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## APPENDIX A - SPECIAL FEATURES

## Measure, CF1R Documentation Requirement <br> General

Battery System kWh, Special feature
Community Solar: kWdc of [utility and project name], Special feature
Controlled-Ventilation Crawlspace (CVC), Not yet implemented
PV System kWdc, Special feature
PV module type: Premium, Special feature
PV module type: Thin Film, Special feature
PV array type: Tracking (one axis), Special feature
PV array type: Tracking (two axis), Special feature
PV power electronics: Microinverters, Special feature
PV power electronics: DC power optimizers, Special feature
PV exception 1: Effective solar access < $80 \mathrm{ft2}$, Special feature
PV exception 2: Smaller of solar access and home area-based size (CZ 15 only), Special feature

PV exception 3: 2 habitable stories, Special feature
PV exception 4: 3 habitable stories, Special feature
PV exception 5: 80-200 ft2 solar ready zone approved before $1 / 1 / 20$, Special feature
PV exception 6: AB 178 Declared emergency area, Special feature
Self-utilization credit, Special feature
Zonal heating controls, Special feature

## Envelope

Insulation above roof deck, Special feature
Advanced wall framing (see opaque surface constructions), Special feature
Insulation below roof deck, Special feature
Building air leakage/reduced infiltration, HERS verification of reported ACH50 value
Ceiling has high level of insulation, Special feature

Cool roof, Special feature
Dynamic glazing, Not yet implemented
Exterior shading device, Not yet implemented
Exposed slab floor in conditioned zone, Special feature
Metal-framed assembly, Special feature
Window overhangs and sidefins, Special feature
Quality insulation installation (QII), HERS verification
High R-value Spray Foam Insulation, HERS verification
Raised heel truss (height above top plate), Special feature
Structurally insulated panel (SIP) assembly, Special feature

## Mechanical

Fan Efficacy Watts/CFM, HERS verification
Minimum Airflow, HERS verification
Central fan ventilation cooling, fixed speed, HERS verification
Central fan ventilation cooling, variable speed, HERS verification
Verified EER, HERS verification
Evaporatively-cooled condenser, HERS verification
Evaporative cooling, indirect, indirect/direct, Not yet implemented
Verified heat pump rated heating capacity, HERS verification
Verified HSPF, HERS verification
Verified SEER, HERS verification
Indoor air quality mechanical ventilation, HERS verification
Indoor air quality, balanced fan, Special feature
Kitchen range hood, HERS verificationNo cooling system installed, Special feature
Pre-cooling credit, Special feature
Verified Refrigerant Charge), HERS verification
Refrigerant charge verification required if a refrigerant containing component is altered, HERS verification

Whole house fan airflow and fan efficacy, HERS verification
Whole house fan, Special feature

## Ducts

Duct design specifies buried duct, HERS verification

Bypass duct conditions in zonal system(s), HERS verification
Duct design specifies deeply buried duct, HERS verification
Duct leakage testing, HERS verification
Ducts located entirely in conditioned space confirmed by duct leakage testing, HERS verification
Ducts in crawl space, Special feature
Duct sealing required if a duct system component, plenum, or air handling unit is altered, HERS verification

Ducts with high level of insulation, Special feature
Low leakage air handling unit, HERS verification
Verified low leakage ducts in conditioned space must meet maximum 25 cfm leakage to outside (RA3.1.4.3.8), HERS verification
New ductwork added is less than 40 ft . in length, Special feature
Non-standard duct leakage target, HERS verification
Non-standard duct location (any location other than attic), Special feature
Verified duct design (RA3.1.4.1.1), HERS verification

## Water Heating

Compact distribution system basic credit, Special feature
Compact distribution system expanded credit, HERS verification
Drain water heat recovery system, HERS verification
Multifamily: Drain water heat recovery system, HERS verification
Multifamily: Recirculating demand control, Special feature
Multifamily: No loops or recirc pump, Special feature
Multifamily: Recirculating with no control (continuous pumping), Special feature
Multifamily: Recirculating with temperature modulation, Special feature
Multifamily: Recirculating with temperature modulation and monitoring, Special feature
Solar water heating credit, Multi-family, Special feature
Central parallel piping, Special feature
Central parallel piping, HERS verification
Pipe Insulation, All Lines, HERS verification
Point of use, Special feature
Recirculating with demand control, occupancy/ motion sensor, Special feature

