DOCKETED

Docket Number:	22-SPPE-01
Project Title:	Bowers Backup Generating Facility
TN #:	248688
Document Title:	BBGF Thermal Plume Analysis DR-45
Description:	N/A
Filer:	Scott Galati
Organization:	DayZenLLC
Submitter Role:	Applicant Representative
Submission Date:	2/7/2023 1:59:57 PM
Docketed Date:	2/7/2023

Vertical Plume Velocity Assessment

Bowers Backup Generating Facility

Santa Clara, California

Submitted to

California Energy Commission

Submitted by

GI PARTNERS

Prepared by

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January 2023

Introduction

This report presents the evaluation of the Bowers Backup Generating Facility (BBGF) source generated plumes from the 32 Tier 4 Cummins diesel engines and 64 rooftop chillers on the effects on airport/aircraft operations. The Normal Y. Mineta San Jose International Airport is located approximately 3.1 kilometers east southeast of the BBGF. This report is based upon an analysis prepared by Atmospheric Dynamics, Inc. in accordance with the California Energy Commission (CEC) application requirements for a Small Power Plant Exemption (SPPE) pursuant to the power plant siting regulations. This analysis is but one part of a larger analysis, which seeks an SPPE Decision from the CEC.

Based on the stack parameter data, an analysis of the potential plume characteristics from the routine operation the diesel engines and rooftop chillers on vertical winds was prepared and compared to the California Energy Commission (CEC) significance criteria of 5.3 meters per second (m/s) for the average vertical plume velocities as described below.

Atmospheric Dynamics, Inc. (ADI) prepared a screening level plume vertical velocity assessment which are based on the calm wind Spillane methodology outlined in the *"Aviation Safety and Buoyant Plumes"* paper (Peter Best, et. al., presented at the Clean Air Conference, Newcastle, New South Wales, Australia, 2003). This methodology is also recognized as a screening tool for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in *"AC 139-5(1) Plume Rise Assessments (CASA, 2012)".*

The aim of this screening assessment is to conservatively determine the potential for turbulence generated by the diesel engines and rooftop chillers waste heat exhaust plumes. Part 139.370 of the Australian Civil Aviation Safety Regulations (1998, 2004) provides that CASA may determine that plume velocities in excess of 4.3 m/s is or will be a potential hazard to aircraft operations. The *Manual of Aviation Meteorology* (Australian Bureau of Meteorology 2003) defines severe turbulence as a vertical wind gust velocity in excess of 10.6 m/s. The assumed critical vertical velocity used as a CEC significance threshold is 5.3 meters per second* (m/s) but it should be noted that the basis of the original CASA derived threshold of 4.3 m/s has been lost in antiquity and that CASA no longer relies on the 1998 and 2004 regulations that established this critical threshold other than to note that a more rigorous analysis, which includes site specific meteorology, should be used if the 4.3 m/s and 10.6 m/s screening thresholds are exceeded. The screening method uses absolute worst-case assumptions of calm winds and neutral atmospheric conditions for the entire vertical extent of the plume to determine these worst-case impacts. It should be noted that these results are extremely conservative in that these worst-case conditions typically only occur during a few hours each year.

The Spillane methodology is generally applied to a limited number of plume source geometry's (turbines, power plant boilers, etc.) with the stacks arranged linearly (in a single straight-line) and separated by distances that typically exceed the individual stack diameters. For the diesel engines, this assumption was maintained. Only one engine stack was modeled consistent with the normal operational testing schedule of the emergency generator engines. For the chiller assessment, a conservative assumption was made in order to use the Spillane methodology on an atypical chiller plume configuration, which is made up of 88 chillers arranged on a two-dimensional surface. Here, the methodology, as described below, assumed that all sixteen chiller cells for each chiller were merged into a single stack with an effective diameter based on the combined area of all sixteen chiller cells. In other words, a single stack was assumed to initially

*For the Puente Power Project (Docket#15-AFC-01, TN#213674, 9/15/2016), "CEC staff … concluded that an average velocity of 5.3 m/s is the appropriate velocity … [for a plume velocity threshold]." The CEC staff "Plume Background Threshold" attached to the docketed document concludes with "…[CEC] staff will use 10.6 m/s peak vertical plume velocity as the new threshold. The altitude at which a plume would have a peak vertical velocity of 10.6 m/s would be the same altitude at which a plume would have an average vertical velocity of half that, 5.3 m/s."

describe the release parameters of the combined chiller cells in each of the 88 individual chillers. The effective plume diameter is appropriate for each individual chiller based on the close proximity and arrangement of the sixteen chiller cells.

Screening Methodology and Vertical Plume Velocity Calculations

The Spillane methodology is based on worst-case calm wind neutral stability conditions to assess the average plume vertical velocity as a function of height. The methodology is based on well-verified laboratory and theoretical treatments of the rise and spread of a buoyant jet, both into a still ambient environment and into a light crosswind. This treatment covers in detail the initial dynamics of the plume as it exits the stack and the entrainment of ambient air into the plume as it rises directly above the stack. In addition to providing clarifications and algebraic solutions to the Spillane methodology, the 2003 Peter Best paper provides additional methodologies that also consider the enhancement of vertical velocities that may occur if the plumes from multiple identical stacks merge and form a higher buoyancy combined plume (referred to here as the enhanced Spillane methodology).

The vertical plume assessment will involve several stages of development. For individual plumes, the stages are:

(a) In the first stage very close to the stack exit, the high plume momentum will result in a short section in which the conditions at the center of the plume are relatively unaffected by ambient and plume buoyancy conditions. This jet phase extends from the stack exit to approximately a distance of 6.25 D above the stack (where D is the stack diameter) in calm conditions. At the end of this stage, the plume-averaged vertical velocity has decreased to half of the stack exit velocity, with a corresponding increase, or doubling, in effective plume diameter.

(b) In the second stage, the plume responds to differences between ambient and plume buoyancy conditions, with much cooler and less turbulent ambient air being entrained into the plume from the outside regions of the plume towards the plume centerline. The momentum and buoyancy of the plume significantly influences plume rise and subsequently the dilution of the stack exhaust to decrease plume vertical velocities. This dilution is very sensitive to ambient wind speed, so the calm wind conditions considered here are extremely conservative.

(c) In the third stage of plume development, plume rise is due entirely to the buoyancy of the plume and continues from some distance until there is an equalization of turbulence conditions within and outside the plume. This final rise is often only achieved at considerable heights/distances from the stack where the effective average vertical velocity is then close to zero. Since there is very little turbulence and near-zero vertical velocities, this stage of plume development is usually not considered for this type of analysis.

