$$

$$Frda_{c,u} = \frac{(Urc_{c,u})}{(Urc_{c,u}+Urr_{c,u})}$$

$$Fsda_{c,u} = \frac{(Usc_{c,u})}{(Usc_{c,u}+Usr_{c,u})}$$

where the fraction $Frda_{c,u}$ of the total return loss $qlr_{c,u}$ goes to the Tair node, and similarly the fraction $Fsda_{c,u}$ of the supply duct loss $qls_{c,u}$ goes to the Tair node.

8.2 MRT Node Balance:

Similar to the above, for each UZ, summing all instantaneous rates of heat transfer to the $Tmrt_u$ node to zero, gives:

$$Tmrt_u = \left(\frac{P1_u + P2_u + P3_u}{S1_u + S2_u}\right) \tag{8-2}$$

The term $P1_u$ represents the radiative heat transfer to Tair from all of the mass nodes adjacent to the interior of the UZ being modeled.

$$P1_u = \sum_{\text{adj mass nodes } i} (cr_{u,i})\cdot(Tm_{u,i})$$

The corresponding term in the denominator is:

$$S1_u = \sum_{\text{adj mass nodes } i} (cr_{u,i})$$

The term $P2_u$ represents the conductance between Tair and Tmrt due the airs absorption of radiation, and usually a larger part due to the Y-Δ transformations.

$$P2_u = \sum_{\text{interior nodes } j} (cx_{u,j})\cdot Tair_u$$

The corresponding term in the denominator is:

$$S2_u = \sum_{\text{interior nodes } j} (cx_{u,j})$$

The term P3 $_u$ is the rate of radiative heat transfer to the Tmrt from the supply and return ducts:

$$P3_u = \sum_c \left(Frun_c \cdot \left(Frdr_{c,u} \cdot qlr_{c,u} + Fsdr_{c,u} \cdot qls_{c,u} \right) \right) \qquad (8\text{-}3)$$

where,

$$Frdr_{c,u} = \frac{(Urr_{c,u})}{(Urc_{c,u} + Urr_{c,u})}$$

$$Fsdr_{c,u} = \frac{(Usr_{c,u})}{(Usc_{c,u} + Usr_{c,u})}$$

where the fraction $Frdr_{c,u}$ of the total return loss $qlr_{c,u}$ that goes to the Tmrt node , and similarly the fraction $Fsda_{c,u}$ for the supply duct loss $qls_{c,u}$ that goes to the Tmrt node.

Because equations (8-1) and (8-2) constitute two linear equations with the two unknowns Tair and Tmrt, they are solved simultaneously to give an explicit equation for Tair (see source code). Given Tair, equation (8-2) is solved for Tmrt. The air and mrt temperatures found are the temperatures predicted for the UZ at the end of the sub-hour time step, and they are in instantaneous equilibrium with all of the internal components of the UZ. They are used as the UZ temperatures for calculating heat transfers during the following sub-hour time step.

9. References

Residential Alternative Calculation Method (ACM), Approval Manual For Compliance with California's 2005 Energy Efficiency Standards, Publication Number: 400-03-003F, Dated Published: October 2004, Effective Date: October 1, 2005

ASHRAE, American Society of Heating, Refrigeration and Air-Conditioning Engineers. *1993 ASHRAE Handbook, Fundamentals*, pp 22.1.

Carroll, J. A., 1980a, "An MRT method of computing radiant energy exchange in rooms," Proceedings of the 2nd Systems Simulation and Economic Analysis Conference, San Diego, CA.

Carroll J.A., J. R. Cllinton, 1980b, "A Thermal Network Model of a Passive Solar House," Proceedings of the 5th National Passive Solar Conference, American Section of ISES.

Carroll J.A., J. R. Cllinton, 1982, " Appendix H: A Thermal Network Model of a Passive Solar House," Final Report USDOE Contract DE AC04-79AL10891. July, 1982.