Recirculation, demand control occupancy/motion, HERS verification
Recirculating with demand control, push button, Special feature
Recirculation, demand control push button, HERS verification
Recirculating with non-demand control (continuous pumping), Special feature
Solar water heating credit, single family, Special feature
Northwest Energy Efficiency Alliance (NEEA) rated heat pump water heater; specific brand/model, or equivalent, must be installed, Special feature

## Additions/Alterations

Verified existing conditions, HERS verification

## APPENDIX B - WATER HEATING CALCULATION METHOD

## B1. Purpose and Scope

This appendix documents the methods and assumptions used for calculating the hourly energy use for residential water heating systems for both the proposed design and the standard design. The hourly fuel and electricity energy use for water heating will be combined with hourly space heating and cooling energy use to come up with the hourly total fuel and electricity energy use to be factored by the hourly TDV energy multiplier. The calculation procedure applies to low-rise single family, low-rise multifamily, and high-rise residential.
Calculations are described below for gas and electric water heaters. The internal water heater modeling is performed within the CSE simulation engine. The compliance modeling rules documented here are implemented in the CBECC-Res ruleset and determine the input values passed to CSE.
When buildings have multiple water heaters, the hourly total water heating energy use is the hourly water heating energy use summed over all water heating systems, all water heaters, and all dwelling units being modeled.
The following diagrams illustrate the domestic hot water (DHW) system types that shall be recognized by the compliance software.

| Option \# | Description | Diagram |
| :---: | :---: | :---: |
| 1 | One distribution system with one or multiple water heaters serving a single dwelling unit. The system might include recirculation loops within the dwelling unit. |  |
| 2 | Two water heaters with independent distribution systems serving a single dwelling unit. One or more of the distribution systems may include a recirculation loop within the dwelling unit. |  |
| 3 | One distribution system without recirculation loop and with one or multiple water heaters serving multiple dwelling units. |  |


| Option \# | Description | Diagram |  |  |
| :---: | :--- | :--- | :--- | :--- |
| 4 | One distribution system with one or <br> multiple recirculation loops and with <br> one or multiple water heaters <br> serving multiple dwelling units. |  |  |  |

## Source: NORESCO for California Energy Commission

## B2. Water Heating Systems

Water heating distribution systems may serve more than one dwelling unit and may have more than one water heating appliance. The energy used by a water heating system is calculated as the sum of the energy used by each individual water heater in the system. Energy used for the whole building is calculated as the sum of the energy used by each of the water heating systems. To delineate different water heating elements several indices are used.
i Used to describe an individual dwelling unit. For instance, CFA $_{i}$ would be the conditioned floor area of the $i^{\text {th }}$ dwelling unit. Nunit is the total number of dwelling units.
j Used to refer to the number of water heaters in a system. NWH is the total number of water heaters.
k Used to refer to a water heating system or distribution system. A building can have more than one system and each system can have more than one water heater.
I Used to refer to the $I^{\text {th }}$ unfired- or indirectly-fired storage tank in the $k^{\text {th }}$ system. $N L_{k}$ is the total number of unfired- or indirectly-fired storage tanks in the $\mathrm{k}^{\text {th }}$ system. Temperature buffering tanks with electric heating (e.g., minitanks) shall not to be treated as unfired or indirectly-fired storage tanks.
CFA - Conditioned floor area, $\mathrm{ft}^{2}$
NFloor - Number of floors in building
Nunit - Number of units in building
NK - Number of water heating systems
$\mathrm{NWH}_{\mathrm{k}}$ - Number of water heaters in $\mathrm{k}^{\text {th }}$ system
NLoop $_{k}$ - Number of recirculation loops in kth system (multi-unit dwellings only)
CFA $A_{i}$ - Conditioned floor area of ith dwelling unit
$C F A U_{k}$ - Average unit conditioned floor area served by kth system, $\mathrm{ft}^{2}$

## B3. Hot Water Consumption

The schedule of hot water use that drives energy calculations is derived from measured data as described in Appendix F (Kruis, 2019). That analysis produced 365 day sets of fixture
water draw events for dwelling units having a range of number of bedrooms. The draws are defined in the file DHWDU.TXT (for single-family) that installs with CBECC-Res. Each draw is characterized by a start time, duration, flow rate, and end use. The flow rates given are the total flow at the point of use (fixture or appliance). This detailed representation allows derivation of draw patterns at 1-minute intervals as is required for realistic simulation of heat pump water heaters.
The fixture flow events are converted to water heater (hot water) draws by (1) accounting for mixing at the point of use and (2) accounting for waste and distribution heat losses:

VSk-VDk $x \int \operatorname{dur} x V Q k x \int \operatorname{hot} V$
Equation 1

## Where

$V S_{k}=\quad$ Hot water draw at the $k^{\text {th }}$ water heating system's delivery point (gal)
$V D_{k}=\quad$ Mixed water draw duration at an appliance or fixture $(\mathrm{min})$ served by the $\mathrm{k}^{\text {th }}$ water heating system, as specified by input schedule
$V Q_{k}=\quad$ Mixed water flow at an appliance or fixture (gpm) served by the $k^{\text {th }}$ water heating system, as specified by input schedule
$f_{\text {hot }}, f_{\text {dur }}, f_{q}=$ End-use-specific factors from the following:
Shower/bath
$\begin{array}{ll}\boldsymbol{f}_{\text {hot }} & \frac{105-T_{\text {inlet }}}{T_{s}-T_{\text {inlet }}} \\ \boldsymbol{f}_{\text {dur }} & W F_{k} \times D L M_{k}\end{array}$
Faucet
$\boldsymbol{f}_{\text {hot }} 0.50$
$\boldsymbol{f}_{\text {dur }} 1$
Clothes washer
$\boldsymbol{f}_{\text {hot }} 0.22$
$\boldsymbol{f}_{\text {dur }} 1$
Dish washer
$\boldsymbol{f}_{\text {hot }} \quad 1 \boldsymbol{f}_{\text {dur }} 1$
$\mathrm{T}_{\mathrm{s}}=\quad \quad \quad$ Hot water supply temperature ( ${ }^{\circ} \mathrm{F}$ ); assumed to be $115{ }^{\circ} \mathrm{F}$
 may be tempered by drain water heat recovery (DWHR).
$W F_{k}=\quad$ Hot water waste factor

- $W F_{k}=0.9$ for within-dwelling-unit pumped circulation systems (see Table $\mathrm{B}-1)$
- $\mathrm{WF}_{\mathrm{k}}=1.0$ otherwise
$\mathrm{DLM}_{\mathrm{k}}=\quad$ Distribution loss multiplier (unitless), see Equation 5
The individual water heater draws are combined to derive the overall demand for hot water.
For each hour of the simulation, all water heater draws are allocated to 1 minute bins using each draw's starting time and duration. This yields a set of $60 \mathrm{VS}_{k, t}$ values for each hour that is used as input to the detailed heat pump water heater (HPWH) and instantaneous water heater models in later sections. For hourly efficiency-based models used for some water heater types, the minute-by-minute values are summed to give an hourly hot water requirement:

$$
G P H_{k}=\sum_{t=1}^{60} V S_{k, t}
$$

## Equation 2

In cases where multiple dwelling units are served by a common water heating system, the dwelling unit draws are summed.
In cases where there are multiple water heating systems within a dwelling unit, the draws are divided equally among the systems. For minute-by-minute draws, this allocation is accomplished by assigning draws to systems in rotation within each end use weighted by the number of fixtures of each type are served by each system. This ensures that some peak draw events within each end use get assigned to each system. Since heat pump water heater performance is non-linear with load (due to activation of resistance backup), allocation of entire events to systems is essential. Note that the assignment scheme allocates draws by end use as opposed to specific draws to specific systems. Explicit draw assignment would require plumbing layout information - capturing that is deemed to impose an unacceptable user input burden.

## B4. Hourly Adjusted Recovery Load

The hourly-adjusted recovery load for the kth water heating system is calculated as:

$$
\begin{equation*}
H A R L_{k}=H S E U_{k}+H R D L_{k}+\sum_{1}^{N L_{k}} H J L_{l} \tag{Equation 3}
\end{equation*}
$$

## where

$\operatorname{HSEU}_{\mathrm{k}}=\quad$ Hourly standard end use at all use points (Btu), see Equation 4 HRDLk = Hourly recirculation distribution loss (Btu), see Equation 14
15 ; $H R D L_{k}$ is non-zero only for multi-family central water heating systems
$N L_{k}=\quad \quad$ Number of unfired or indirectly-fired storage tanks in the $k^{\text {th }}$ system
$H J_{\mathrm{I}}=\quad$ Tank surface losses of the $\mathrm{l}^{\text {th }}$ unfired tank of the $\mathrm{k}^{\text {th }}$ system (Btu), see Equation 4345