In the second stage of development, the analytical solution of the governing equations under these conditions is given by:

$$a = 0.16(z - z_v)$$
$$V = \{(Va)o^3 + 0.12Fo\,[\,(z - zv)^2 - (6.25D - zv)^2]\}^{1/3} / a$$

Where the subscript 'o' refers to values of the parameters at the stack outlet and the variables are:

a	plume radius (m)
V	average vertical velocity (m/s)
z	height above stack top (m)
z_v	virtual source height (m)
D	stack diameter (m)
F_o	buoyancy flux evaluated at the stack outlet (m^4s^{-3})

These are the two primary equations governing the growth of a single plume in the second stage of development under neutral calm wind conditions. Additional equations governing the first stage of single plume development as well as the interaction of multiple plumes in the second stage of development are discussed in detail in the Best paper.

For multiple stacks in the enhanced Spillane methodology, the equations governing the second stage are calculated from the point when the plumes begin to merge until they are fully merged. The plume merging begins at the height where the plume diameters equal the stack separations and the plumes are fully merged at the height where the plume diameters are equal to 2d(N-1)/2 for three or more stacks or 2d for two stacks. At the fully merged height, the merged plume diameter and velocity is enhanced by the fourth root of the number of stacks. Above the fully merged plume height, the enhanced plume diameter and plume velocities follow the regular equations given for the second stage. Below the fully merged plume height for the merging phase, plume velocities are linearly interpolated by height from the single plume velocity at the height where the plumes begin to merge to the enhanced plume velocity at the fully merged plume height.

Vertical Plume Velocity Calculations for the Diesel Engines

The BBGF is comprised of 44 individual large and one (1) small diesel emergency generator stacks. The small diesel emergency generator was not assessed as it would have smaller plume vertical velocities. Generator stack parameter data (plume exit velocity, plume exit temperature and stack exit diameter) were provided by Cummins. Only one (1) engine will be tested during any one hour. While the engines will be tested at minimum loads, the 100 percent load case was utilized for the worst-case plume analysis. For the engine analysis, two ambient conditions were considered: 41.0°F, the minimum monthly mean of daily minimum temperatures, and 84.3°F, the maximum monthly mean of daily maximum temperatures for the San Jose Airport (*"Climatology of the United States No. 81 – Monthly Station Normals of Temperature, Precipitation, and Heating and Cooling Degree Days 1971-2000 – California"*, February 2002, and *"Climatology of the United States No 20 – Monthly Station Climate Summaries, 1971-2000 – California"*, February 2004). These data is summarized in Table 1.

Table 1 Cummins Diesel Stack Characteristics for Vertical Plume Velocity Analysis		
Case #	**1**	**2**
Ambient Temperature (°F)*	41.0	41.0
Stack Diameter (m)	0.559	0.559
Exhaust Velocity (m/s)*	43.89	43.89
Exhaust Temperature (K)*	762.04	762.04
Stack Release Height (m)	15.24	15.24
Stack Buoyancy Flux (m^4/s^3)	21.34	20.29
*Stack data provided by Cummins at 100% load		

Screening level vertical plume velocity assessments were made for two ambient temperatures with calm winds and neutral atmospheric conditions for the cases presented in Table 1 which are based on 100 percent load. The results based on the two ambient conditions are presented in Table 2 and the output from the calculation spreadsheet provided in Attachment A.

The initial jet phase extends to a height of about 62 feet above grade level (ft-agl) for both cases. After the jet phase, plume temperature buoyancy characteristics modeled in the Spillane methodology cause a uniform decrease in plume-averaged vertical velocities, with the critical plume-averaged vertical velocity of 5.3 m/s occurring at about 89 ft-agl for both cases.

Table 2 Diesel Engine Vertical Plume Velocity Analysis Results for Reference Height		
Case #	**1**	**2**
Ambient Temperature (°F)	41.0	84.3
Single Plume Results:		
Plume-Averaged Vertical Velocity at 200 feet-agl (m/s)	2.48	2.45
Height of 5.3 m/s Plume-Averaged Vertical Velocity (feet-agl)	88.3	88.9

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the diesel engine always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above about 89 ft-agl. Even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities, so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear when realistically, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Vertical Plume Velocity Calculations for the Rooftop Chillers

The 63 rooftop chillers are each comprised of 20 individual cells, with a cell fan diameter of 34 inches. The 63 chillers are generally arranged 21 along the longer building length (averaging 20 feet between adjacent chillers) by three (3) along the shorter building width (averaging 45 feet between adjacent chillers). Based on the groupings of chillers, the single and merged plumes were based on the 63 (3x21) chiller arrangement my merging plumes along the length (3 merged stacks) and width (21 merged stacks). Chiller stack parameter data (exit velocity and temperature) were provided by the applicant. An effective stack diameter for all 20 cells was utilized for each chiller. The chillers will utilize variable speed fans and the number of fans that are operational are dependent upon ambient temperature and plant load. However, to be conservative, all chillers/cells were assumed to be operating at full load. These data are summarized in Table 3 for the same ambient temperatures used for the engine analysis.

Table 3
Chiller Stack Characteristics for Vertical Plume Velocity Analysis

Case #	1	2
Ambient Temperature (°F)*	41.0	84.3
Effective Stack Diameter (m)**	3.86	3.86
Exhaust Velocity (m/s)*	8.06	8.06
Exhaust Temperature (K)*	289.26	313.32
Stack Release Height (m)	31.35	31.35
Stack Buoyancy Flux (m^4/s^3)	11.33	10.45

*Chiller stack data provided by the applicant
** Calculated value based on the cell diameter of 34 inches multiplied by the square of the number of operating cells, or D_{eff} = 34"*$\sqrt{20}$

The Spillane methodology was originally developed to treat multiple individual stacks that are arranged along a linear x or y direction, but not both directions at once, with stack separations much greater than the stack diameters, typical of boilers/turbines at large power plants. As noted above, the 63 chillers are generally arranged in a 3 x 21 pattern. Therefore, the enhanced Spillane methodology was based on calculating the total merging height for the largest linear direction of chiller placements (which is 21 chillers spaced 20.0 feet apart along the longer length of the building). The largest grouping of 63 (3x21) chillers were considered in the calculation of vertical velocity plume enhancement (both at and above the totally merged height, and for the interpolation down to the plume touching height.

Screening level vertical plume velocity assessments were made for the same ambient temperatures with calm winds and neutral atmospheric conditions as was done for the emergency generator engines. The results are presented in Table 4 and the output from the calculation spreadsheets are provided in Attachment A.

The initial jet phase extends to a height of about 182.5 ft-agl for both cases. The critical plume-averaged vertical velocity of 5.3 m/s occurs in the jet phase at about 157 ft-agl for both cases. The plumes touch (begin to merge) at about 167 ft-agl and are fully merged at about 1,356 ft-agl for both cases. Under the enhanced Spillane methodology, the merged plume-averaged vertical velocities never approach 5.3 m/s (either above the totally merged height or when interpolated down to the touching height).