Carroll, J. A., 1981, "A Comparison of Radiant Interchange Algorithms," Proceedings of the 3d Annual Systems Simulation and Economics Analysis/Solar Heating and Cooling Operational Results Conference, Reno. Solar Engineering, Proceedings of the ASME Solar division.

Clear, R.D., L. Gartland and F.C. Winkelmann, 2001, "An Empirical Correlation for the Outside Convection Air Film Coefficient for Horizontal Roofs," Lawrence Berkeley National Laboratory, Jan 2001.

Francisco, P.W., and L. Palmiter, 1999, revision 2003, "Improvements to ASHRAE Standard 152P," U.S. DOE Subcontract 324269-AU1.

Martin, Marlo and Berdahl, Paul, "Characteristics of Infrared Sky Radiation in the United States." Solar Energy Vol. 33, No. ¾, pp 321-336, 1984.

Mills,A.F. , 1992, "Heat Transfer" Irwin Press, Boston.

Palmiter, L. and T. Bond. 1992, "Impact of Mechanical Systems on Ventilation and Infiltration in Homes," ACEEE.

Palmiter, L., and T. Bond. 1991a, "Modeled and Measured Infiltration: A Detailed Case Study of Four Electrically Heated Homes, " Electric Power Institute report CU-73227, Palo Alto, California.

Palmiter, L., and T. Bond. 1991b, "Interaction of Mechanical Systems and Natural Infiltration, " AIVC, Ottawa, Canada.

Palmiter, L., and T. Bond. 1991a, "Modeled and Measured Infiltration: A Detailed Case Study of Four Electrically Heated Homes, " Electric Power Institute report CU-73227, Palo Alto, Calif.

Palmiter, L., and T. Bond. 1991b, "Interaction of Mechanical Systems and Natural Infiltration, " AIVC, Ottawa, Canada.

Palmiter, L. and P.W. Francisco, 1996, "Modeled and Measured Infiltration: Phase III-A detailed Study of Three Homes". TR-106228, EPRI WO3512-12.

Palmiter, L, and E. Kruse, 2003, "Derivation of Duct Efficiency Equations in ASHRAE Standard 152", ASHRAE Seminar, Feb. 2003. Available from author at Ecotope Inc.

Sebald, A. V., 1985," Efficient Simulation of Large Controlled Passive Solar Systems: Forward Differencing in Thermal Networks", Solar Energy Vol. 34, No. 3.

Walker, I.S., T.W. Forest and D.J. Wilson (1995), “A Simple Calculation Method for Attic Ventilation Rates”, Proc. 16th AIVC Conference, Vol. 1, pp. 221-232, Air Infiltration and Ventilation Centre, Coventry, UK.

Other references found useful during the program development:

ASTM C 1340-99, "Standard Practice for Estimation of Heat Gain or Loss Through Ceilings Under Attics Containing Radiant Barriers by Use of a Computer Program," and ASTM 1340 revision "Appendix X1:

Computer Program with Quasi-steady Model for HVAC Ducts in Attics." ; Fortran listing: RB-DUCT (ver 4-03-03).

Deru, M., and P. Burns. 2003, "Infiltration and Natural Ventilation Model for Whole Building Energy Simulation of Residential Buildings," NREL/CP-550-33698; ASHRAE preprint, Kansas City, Missouri.

Palmiter, L., E. Kruse, and P. Francisco, 2004, "Duct Efficiency Under Full-load or Modulating Conditions: Implications for Heat Pump Performance," ACEEE.

Parker, D.S., P. Fairey and L. Gu, 1993. "Simulation of the Effect of Duct Leakage and Heat Transfer on Residential Space Cooling Use," Energy and Buildings, 20, Elsevier Sequoia, Netherlands.

Parker, D., P. Broman, J. Grant, L. Gu, M. Anello, R. Vieira and H. Henderson, 1999, "EnergyGauge USA: A Residential Building Energy Design Tool." Proceedings of Building Simulation '99, Kyoto, Japan. International Building Performance Simulation Association, Texas A&M University, College Station, TX, September 1999.