Equation 4 calculates the hourly standard end use (HSEU). The heat content of the water delivered at the fixture is the draw volume in gallons (GPH) times the temperature rise DT (difference between the cold water inlet temperature and the hot water supply temperature) times the heat required to elevate a gallon of water $1^{\circ} \mathrm{F}$ (the 8.345 constant).

$$
H S E U_{k}=8.345 \times G P H_{k} \times\left(T_{s}-T_{i n l e t}\right)
$$

## where

$\operatorname{HSEU}_{\mathrm{k}}=\quad$ Hourly standard end use (Btu)
$\mathrm{GPH}_{\mathrm{k}}=\quad$ Hourly hot water consumption (gallons) from Error! Reference source not found.
Equation 5 calculates the distribution loss multiplier (DLM) which combines the standard distribution loss multiplier (SDLM), which depends on the floor area of the dwelling unit and the distribution system multiplier (DSM).

$$
D L M_{k}=1+\left(S D L M_{k}-\mathbf{1}\right) \times D S M_{k} \quad \text { Equation } 5
$$

where
DLM ${ }_{k}=\quad$ Distribution loss multiplier (unitless)
$\mathrm{SDLM}_{\mathrm{k}}=\quad$ Standard distribution loss multiplier (unitless), see Equation 6
DSM $_{k}=\quad$ Distribution system multiplier (unitless), see Section 3.2Several relationships depend on CFA ${ }_{k}$, the floor area served (see below).

Equation 6 calculates the standard distribution loss multiplier (SDLM) based on dwelling unit floor area. Note that in Equation 6, that floor area $\mathrm{CFAU}_{k}$ is capped at $2500 \mathrm{ft}^{2}$--without that limit, Equation 6 produces unrealistic SDLM ${ }_{k}$ values for large floor areas.

$$
\mathrm{SDLM}_{\mathrm{k}}=1.0032=0.0001864 \times \mathrm{CFAU}_{\mathrm{k}}-0.00000002165 \times \mathrm{CFAU}_{\mathrm{k}}^{2} \quad \text { Equation } 6
$$

where
SDLM $_{\mathrm{k}}=$ Standard distribution loss multiplier (unitless).
$C F A U_{k}=$ Dwelling unit conditioned floor area (ft2) served by the kth system, calculated using methods specified in Equation 7.
Single dwelling unit,

$$
C F A U_{k}=C F A / N K
$$

For multiple dwelling units served by a central system:

$$
C F A U_{k}=\frac{\sum_{\text {all units served by system } \mathrm{k}} C F A_{i}}{\text { Nunit }_{k}}
$$

Alternatively, if the system-to-unit relationships not known:

$$
C F A U_{k}=\frac{\sum_{\text {all units served by any central system }} C F A_{i}}{\text { Number of units served by any central system }}
$$

Equation 7
Method WH-

## CFAU

Note: "Method" designations are invariant tags that facilitate cross-references from comments in implementation code.
When a water heating system has more than one water heater, the total system load is assumed to be shared equally by each water heater, as shown in Equation 8.

$$
H A R L_{j}=\frac{H A R L_{k}}{N W H_{k}}
$$

where
$\operatorname{HARL}_{j}=\quad$ Hourly adjusted recovery load for the $j^{\text {th }}$ water heater of the $k^{\text {th }}$ system (Btu)
HARL $_{k}=\quad$ Hourly adjusted total recovery load for the $k^{\text {th }}$ system (Btu)
$\mathrm{NWH}_{\mathrm{k}}=\quad$ The number of water heaters in the $\mathrm{k}^{\text {th }}$ system

## B4.1. Distribution Losses within the Dwelling Unit

The distribution system multiplier (DSM, unitless) is an adjustment for alternative water heating distribution systems within the dwelling unit. A value of 1.00 for "standard" distribution systems, defined as a non-recirculating system with the following mandatory requirements:
The full length of distribution piping is insulated in accordance with Section 150.0(j)2.

$$
D S M_{k}=A D S M_{k} \times C F_{k}
$$

Equation 9

## Where

$A D S M_{k}=$ Assigned Distribution System Multiplier, see below.
$C F_{\mathrm{k}}=\quad$ Compactness factor (unitless), default value is 1.0; calculated according to Section 5.6.2.4 of the Residential Compliance Manual.

ADSM values for alternative distribution systems are given in Table B-1. Improved ADSM values are available for cases where voluntary HERS inspections are completed, as per the eligibility criteria shown in Reference Residential Appendix RA4.4. Detailed descriptions of all of the distribution system measures are found in Residential Appendix RA 4.4.