Table 4
Chiller Vertical Plume Velocity Analysis Results for Reference Height

Case #	1	2
Ambient Temperature (°F)	41.0	84.3
Single Plume Results:		
Height of 5.3 m/s Plume-Averaged Vertical Velocity (Within the Jet Phase, feet-agl)	157.6	157.6
Merged Plume Results:		
Plume-Averaged Vertical Velocity at 1,000 feet-agl (m/s)	3.97	3.89

From these results and for each ambient condition, the vertical plume velocities are less than the threshold value of 5.3 m/s for all heights above about 158 ft-agl and above for the chillers. The heights at which plume-averaged vertical velocities exceed 5.3 m/s only occur during the jet phase for both cases. These cases also represent worst-case conditions of calm winds at all levels of a neutral atmosphere.

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the chillers always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above about 157.6 ft-agl. Even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear when realistically, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Attachment A
Spillane Method Plume Velocity Calculations

SINGLE Plume Average Vertical Velocities for WBGF Large Emer.Gen Engine, 100% Load, and Maximum Stack Height - Winter Min*

***"Aviation Safety and Buoyant Plumes*," Peter Best, et. al.**

***"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Maximum Stack Height h_s	15.24	meters	50	feet-inches	λ	1.11		
Stack Diameter D	0.5588	meters	22	inches	λ_o	~1.0		
Stack Velocity V_{exit}	43.89	m/s	143.99	ft/sec				
Volumetric Flow	10.76	cu.m/sec	22,806	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	762.04	Kelvins	912	°F				
Initial Stack Buoyancy Flux F_o	21.3430	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	3.493	meters*	11.5	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	18.733	meters	**61.5**	feet				"
Vertical Velocity V_{jet}	21.945	m/s	72.00	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	1.118	meters	3.7	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	1.382	meters*	4.5	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	16.622	meters	54.5	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6042
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a			Sect.2.1(6)
Product $(Va)_o$	7.409	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height	**200.0**	**feet**		60.96	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	45.720	meters*	150.0	feet*				
Plume Top-Hat Diameter 2a'	14.188	meters	46.5	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**2.478**	**m/s**	8.13	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV > Top of Jet (Spillane)**	
Find Height above Stack z_{crit}	11.687	meters	38.3	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	26.927	meters	**88.3**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=		-4.2000
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=		-648.25
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet		gives the real solution x = z-zv =		10.3047
							or z(m/above stack) =	11.687
							z(ft/above ground) =	88.3

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)				
Stack.Rel.Ht = 50.0	**0.00**	**0.279**	**43.89**					
55.0	1.52	0.401	34.35				**Jet Phase Eqs:**	**5 foot Intervals**
60.0	3.05	0.522	24.81				Linearly interpolated from Stack Rel.Ht to Top of Jet	
Top of jet = 61.5	**3.51**	**0.559**	**21.95**				**Spillane Equations:**	
70.0	6.10	0.754	10.18	363.00			$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
80.0	9.14	1.242	6.60	326.43			$a = 0.16(z-z_v)$	**10 foot Intervals**
90.0	12.19	1.730	5.12	310.22			$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Spillane 5.3 m/s Height = 88.3	**11.69**	**1.649**	**5.30**	312.24				
100.0	15.24	2.217	4.33	301.20				
110.0	18.29	2.705	3.85	295.60				
120.0	21.34	3.193	3.52	291.85				**Max<5.30 m/s**
130.0	24.38	3.680	3.27	289.22				
140.0	27.43	4.168	3.09	287.30				
150.0	30.48	4.656	2.94	285.86				
200.0	45.72	7.094	2.48	282.09				**50 foot Intervals**
250.0	60.96	9.532	2.22	280.58				
300.0	76.20	11.971	2.05	279.82				
350.0	91.44	14.409	1.92	279.38				
400.0	106.68	16.848	1.82	279.10				
450.0	121.92	19.286	1.74	278.91				
500.0	137.16	21.724	1.67	278.77				
600.0	167.64	26.601	1.56	278.60				**100 foot Intervals**
700.0	198.12	31.478	1.47	278.49				
800.0	228.60	36.355	1.40	278.41				
900.0	259.08	41.232	1.34	278.36				
1000.0	289.56	46.108	1.30	278.33				
1100.0	320.04	50.985	1.25	278.30				
1200.0	350.52	55.862	1.21	278.28				
1300.0	381.00	60.739	1.18	278.26				
1400.0	411.48	65.616	1.15	278.25				
1500.0	441.96	70.492	1.12	278.24				
1600.0	472.44	75.369	1.10	278.23				
1700.0	502.92	80.246	1.08	278.22				
1800.0	533.40	85.123	1.06	278.21				
1900.0	563.88	90.000	1.04	278.21				
2000.0	594.36	94.876	1.02	278.20				

*Winter Min = Monthly Mean of Minimum Daily Temperatures for 1971-2000 (Lowest in December)

NOAA Sources: Climatography of the United States No.81 *"Monthly Station Normals of Temperatures, Precipitation, and Heating and Cooling Degree Days, 1971-2000 California"* and Climatography of the United States No. 20 *"Monthly Station Climate Summaries, 1971-2000 California"*

SINGLE Plume Average Vertical Velocities for Single WBGF Large Emer.Gen Engine, 100% Load, and Maximum Stack Height - Summer Max*

"***Aviation Safety and Buoyant Plumes*** **," Peter Best, et. al.**

"***The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia*** **," Dr. K.T. Spillane**

Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Maximum Stack Height h_s	15.24	meters	50	feet-inches	λ 1.11	
Stack Diameter D	0.5588	meters	22	inches	λ_o ~1.0	
Stack Velocity V_{exit}	43.89	m/s	143.99	ft/sec		
Volumetric Flow	10.76	cu.m/sec	22,806	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	762.04	Kelvins	912	°F		
Initial Stack Buoyancy Flux F_o	20.2818	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	3.493	meters*	11.5	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	18.733	meters	**61.5**	feet		"
Vertical Velocity V_{jet}	21.945	m/s	72.00	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	1.118	meters	3.7	feet	$2a_{jet}$ = 2D — Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	1.293	meters*	4.2	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	16.533	meters	54.2	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6297
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	Sect.2.1(6)
Product $(Va)_o$	7.722	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$	
Solve for plume-averaged vertical velocity at height	**200.0**	**feet**		60.96	meters above ground ($z'+h_s$)	
Gives the following Height above Stack z'	45.720	meters*	150.0	feet*		
Plume Top-Hat Diameter 2a'	14.217	meters	46.6	feet	$2a'=2*0.16(z'-z_v)$	Sect.2/Eq.6
Vertical Velocity V	**2.445**	**m/s**	8.02	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$	Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**				**Critical VV > Top of Jet (Spillane)**
Find Height above Stack z_{crit}	11.866	meters	38.9	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)	
Height above Ground $z_{crit}+h_s$	27.106	meters	**88.9**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where	
					a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=	-3.9912
Interpolated Height of critical vertical velocity in Jet Phase:					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=	-735.79
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet		http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	gives the real solution x = z-zv =	10.5731
					or z(m/above stack) =	11.866
					z(ft/above ground) =	88.9