Walker, I.S. and D.J. Wilson. 1994, " Practical Methods for Improving Estimates of Natural Ventilation Rates," Proc. 15th AIVC Conference, Buxton, U.K., 1994: 517-525.

Walker, I.S., and D. J. Wilson, 1990, "The Alberta Air Infiltration Model, AIM-2". The University of Alberta, Dept. of Mechanical Engineering, Tech Report 71.

Walker, I. S., and D. J. Wilson, 1994, "Practical Methods for Improving Estimates of Natural Ventilation Rates". Proceedings of the 15th AIVC Conference, Buxton , UK.

Walker, I. S., T. W. Forest, and D. J. Wilson, 2005, "An Attic-interior Infiltration and Interzone Transport Model of a House," Building and Environment 40 (2005) 701-718.

Walker, Iain, 2005, " REGCAP Model Outline."

Appendix A. <u>Derivation of Duct Loss Equations</u>

This derivation is for one zone only, and the nomenclature is specific to this appendix alone.

Heat transfer through the duct walls can be illustrated in the electrical analogy in Fig. **A-**1. The first node on the left represents the temperature of the air in the duct (T_d) and is connected to the temperate on the surface of the duct (T_s) by the conductance through the duct wall (U_d). The convective heat transfer coefficient (h_c) connects the surface temperature to the duct zone air temperature (T_a). The radiation heat transfer coefficient (h_r) connects the surface temperature to the duct zone radiant temperature (T_r).

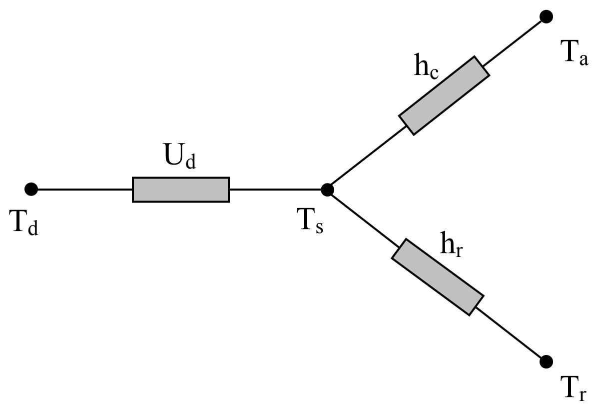

Figure A-1. Electrical analogy of heat transfer through a duct wall

The temperatures of the duct zone are assumed to be constant; the duct surface temperature is not. The duct surface temperature can be removed from the analysis by using a Y-Δ transform. Fig. **A-**2 shows the result of this transformation with direct connections between the duct air temperature, the duct zone radiant and air temperatures through combined coefficients defined in eqs. 1.

$$U_r = \frac{U_d h_r}{D} \qquad U_c = \frac{U_d h_c}{D} \qquad U_x = \frac{h_c h_r}{D} \qquad \textbf{(A-1)}$$

where

$$D = U_d + h_c + h_r$$

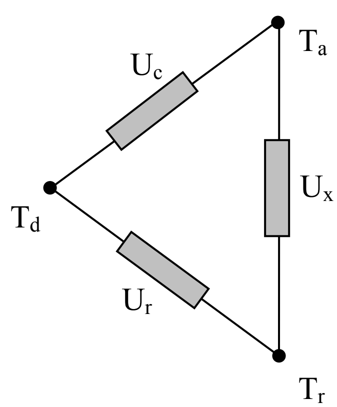

Figure A-2. Heat transfer through a duct wall with surface temperature removed

Using an energy balance, the rate of change of heat flow along the length (x) of duct must equal the heat flow through the duct wall, or

$$-mc_p \frac{dT_d(x)}{dx} = U_c P(T_d(x) - T_a) + U_r P(T_d(x) - T_r) \qquad (\textbf{A-}2)$$

where

mc_p = capacitance flow rate of the air in the duct
T_d = temperature of air in the duct
U_c = equivalent heat transfer coefficient (see eq. **A-**1)
P = perimeter of duct
T_a = temperature of air in duct zone
U_r = equivalent heat transfer coefficient (see eq. **A-**1)
T_r = radiant temperature in duct zone