Table B-1. Distribution System Multipliers within a Dwelling Unit with One or More Water Heaters

| Distribution System Types | Assigned <br> Distributio <br> n System <br> Multiplier <br> (ADSM) | Syste <br> m <br> Type 1 <br> and 2 | System <br> Type 3 <br> and 4 |
| :--- | :---: | :---: | :---: |
| No HERS Inspection Required | 1.0 | Yes | Yes |
| Trunk and Branch -Standard (STD) | 1.10 | Yes |  |
| Central Parallel Piping (PP) | 0.30 | Yes |  |
| Point of Use (POU) | $9.0^{*}$ | Yes |  |
| Recirculation: Non-Demand Control Options (R- <br> ND) | $1.75^{*}$ | Yes |  |
| Recirculation with Manual Demand Control (R- <br> DRmc) | $2.60^{*}$ | Yes |  |
| Recirculation with Motion Sensor Demand Control <br> (R-DRsc) | 0.85 | Yes | Yes |
| Optional Cases: HERS Inspection Required | 1.00 | Yes |  |
| Pipe Insulation (PIC-H) | 0.70 | Yes |  |
| Central Parallel Piping with 5' maximum length <br> (PP-H) | $1.60^{*}$ | Yes |  |
| Compact Design (CHWDS-H) | $2.40^{*}$ | Yes |  |
| Recirculation with Manual Demand Control (R- <br> DRmc-H) | Recirculation with Motion Sensor Demand Control <br> (RDRsc-H) |  |  |

*Recirculation ADSMs reflect impact of reduced hot water consumption associated with recirculation systems.
Source: California Energy Commission

## B4.2. Cold Water Inlet Temperature

The water heater inlet temperature is assumed to vary daily and depends on mains water temperature, drain water heat recovery, and solar preheating.
For each day of the year, $\mathrm{T}_{\text {mains }}$ is calculated as follows:

$$
\text { Tmain }=\text { Tground } x 0.65+\text { Tavg31 } \times 0.035
$$

$\mathrm{T}_{\text {avg31 }}=\quad$ Outdoor dry-bulb temperature averaged over all hours of the previous 31 days (note for January days, weather data from December will be used)
$\mathrm{T}_{\text {ground }}=\quad$ Ground temperature ( ${ }^{\circ} \mathrm{F}$ ) for current day of year, calculated using Error! Reference source not found.
each day ( $q=1$ TO 365)
$T_{\text {ground }}(\theta)=$
TyrAve $-0.5 \times($ TyrMax - TyrMin $) \times \operatorname{COS}(2 \times \pi \times((\theta-1) / P B)-P O-P H I) \times G M$
Equation 11
where
TyrAve $=$ average annual temperature, ${ }^{\circ} \mathrm{F}$
TyrMin $=$ the lowest average monthly temperature, ${ }^{\circ} \mathrm{F}$
TyrMax $=$ the highest average monthly temperature, ${ }^{\circ} \mathrm{F}$
$\mathrm{PB}=365$
$\mathrm{PO}=0.6$
DIF $\quad=0.025 \mathrm{ft}^{2} / \mathrm{hr}$
BETA $=\operatorname{SQR}(\mathrm{p} /($ DIF*PB*24)$) * 10$
$X B=E X P(-B E T A)$
$C B=\operatorname{COS}(B E T A)$
$\mathrm{SB}=\operatorname{SIN}(B E T A)$
$\mathrm{GM}=\operatorname{SQR}((\mathrm{XB} * \mathrm{XB}-2 . * X B * \mathrm{CB}+1) /(2 . * B E T A * B E T A))$
PHI $=\operatorname{ATN}\left(\left(1 .-X B^{*}(C B+S B)\right) /\left(1 .-X B^{*}(C B-S B)\right)\right)$
The water heater inlet temperature, $T_{\text {inlet, }}$, is calculated as follows:

$$
T_{\text {inlet }}=\left(1-S S F_{k}\right)\left(T_{\text {mains }}+\Delta T_{d w h r}\right)+S S F_{k} \times T_{S}
$$

Equation 11
where
SSF ${ }_{k}=$ Solar savings fraction for kth system (see below), unitless
$\Delta T_{d w h r}=\quad$ Water temperature increase due to drain water heat recovery, ${ }^{\circ} \mathrm{F}$ ( 0 if no DWHR). See Section B4.3.
$T_{s}=$ Hot water supply temperature
All water heaters in a water heating system are assumed to have the same $T_{\text {inlet }}$.
The hourly solar savings fraction for the kth water heating system, $\mathrm{SSF}_{\mathrm{k}}$, is the fraction of the total water heating load that is provided by solar hot water heating. The annual average value for SSF is provided from the results generated by the Energy Commission approved calculations approaches for the OG-100 and OG-300 test procedure. A Commission-approved
method shall be used to convert the annual average value for SSF to hourly SSFk values for use in compliance calculations.