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 50.0	***0.00***	**0.279**	**43.89**			
55.0	1.52	0.401	34.35		**Jet Phase Eqs:**	**5 foot Intervals**
60.0	3.05	0.522	24.81		Linearly interpolated from Stack Rel.Ht to Top of Jet	
Top of jet = 61.5	***3.51***	***0.559***	***21.95***		**Spillane Equations:**	
70.0	6.10	0.768	10.36	385.09	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
80.0	9.14	1.256	6.71	350.11	$a = 0.16(z-z_v)$	**10 foot Intervals**
90.0	12.19	1.744	5.18	334.39	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Spillane 5.3 m/s Height = 88.9	**11.87**	**1.692**	**5.30**	335.64		
100.0	15.24	2.232	4.36	325.56		
110.0	18.29	2.719	3.85	320.00		
120.0	21.34	3.207	3.51	316.26		**Max<5.30 m/s**
130.0	24.38	3.695	3.26	313.61		
140.0	27.43	4.182	3.07	311.66		
150.0	30.48	4.670	2.91	310.19		
200.0	45.72	7.108	2.45	306.31		**50 foot Intervals**
250.0	60.96	9.547	2.19	304.75		
300.0	76.20	11.985	2.02	303.96		
350.0	91.44	14.424	1.89	303.50		
400.0	106.68	16.862	1.79	303.21		
450.0	121.92	19.300	1.71	303.01		
500.0	137.16	21.739	1.64	302.86		
600.0	167.64	26.616	1.53	302.68		**100 foot Intervals**
700.0	198.12	31.492	1.45	302.56		
800.0	228.60	36.369	1.38	302.49		
900.0	259.08	41.246	1.32	302.44		
1000.0	289.56	46.123	1.27	302.40		
1100.0	320.04	51.000	1.23	302.37		
1200.0	350.52	55.876	1.19	302.35		
1300.0	381.00	60.753	1.16	302.33		
1400.0	411.48	65.630	1.13	302.31		
1500.0	441.96	70.507	1.11	302.30		
1600.0	472.44	75.384	1.08	302.29		
1700.0	502.92	80.260	1.06	302.28		
1800.0	533.40	85.137	1.04	302.28		
1900.0	563.88	90.014	1.02	302.27		
2000.0	594.36	94.891	1.00	302.27		

*Summer Max = Monthly Mean of Maximum Daily Temperatures for 1971-2000 (Highest in July)

NOAA Sources: Climatography of the United States No.81 *"Monthly Station Normals of Temperatures, Precipitation, and Heating and Cooling Degree Days, 1971-2000 California"* and Climatography of the United States No. 20 *"Monthly Station Climate Summaries, 1971-2000 California"*

SINGLE/Approximated Plume Average Vertical Velocities for WBGF Chillers using CEC Staff Methodology - Winter Min*

Based on 63 chillers w/ 20 cells/chiller. Calc' eff.diam for each chiller wiith each cell at 34" ID (200,110 ACFM total for each chiller).

***"Aviation Safety and Buoyant Plumes,"* Peter Best, et. al.**

***"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia,"* Dr. K.T. Spillane**

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F		0.3048 meters/feet	
Plume Exit Conditions:					Gravity g	9.81 m/s^2	
Stack Height h_s	31.50	meters	103.35	feet	λ	1.11	
Individual Chiller Stack Diameter D	3.8621	meters	152.1	inches	λ_o	~1.0	
Stack Velocity V_{exit}	8.06	m/s	26.45	ft/sec	$4Vol/(60\pi D^2)$		
Individual Chiller Volumetric Flow	94.44	cu.m/sec	200,110	ACFM	$\pi V_{exit}D^2/4$		Sect.2/¶1
Stack Potential Temp θ_s	289.26	Kelvins	61.0	°F			
Initial Stack Buoyancy Flux F_o	11.3279	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$		Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)		
Number of Chillers n	63			2.817	Multiple Stack Multiplication Factor ($n^{0.25}$)		
Conditions at End (Top) of Jet Phase:							
Height above Stack z_{jet}	24.138	meters*	79.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top		Sect.3/¶1
Height above Ground $z_{jet}+h_s$	55.639	meters	**182.5**	feet			"
Vertical Velocity V_{jet}	4.031	m/s	13.22	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$		"
Plume Top-Hat Diameter $2a_{jet}$	7.724	meters	25.3	feet	$2a_{jet}$ = 2D	Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase							
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:							
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height		Sect.2/Eq.6
Virtual Source Height z_v	0.468	meters*	1.5	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z_v+h_s	31.969	meters	104.9	feet		where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9806
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a		Sect.2.1(6)
Product $(Va)_o$	15.265	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$		
Single Chiller Results:							
Solve for plume-averaged vertical velocity at height		**940.0**	**feet**	286.512	meters above ground ($z'+h_s$)		
Gives the following Height above Stack z'	255.011	meters*	836.7	feet*			
Plume Top-Hat Diameter 2a'	81.454	meters	267.2	feet	$2a'=2*0.16(z'-z_v)$		Sect.2/Eq.6
Vertical Velocity V	**1.104**	**m/s**	3.62	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$		Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**			**Critical VV < Top of Jet**	
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)		
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where		
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=	-2.22917
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=	-4584.19
Find Height above Stack z_{crit}	16.537	meters	54.3	feet			http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	48.039	meters	**157.6**	feet		gives the real solution x = z-zv =	17.3892
						or z(m/above stack) =	17.857
						z(ft/above ground) =	161.9

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 103.4	**0.00**	**1.931**	**8.06**			
120.0	5.07	2.337	7.21		**Jet Phase Eqs:**	**20 ft Intervals**
140.0	11.17	2.825	6.20		Linearly interpolated from Stack Rel.Ht to Top of Jet	
160.0	17.27	3.312	5.18		**Spillane Equations:**	
Single Jet 5.3 m/s Height = 157.6	**16.54**	**3.254**	**5.30**			
180.0	23.36	3.800	4.16		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
Top of Single jet = 182.5	**24.14**	**3.862**	**4.03**		$a = 0.16(z-z_v)$	
200.0	29.46	4.639	3.40	282.49	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
220.0	35.55	5.614	2.93	281.71	**CEC Staff Equation:**	
240.0	41.65	6.589	2.61	280.97	$V_{mp}=n^{0.25}V_{sp}$	
260.0	47.75	7.565	2.38	280.45	**Brigg's Equation:**	
280.0	53.84	8.540	2.20	280.06	$V_{Brigg's} = (2/3) \times 1.6^{(3/2)} \times F_{mp}^{(1/2)} \times u^{(-1/2)} \times z^{(-1/2)}$	
300.0	59.94	9.515	2.07	279.77	where $F_{mp} = nF_{sp}$	
320.0	66.03	10.491	1.96	279.54		
350.0	75.18	11.954	1.83	279.36		
400.0	90.42	14.392	1.67	279.15		**50 ft Intervals**
450.0	105.66	16.831	1.55	278.91	**Max<5.3 m/s**	
500.0	120.90	19.269	1.47	278.74		
550.0	136.14	21.707	1.40	278.63		
600.0	151.38	24.146	1.34	278.55		
650.0	166.62	26.584	1.29	278.48		
700.0	181.86	29.023	1.25	278.44		
800.0	212.34	33.899	1.18	278.40		
900.0	242.82	38.776	1.12	278.34		**100 ft Intervals**
1000.0	273.30	43.653	1.08	278.30		
1100.0	303.78	48.530	1.04	278.28		
1200.0	334.26	53.407	1.00	278.26		
1300.0	364.74	58.283	0.97	278.24		
1400.0	395.22	63.160	0.95	278.23		
1500.0	425.70	68.037	0.92	278.22		
2000.0	578.10	92.421	0.83	278.21		
2500.0	730.50	116.805	0.77	278.19		**500 ft Intervals**
3000.0	882.90	141.189	0.72	278.17		
3500.0	1035.30	165.573	0.68	278.17		
4000.0	1187.70	189.957	0.65	278.16		
4500.0	1340.10	214.341	0.63	278.16		
5000.0	1492.50	238.725	0.61	278.16		
*Winter Min = Monthly Mean of Minimum Daily Temperatures for 1971-2000 (Lowest in Dec				278.16		
NOAA Sources: Climatography of the United						