Regrouping by temperature terms

$$mc_p \frac{dT_d(x)}{dx} = -(U_c P + U_r P)T_d(x) + U_c PT_a + U_r PT_r \qquad (\text{A-3})$$

and dividing through by the quantity $(U_c P + U_r P)$ gives

$$\frac{mc_p}{(U_c P + U_r P)} \frac{dT_d(x)}{dx} = -T_d(x) + T_{amb} \qquad (\text{A-4})$$

where

$$T_{amb} = \frac{U_c P}{(U_c P + U_r P)} T_a + \frac{U_r P}{(U_c P + U_r P)} T_r \qquad (\text{A-5})$$

Let $y(x)$ be

$$y(x) = T_{amb} - T_d(x) \qquad (\text{A-6})$$

The derivative of which is

$$dy = -dT_d \qquad (\text{A-7})$$

Substituting eqs. A-6 and A-7 into eq. A-4 gives

$$-\frac{mc_p}{(U_c + U_r)P} \frac{dy}{dx} = y(x) \qquad (\text{A-8})$$

Rearranging

$$\frac{1}{y(x)}dy = -\frac{(U_c + U_r)P}{mc_p}dx \tag{A-9}$$

and integrating from entrance ($x = 0$) to exit ($x = L$)

$$\int_0^L \frac{1}{y(x)}dy = \int_0^L -\frac{(U_c + U_r)P}{mc_p}dx \tag{A-10}$$

gives

$$\ln y(L) - \ln y(0) = -\frac{(U_c + U_r)PL}{mc_p} \tag{A-11}$$

Recalling the definition in eq. A-6 and replacing the product of the perimeter and length with the surface area (A) of the duct, and a bit of manipulation yields the following relationships

$$\frac{y(L)}{y(0)} = \frac{T_d(L) - T_{amb}}{T_d(0) - T_{amb}} = \exp\left(-\frac{(U_c + U_r)A}{mc_p}\right) \tag{A-12}$$

Let

$$\beta = \exp\left(-\frac{(U_c + U_r)A}{mc_p}\right) \tag{A-13}$$

Then

$$\frac{T_d(L) - T_{amb}}{T_d(0) - T_{amb}} = \beta \tag{A-14}$$

Solving for the exit temperature gives

$$T_d(L) = \beta(T_d(0) - T_{amb}) + T_{amb} \tag{A-15}$$

The temperature change in length L of duct is

$$T_d(0) - T_d(L) = -\beta(T_d(0) - T_{amb}) - T_{amb} + T_d(0) \tag{A-16}$$

This can be rewritten as

$$T_d(0) - T_d(L) = (1-\beta)(T_d(0) - T_{amb}) \tag{A-17}$$

Let ε be the sensible heat exchanger effectiveness

$$\varepsilon = (1-\beta) \tag{A-18}$$

Then the conduction loss from the duct to the duct zone can then be written as

$$Q_{loss} = mc_p(T_d(0) - T_d(L)) = \varepsilon\, mc_p(T_d(0) - T_{amb}) \qquad \text{(A-19)}$$

Appendix B. Hourly Calculation of Infrared Sky Radiation

Net Infrared Radiation

The following algorithms are a modified version of those found in Martin and Berdahl (1984). The terminology and notation follow that of Martin and Berdahl as well.

The weather data used for input to these calculations provides hourly values for cloud cover, opaque cloud cover, cloud ceiling height, dew point, and air temperature. Note also that the weather data provides temperatures in Celsius.

The net infrared radiation absorbed by a graybody horizontal surface is

$$q_{net} = \varepsilon_{surf}(\sigma T_{sky}^4 - \sigma T_{surf}^4) \qquad \text{(B-1)}$$

where

ε_{surf} = emissivity of the horizontal graybody surface

T_{surf} = horizontal graybody surface temperature (Kelvin)

T_{sky} = equivalent blackbody sky temperature (Kelvin)

σ = Stefan-Boltzmann constant in SI units

The term on the right involving Tsurf is the radiation leaving the surface, while the term involving Tsky is the downward radiation from the sky.