MERGED (along length) Plume Average Vertical Velocities for WBGF Chillers using CEC Staff Methodology - Winter Min*						
	"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.					
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged*					
		***Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**				
Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	31.50	meters	103.35	feet	λ 1.11	
Individual Stack Diameter D	3.86213661	meters	152.1	inches	λ_o ~1.0	
Stack Velocity V_{exit}	8.06	m/s	26.45	ft/sec	$4Vol/(60\pi D^2)$	
Individual Volumetric Flow	94.44	cu.m/sec	200,110	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	289.26	Kelvins	61.0	°F		
Initial Stack Buoyancy Flux F_o	11.33	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Total Number of Stacks n	63					
Average Adjacent Stack Separation d	13.72	meters	45.0	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**	
Number of Stacks along Orientation N	3				**plume velocities increased by $N^{0.25}$ at the height where plumes**	
					fully merged (interp. below ht, single merged stack above ht)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	24.138	meters*	79.2	feet*	$z_{jet} = 6.25D$, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	55.639	meters	**182.5**	feet		"
Vertical Velocity V_{jet}	**4.031**	m/s	13.22	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	7.724	meters	25.3	feet	$2a_{jet} = 2D$ Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	0.468	meters*	1.5	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	31.969	meters	104.9	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9806
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	Sect.2.1(6)
Product $(Va)_o$	15.265	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$	
Plume Merging - Based on Single Plume Calculations where:						Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	13.720	meters	45.0	feet	$2a_{touch}=d$, (or $a_{touch}=d/2$)	
Height above Stack z_{touch}	43.343	meters*	142.2	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{touch}+h_s$	74.844	meters	**245.6**	feet		
Vertical Velocity V_{touch}	**2.541**	m/s	8.3	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**27.440**	meters	90.0	feet	$2a_{full}=2d(N-1)/2$, (or $a_{full}=d(N-1)/2$) **FOR 2 STACKS, $2a_{full}=2d$**	
Height above Stack z_{full}	86.218	meters*	282.9	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{full}+h_s$	117.719	meters	**386.2**	feet		
Vertical Velocity V_{full}	1.705	m/s	5.6	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o [(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$	
Product $(V^3a)_{full}$	68	m^4/s^3				
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):						
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height	
Revised Merged Plume **Radius** a_m	38.654	meters	126.8	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs	
Revised Merged Plume Velocity V_m	**4.802**	m/s	15.76	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs	
Revised Virtual Source Height z_{full}	86.218	meters*	282.9	feet*	Height above stack where Total Merging Occurs (shown above)	
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation	
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$	
Multiple Plume Calculations					for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height		**940.0**	**feet**	286.512	meters above ground $(z+h_s)$	
Gives the following Height above Stack z	255.011	meters*	836.7	feet*	REGULAR EQNS	
Plume Top-Hat Radius a	65.660	meters	215.4	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**	
Vertical Velocity V	**4.025**	**m/s**	13.20	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**	
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**	
					V'=single plume values **if $z<z_{touch}$**	
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s**		BEFORE TOUCHING	**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	JET	meters	JET	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**	
Height above Ground $z_{crit}+h_s$	JET	meters	**JET**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**	

Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	Vert. Vel(m/s)		
				Single Plume Eqns (see Single Plume spreadsheet)	
				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
				$a = 0.16(z-z_v)$	
Begin Merging (touch) = 245.6	***43.36***	***6.860***	***2.54***	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
260.0	47.75	#N/A	2.77	*Interpolated Layer Eqns*	**20 ft Intervals**
280.0	53.84	#N/A	3.09	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
300.0	59.94	#N/A	3.42		
320.0	66.03	#N/A	3.74		
340.0	72.13	#N/A	4.06		
360.0	78.23	#N/A	4.38		
380.0	84.32	#N/A	4.70		
400.0	90.42	#N/A	5.02		
End Merging (full/mp) = 386.2	***86.21***	***38.654***	***4.80***	*Merged Plume Eqns*	
450.0	105.66	41.764	4.68	$V=\{n(V^3a)_{full}/a\}^{1/3}$	**50 ft Intervals**
500.0	120.90	44.202	4.59	$a=a_m+0.16(z-z_{full})$	
550.0	136.14	46.641	4.51		
600.0	151.38	49.079	4.43		
650.0	166.62	51.518	4.36		
700.0	181.86	53.956	4.30		**100 ft Intervals**
800.0	212.34	58.833	4.17		
900.0	242.82	63.710	4.07		
1000.0	273.30	68.586	3.97		
1100.0	303.78	73.463	3.88		
1200.0	334.26	78.340	3.79		
1300.0	364.74	83.217	3.72		
1400.0	395.22	88.094	3.65		
1500.0	425.70	92.970	3.58		**500 ft Intervals**
2000.0	578.10	117.354	3.32		
2500.0	730.50	141.738	3.11		
3000.0	882.90	166.122	2.95		
3500.0	1035.30	190.506	2.82		
4000.0	1187.70	214.890	2.71		
4500.0	1340.10	239.274	2.62		
5000.0	1492.50	263.658	2.53		

MERGED (along width) Plume Average Vertical Velocities for WBGF Chillers using CEC Staff Methodology - Winter Min*

"Aviation Safety and Buoyant Plumes," Peter Best, et. al.

"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia," Dr. K.T. Spillane

Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	31.50	meters	103.35	feet	λ 1.11	
Individual Stack Diameter D	3.86213661	meters	152.1	inches	λ_o ~1.0	
Stack Velocity V_{exit}	8.06	m/s	26.45	ft/sec	$4Vol/(60\pi D^2)$	
Individual Volumetric Flow	94.44	cu.m/sec	200,110	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	289.26	Kelvins	61.0	°F		
Initial Stack Buoyancy Flux F_o	11.33	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Total Number of Stacks n	63					
Average Adjacent Stack Separation d	6.10	meters	20.0	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**	
Number of Stacks along Orientation N	21				**plume velocities increased by $N^{0.25}$ at the height where plumes**	
					fully merged (interp. below ht, single merged stack above ht)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	24.138	meters*	79.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	55.639	meters	**182.5**	feet		"
Vertical Velocity V_{jet}	**4.031**	m/s	13.22	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	7.724	meters	25.3	feet	$2a_{jet}$ = 2D Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	0.468	meters*	1.5	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	31.969	meters	104.9	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9806
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	Sect.2.1(6)
Product $(Va)_o$	15.265	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$	
Plume Merging - Based on Single Plume Calculations where:						Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	6.100	meters	20.0	feet	$2a_{touch}$=d, (or a_{touch}=d/2)	
Height above Stack z_{touch}	19.531	meters*	64.1	feet*	$z_{touch} = z_v$+d/(2*0.16), meters*=meters above stack top	
Height above Ground $z_{touch}+h_s$	51.032	meters	**167.4**	feet		
Vertical Velocity V_{touch}	**4.876**	m/s	16.0	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**122.000**	meters	400.3	feet	$2a_{full}$=2d(N-1)/2, (or a_{full}=d(N-1)/2) **FOR 2 STACKS, $2a_{full}$=2d**	
Height above Stack z_{full}	381.718	meters*	1252.4	feet*	$z_{full} = z_v$+2d/(2*0.16), meters*=meters above stack top	
Height above Ground $z_{full}+h_s$	413.219	meters	**1355.7**	feet		
Vertical Velocity V_{full}	0.959	m/s	3.2	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o [(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$	
Product $(V^3a)_{full}$	54	m^4/s^3				
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):						
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height	
Revised Merged Plume **Radius** a_m	171.856	meters	563.8	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs	
Revised Merged Plume Velocity V_m	**2.703**	m/s	8.87	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs	
Revised Virtual Source Height z_{full}	381.718	meters*	1252.4	feet*	Height above stack where Total Merging Occurs (shown above)	
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation	
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$	
Multiple Plume Calculations					for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height	**940.0**	**feet**	286.512		meters above ground $(z+h_s)$	
Gives the following Height above Stack z	255.011	meters*	836.7	feet*	LESS THAN TOP OF MERGING PHASE-INTERPOLATE	
Plume Top-Hat Radius a	#N/A	meters	#N/A	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**	
Vertical Velocity V	**3.463**	**m/s**	11.36	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**	
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**	
					V'=single plume values **if $z<z_{touch}$**	
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s**			BEFORE TOUCHING	**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	JET	meters	JET	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**	
Height above Ground $z_{crit}+h_s$	JET	meters	**JET**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**	

Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:

Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**Vert. Vel(m/s)**		
				Single Plume Eqns (see Single Plume spreadsheet)	
				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
				$a = 0.16(z-z_v)$	
Begin Merging (touch) = 167.4	***19.52***	***3.050***	***4.88***	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
180.0	23.36	#N/A	4.85	*Interpolated Layer Eqns*	**20 ft Intervals**
200.0	29.46	#N/A	4.82	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
220.0	35.55	#N/A	4.78		
240.0	41.65	#N/A	4.74		
260.0	47.75	#N/A	4.71		
280.0	53.84	#N/A	4.67		
300.0	59.94	#N/A	4.63		**50 ft Intervals**
350.0	75.18	#N/A	4.54		
400.0	90.42	#N/A	4.45		
450.0	105.66	#N/A	4.36		
500.0	120.90	#N/A	4.27		
550.0	136.14	#N/A	4.18		
600.0	151.38	#N/A	4.08		
650.0	166.62	#N/A	3.99		
700.0	181.86	#N/A	3.90		
800.0	212.34	#N/A	3.72		**100 ft Intervals**
900.0	242.82	#N/A	3.54		
1000.0	273.30	#N/A	3.35		
1100.0	303.78	#N/A	3.17		
1200.0	334.26	#N/A	2.99		
End Merging (full/mp) = 1355.7	***381.72***	***171.856***	***2.70***	*Merged Plume Eqns*	
1300.0	364.74	169.139	2.72	$V=\{n(V^3a)_{full}/a\}^{1/3}$	
1400.0	395.22	174.016	2.69	$a=a_m+0.16(z-z_{full})$	
1500.0	425.70	178.893	2.67		
2000.0	578.10	203.277	2.56		
2500.0	730.50	227.661	2.46		**500 ft Intervals**
3000.0	882.90	252.045	2.38		
3500.0	1035.30	276.429	2.31		
4000.0	1187.70	300.813	2.24		
4500.0	1340.10	325.197	2.19		
5000.0	1492.50	349.581	2.13		