This can be rewritten as

$$q_{net} = \varepsilon_{surf}\sigma(T_{air}^4 - T_{surf}^4 + T_{sky}^4 - T_{air}^4) \qquad \text{(B-2)}$$

where

T_{air} = dry bulb temperature of the outdoor air (Kelvin)

Equation B-2 can be regrouped into the following form,

$$q_{net} = q_{std} + q_{sky} = \varepsilon_{surf}\sigma(T_{air}^4 - T_{surf}^4) + \varepsilon_{surf}\sigma(T_{sky}^4 - T_{air}^4) \qquad \text{(B-3)}$$

The first term on the right is the standard method of calculating radiation heat transfer on the surface of buildings (see ASHRAE 1993) and the second is an additional factor to account for radiation from the sky. Therefore, the omission of the sky radiation term in the standard calculations is an error based on an assumption that radiation is only to the air temperature. Q_{sky} will be zero or negative because the sky temperature is always less than or equal to the air temperature.

Sky Emissivity

By definition, the effective black body emissivity of the sky is $\varepsilon_{sky} = \frac{T_{sky}^4}{T_{air}^4}$. Following Martin and Berdahl, the value of ε_{sky} can be estimated as follows.

The clear sky emissivity (ε_0) is a function of the dew point temperature and the atmospheric pressure.

$$\varepsilon_0 = .711 + .56\frac{T_{dew}}{100} + .73\left(\frac{T_{dew}}{100}\right)^2 + .013\cos(\pi/12 * hr) + .00012(P_{atm} - 1000) \qquad \text{(B-4)}$$

where
T_{dew} = the dew point temperature in Celsius
hr = hour of day (1 to 24)
P_{atm} = atmospheric pressure in millibar

The effect of cloud cover must also be considered. The TMY2, WYEC, and EPW weather data provide overall and opaque cloud fractions in tenths. Thin cloud fraction (n_{th}) in tenths can be determined from the data using

$$n_{th} = n - n_{op} \qquad \text{(B-5)}$$

where
n = the cloud fraction
n_{op} = the opaque cloud fraction

The cloud factor (Γ), used to adjust the emissivity of a cloudy sky, is dependent on the cloud base temperature; however, this data is not typically measured at the weather stations. Using the more commonly measured cloud ceiling height, the cloud factor can be approximated using the following equations. This factor differs for opaque and thin clouds.

The opaque cloud factor varies with cloud ceiling height,

if cloud ceiling height > 21,000 m,

$$\Gamma_{op} = \exp(-8000 / h_0) \qquad \text{(B-6)}$$

else,

$$\Gamma_{op} = \exp(-h_{op} / h_0) \qquad \text{(B-7)}$$

The thin cloud factor is expressed as

$$\Gamma_{th} = \exp(-8000 / h_0) \qquad \text{(B-8)}$$

where
h_0 = 8200 m
h_{op} = opaque cloud ceiling height from weather data

The sky emissivity (ε_{sky}) can now be calculated as the sum of the clear sky emissivity and the various cloud emissivities.

$$\varepsilon_{sky} = \varepsilon_0 + (1-\varepsilon_0)\sum_i n_i \varepsilon_{c,i} \Gamma_i \qquad \text{(B-9)}$$

$$\varepsilon_{\text{sky}} = \varepsilon_0 + (1-\varepsilon_0)(n_{op}\varepsilon_{op}\Gamma_{op} + n_{th}\varepsilon_{th}\Gamma_{th}) \qquad \text{(B-10)}$$

where

ε_{op} = the opaque cloud emittance: assume 1

ε_{th} = the thin cloud emittance: assume 0.4

Sky Radiation

Using the definition previously given for ε_{sky}, the q_{sky} term for a horizontal surface can be calculated as

$$q_{sky} = \sigma(\varepsilon_{sky} - 1)(T_{air})^4 \qquad \text{(B-11)}$$

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