SINGLE/Approximated Plume Average Vertical Velocities for WBGF Chillers using CEC Staff Methodology - Summer Max*								
Based on 63 chillers w/ 20 cells/chiller. Calc' eff.diam for each chiller wiith each cell at 34" ID (220,110 ACFM total for each chiller).	***"Aviation Safety and Buoyant Plumes,"*** **Peter Best, et. al.**							
	"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume							
		from a Gas-Turbine Power Station at Oakey, Queensland, Australia," **Dr. K.T. Spillane**						
Ambient Conditions					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F		0.3048	meters/feet	
Plume Exit Conditions					Gravity g	9.81	m/s^2	
Stack Height h_s	31.50	meters	103.35	feet	λ	1.11		
Individual Chiller Stack Diameter D	3.8621	meters	152.1	inches	λ_o	~1.0		
Stack Velocity V_{exit}	8.06	m/s	26.45	ft/sec	$4Vol/(60\pi D^2)$			
Individual Chiller Volumetric Flow	94.44	cu.m/sec	200,110	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	313.32	Kelvins	104.3	°F				
Initial Stack Buoyancy Flux F_o	10.4540	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Number of Chillers n	63			2.817	Multiple Stack Multiplication Factor ($n^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	24.138	meters*	79.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	55.639	meters	**182.5**	**feet**				"
Vertical Velocity V_{jet}	4.031	m/s	13.22	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	7.724	meters	25.3	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	0.432	meters*	1.4	feet*	$6.25D[1-(\theta_a/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	31.933	meters	104.8	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_a/\theta_s)^{1/2}$=	0.9821
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o[(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	15.289	m^2/s			$V_{exit}D/2(\theta_a/\theta_s)^{1/2}$			
Single Chiller Results								
Solve for plume-averaged vertical velocity at height		**1,000.0**	**feet**	304.8	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	273.299	meters*	896.7	feet*				
Plume Top-Hat Diameter 2a'	87.318	meters	286.5	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**1.050**	**m/s**	3.44	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	**feet**	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=		-2.05720
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=		-4704.55
Find Height above Stack z_{crit}	16.537	meters	54.3	feet			http://www.1728.org/cubic.htm	
Height above Ground $z_{crit}+h_s$	48.039	meters	**157.6**	**feet**			gives the real solution x = z-zv =	17.4707
							or z (m/above stack) =	17.902
							z (ft/above ground) =	162.1
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet)	**(meters)**	**Plume**	**Single Stk**	**Plume**				
above ground	**above stack**	**Radius(m)**	**VertVel(m/s)**	**Temp(K)**				
Stack.Rel.Ht = 103.4	**0.00**	**1.931**	**8.06**					
120.0	5.07	2.337	7.21				**Jet Phase Eqs:**	**20 ft Intervals**
140.0	11.17	2.825	6.20				Linearly interpolated from Stack Rel.Ht to Top of Jet	
160.0	17.27	3.312	5.18				**Spillane Equations:**	
Single Jet 5.3 m/s Height = 157.6	**16.54**	**3.254**	**5.30**					
180.0	23.36	3.800	4.16				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
Top of Single jet = 182.5	**24.14**	**3.862**	**4.03**				$a = 0.16(z-z_v)$	
200.0	29.46	4.644	3.40	306.56			$\theta_p=\theta_a(1+(1-(\theta_a/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
220.0	35.55	5.620	2.92	305.78			**CEC Staff Equation:**	
240.0	41.65	6.595	2.59	305.04			$V_{mp}=n^{0.25}V_{sp}$	
260.0	47.75	7.570	2.36	304.53			**Brigg's Equation:**	
280.0	53.84	8.546	2.18	304.15			$V_{Briggs} = (2/3) \times 1.6^{(2/2)} \times F_{mp}^{(1/2)} \times u^{(-1/2)} \times z^{(-1/2)}$	
300.0	59.94	9.521	2.04	303.85			where $F_{mp} = nF_{sp}$	
320.0	66.03	10.497	1.93	303.62				
350.0	75.18	11.960	1.79	303.44				
400.0	90.42	14.398	1.63	303.23				**50 ft Intervals**
450.0	105.66	16.836	1.52	302.98			**Max<5.3 m/s**	
500.0	120.90	19.275	1.43	302.82				
550.0	136.14	21.713	1.36	302.70				
600.0	151.38	24.152	1.31	302.62				
650.0	166.62	26.590	1.26	302.55				
700.0	181.86	29.028	1.22	302.50				
800.0	212.34	33.905	1.15	302.46				
900.0	242.82	38.782	1.09	302.41				**100 ft Intervals**
1000.0	273.30	43.659	1.05	302.37				
1100.0	303.78	48.536	1.01	302.34				
1200.0	334.26	53.412	0.98	302.32				
1300.0	364.74	58.289	0.95	302.30				
1400.0	395.22	63.166	0.92	302.29				
1500.0	425.70	68.043	0.90	302.28				
2000.0	578.10	92.427	0.81	302.27				
2500.0	730.50	116.811	0.75	302.25				**500 ft Intervals**
3000.0	882.90	141.195	0.70	302.24				
3500.0	1035.30	165.579	0.67	302.23				
4000.0	1187.70	189.963	0.64	302.22				
4500.0	1340.10	214.347	0.61	302.22				
5000.0	1492.50	238.731	0.59	302.22				
*Winter Min = Monthly Mean of Minimum Daily Temperatures for 1971-2000 (Lowest in De				302.22				
NOAA Sources: Climatography of the United								

MERGED (along length) Plume Average Vertical Velocities for WBGF Chillers using CEC Staff Methodology - Summer Max*								
	"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged*							
		***Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Stack Height h_s	31.50	meters	103.35	feet	λ	1.11		
Individual Stack Diameter D	3.86213661	meters	152.1	inches	λ_o	~1.0		
Stack Velocity V_{exit}	8.06	m/s	26.45	ft/sec	$4Vol/(60\pi D^2)$			
Individual Volumetric Flow	94.44	cu.m/sec	200,110	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	313.32	Kelvins	104.3	°F				
Initial Stack Buoyancy Flux F_o	10.45	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Total Number of Stacks n	63							
Average Adjacent Stack Separation d	13.72	meters	45.0	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**			
Number of Stacks along Orientation N	3				**plume velocities increased by $N^{0.25}$ at the height where plumes**			
					fully merged (interp. below ht, single merged stack above ht)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	24.138	meters*	79.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	55.639	meters	**182.5**	feet				"
Vertical Velocity V_{jet}	**4.031**	m/s	13.22	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	7.724	meters	25.3	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	0.432	meters*	1.4	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	31.933	meters	104.8	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9821
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	15.289	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$			
Plume Merging - Based on Single Plume Calculations where:								Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	13.720	meters	45.0	feet	$2a_{touch}$=d, (or a_{touch}=d/2)			
Height above Stack z_{touch}	43.307	meters*	142.1	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top			
Height above Ground $z_{touch}+h_s$	74.808	meters	**245.4**	feet				
Vertical Velocity V_{touch}	**2.521**	m/s	8.3	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**27.440**	meters	90.0	feet	$2a_{full}=2d(N-1)/2$, (or $a_{full}=d(N-1)/2$) **FOR 2 STACKS, $2a_{full}$=2d**			
Height above Stack z_{full}	86.182	meters*	282.7	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top			
Height above Ground $z_{full}+h_s$	117.683	meters	**386.1**	feet				
Vertical Velocity V_{full}	1.673	m/s	5.5	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o [(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$			
Product $(V^3a)_{full}$	64	m^4/s^3						
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):								
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height			
Revised Merged Plume **Radius** a_m	38.654	meters	126.8	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs			
Revised Merged Plume Velocity V_m	**4.713**	m/s	15.46	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs			
Revised Virtual Source Height z_{full}	86.182	meters*	282.7	feet*	Height above stack where Total Merging Occurs (shown above)			
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation			
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$			
Multiple Plume Calculations							for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height		**1,000.0**	**feet**	304.8	meters above ground $(z+h_s)$			
Gives the following Height above Stack z	273.299	meters*	896.7	feet*	REGULAR EQNS			
Plume Top-Hat Radius a	68.592	meters	225.0	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**			
Vertical Velocity V	**3.893**	**m/s**	12.77	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**			
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**			
					V'=single plume values **if $z<z_{touch}$**			
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s**		BEFORE TOUCHING		**Critical VV < Top of Jet**	
Find Height above Stack z_{crit}	JET	meters	JET	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**			
Height above Ground $z_{crit}+h_s$	JET	meters	**JET**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**			

Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:				*Single Plume Eqns (see Single Plume spreadsheet)*	
Height (feet)	**(meters)**	**Plume**	**Vert.**	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
above ground	**above stack**	**Radius(m)**	**Vel(m/s)**	$a = 0.16(z-z_v)$	
Begin Merging (touch) = 245.4	***43.30***	***6.860***	***2.52***	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
260.0	47.75	#N/A	2.75	*Interpolated Layer Eqns*	**20 ft Intervals**
280.0	53.84	#N/A	3.06	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
300.0	59.94	#N/A	3.37		
320.0	66.03	#N/A	3.68		
340.0	72.13	#N/A	3.99		
360.0	78.23	#N/A	4.31		
380.0	84.32	#N/A	4.62		
400.0	90.42	#N/A	4.93		
End Merging (full/mp) = 386.1	***86.18***	***38.654***	***4.71***	*Merged Plume Eqns*	
450.0	105.66	41.770	4.59	$V=\{n(V^3a)_{full}/a\}^{1/3}$	**50 ft Intervals**
500.0	120.90	44.208	4.51	$a=a_m+0.16(z-z_{full})$	
550.0	136.14	46.647	4.43		
600.0	151.38	49.085	4.35		
650.0	166.62	51.523	4.28		
700.0	181.86	53.962	4.22		**100 ft Intervals**
800.0	212.34	58.839	4.10		
900.0	242.82	63.715	3.99		
1000.0	273.30	68.592	3.89		
1100.0	303.78	73.469	3.81		
1200.0	334.26	78.346	3.72		
1300.0	364.74	83.223	3.65		
1400.0	395.22	88.099	3.58		
1500.0	425.70	92.976	3.52		**500 ft Intervals**
2000.0	578.10	117.360	3.25		
2500.0	730.50	141.744	3.06		
3000.0	882.90	166.128	2.90		
3500.0	1035.30	190.512	2.77		
4000.0	1187.70	214.896	2.66		
4500.0	1340.10	239.280	2.57		
5000.0	1492.50	263.664	2.49		

MERGED (along width) Plume Average Vertical Velocities for WBGF Chillers using CEC Staff Methodology - Summer Max*						
	***"Aviation Safety and Buoyant Plumes*," Peter Best, et. al.**					
	"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged					
		***Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**				
Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	31.50	meters	103.35	feet	λ 1.11	
Individual Stack Diameter D	3.86213661	meters	152.1	inches	λ_o ~1.0	
Stack Velocity V_{exit}	8.06	m/s	26.45	ft/sec	$4Vol/(60\pi D^2)$	
Individual Volumetric Flow	94.44	cu.m/sec	200,110	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	313.32	Kelvins	104.3	°F		
Initial Stack Buoyancy Flux F_o	10.45	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Total Number of Stacks n	63					
Average Adjacent Stack Separation d	6.10	meters	20.0	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**	
Number of Stacks along Orientation N	21				**plume velocities increased by $N^{0.25}$ at the height where plumes**	
					fully merged (interp. below ht, single merged stack above ht)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	24.138	meters*	79.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	55.639	meters	**182.5**	feet		"
Vertical Velocity V_{jet}	**4.031**	m/s	13.22	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	7.724	meters	25.3	feet	$2a_{jet}$ = 2D Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	0.432	meters*	1.4	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	31.933	meters	104.8	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9821
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	Sect.2.1(6)
Product $(Va)_o$	15.289	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$	
Plume Merging - Based on Single Plume Calculations where:						Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	6.100	meters	20.0	feet	$2a_{touch}$=d, (or a_{touch}=d/2)	
Height above Stack z_{touch}	19.494	meters*	64.0	feet*	$z_{touch} = z_v$+d/(2*0.16), meters*=meters above stack top	
Height above Ground $z_{touch}+h_s$	50.995	meters	**167.3**	feet		
Vertical Velocity V_{touch}	**4.893**	m/s	16.1	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**122.000**	meters	400.3	feet	$2a_{full}$=2d(N-1)/2, (or a_{full}=d(N-1)/2) **FOR 2 STACKS, $2a_{full}$=2d**	
Height above Stack z_{full}	381.682	meters*	1252.2	feet*	$z_{full} = z_v$+2d/(2*0.16), meters*=meters above stack top	
Height above Ground $z_{full}+h_s$	413.183	meters	**1355.6**	feet		
Vertical Velocity V_{full}	0.934	m/s	3.1	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o [(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$	
Product $(V^3a)_{full}$	50	m^4/s^3				
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):						
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height	
Revised Merged Plume **Radius** a_m	171.856	meters	563.8	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs	
Revised Merged Plume Velocity V_m	**2.633**	m/s	8.64	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs	
Revised Virtual Source Height z_{full}	381.682	meters*	1252.2	feet*	Height above stack where Total Merging Occurs (shown above)	
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation	
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$	
Multiple Plume Calculations					for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height	**1,000.0**	**feet**	304.8		meters above ground $(z+h_s)$	
Gives the following Height above Stack z	273.299	meters*	896.7	feet*	LESS THAN TOP OF MERGING PHASE-INTERPOLATE	
Plume Top-Hat Radius a	#N/A	meters	#N/A	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**	
Vertical Velocity V	**3.309**	**m/s**	10.86	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**	
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**	
					V'=single plume values **if $z<z_{touch}$**	
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s**			BEFORE TOUCHING	**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	JET	meters	JET	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**	
Height above Ground $z_{crit}+h_s$	JET	meters	**JET**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**	
Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:					*Single Plume Eqns (see Single Plume spreadsheet)*	
Height (feet)	**(meters)**	**Plume**	**Vert.**		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
above ground	**above stack**	**Radius(m)**	**Vel(m/s)**		$a = 0.16(z-z_v)$	
Begin Merging (touch) = 167.3	***19.49***	***3.050***	***4.89***		$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
180.0	23.36	#N/A	4.87		*Interpolated Layer Eqns*	**20 ft Intervals**
200.0	29.46	#N/A	4.83		$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
220.0	35.55	#N/A	4.79			
240.0	41.65	#N/A	4.76			
260.0	47.75	#N/A	4.72			
280.0	53.84	#N/A	4.68			
300.0	59.94	#N/A	4.64			**50 ft Intervals**
350.0	75.18	#N/A	4.55			
400.0	90.42	#N/A	4.45			
450.0	105.66	#N/A	4.36			
500.0	120.90	#N/A	4.26			
550.0	136.14	#N/A	4.17			
600.0	151.38	#N/A	4.07			
650.0	166.62	#N/A	3.98			
700.0	181.86	#N/A	3.88			
750.0	197.10	#N/A	3.78			
800.0	212.34	#N/A	3.69			**100 ft Intervals**
850.0	227.58	#N/A	3.59			
900.0	242.82	#N/A	3.50			
950.0	258.06	#N/A	3.40			
1000.0	273.30	#N/A	3.31			
1100.0	303.78	#N/A	3.12			
1200.0	334.26	#N/A	2.93			
End Merging (full/mp) = 1355.6	***381.69***	***171.856***	***2.63***		*Merged Plume Eqns*	
1400.0	395.22	174.022	2.62		$V=\{n(V^3a)_{full}/a\}^{1/3}$	
1500.0	425.70	178.899	2.60		$a=a_m+0.16(z-z_{full})$	
2000.0	578.10	203.283	2.49			**500 ft Intervals**
2500.0	730.50	227.667	2.40			
3000.0	882.90	252.051	2.32			
3500.0	1035.30	276.435	2.25			
4000.0	1187.70	300.819	2.18			
4500.0	1340.10	325.203	2.13			