## B4.3. Drain Water Heat Recovery

Drain water heat recovery (DWHR) devices are heat exchangers that transfer heat from warm drain water to incoming cold (mains) water. These operate on draws where supply and drain flow are simultaneous - for example, showers (as opposed to dishwashers). In CBECC-Res, only shower draws support DWHR. Several plumbing configurations are possible.

Heat exchanger output is connected to both shower water heater cold sides. Best performance.

Designated "Equal flow" because the heat exchanger potable and drain sides have the same flow (assuming no simultaneous other draws)


Heat exchanger output is connected to water heater cold side.

Designated "Unequal flow water heater"


Heat exchanger output is connected to shower cold side. Easiest to construct.

Designated "Unequal flow shower"


In practice, there are many combination plumbing configurations that are possible. For example, only some showers may drain via DWHR devices or more than one shower may drain via a shared DHWR device. CBECC-Res input structure allows flexible specification of such arrangements.
The drain water heat recovery temperature increase, $\Delta T_{d w h r, ~ i s ~ m o d e l e d ~ w i t h i n ~ C S E ~ u s i n g ~}^{\text {in }}$ effectiveness derived using correlations presented in:

- Drain Water Heat Recovery - Final Report. Measure Number: 2019-RES-DHW2-F. Available at http://title24stakeholders.com/wp-content/uploads/2017/09/2019-T24-CASE-Report_DWHR_Final_September-2017.pdf
- Explanation of Drain Water Heat Recovery Calculations. NegaWatt Consulting. Dec. 13, 2017.
DWHR is supported only for shower draws. Based on experimental data, the effectiveness correlation is function of potable water flow rate, potable water entering temperature, and drain water flow rate, as shown here:

$$
\begin{aligned}
t_{p i} & =\min \left(t_{\text {mains }}, 81\right) \\
f_{t} & =\left(-3.06 \times 10^{-5} t_{p i}{ }^{2}+4.96 \times 10^{-3} t_{p i}+0.281\right) / 0.466 \\
f_{v} & =-6.98484455 \times 10^{-4} v_{p}{ }^{4}+1.28561447 \times 10^{-2} v_{p}{ }^{3}-7.02399803 \times 10^{-2} v_{p}{ }^{2} \\
& +1.33657748 \times 10^{-2} v_{p}+1.23339312 \\
\varepsilon & =\left[0,\left(1+0.3452 \ln \left(v_{d} / v_{p}\right) f_{t} f_{v} \varepsilon_{\text {rated }}, 0.95\right]\right.
\end{aligned}
$$

Equation 12

## where

$t_{p i}=$ DWHR potable water inlet temperature, ${ }^{\circ} \mathrm{F}$
$v_{p}=$ Potable volume flow rate, gpm. $v_{p}$ depends on the plumbing configuration and is various combinations of the fixture hot water draw, the fixture cold water draw, and the total hot water draw.
$v_{d}=$ Drain volume flow rate, gpm. The drain volume is equal to the total (mixed) draws of fixture(s) evaluated not including four (see Equation 1) since no heat can be recovered during warmup.
$e=$ DWHR effectiveness under current conditions, unitless
$e_{\text {rated }}=\quad$ DWHR rated effectiveness $=$ efficiency $/ 100$, rated at CSA B55.1 conditions ( 9.5 Ipm, equal flow)
The effectiveness, $e$, is used to calculate the potable water temperature increase.

$$
\Delta T_{d w h r}=\frac{\varepsilon \min \left(v_{p}, v_{d}\right)\left(t_{d}-t_{p i}\right)}{v_{p}}
$$

where
$t_{d}=$ DWHR drain-side entering temperature, ${ }^{\circ} \mathrm{F}=$ shower use temperature $\left(105{ }^{\circ} \mathrm{F}\right)-4.6$ ${ }^{\circ} \mathrm{F}$. The latter adjustment approximates heat loss between the shower and the DWHR device.

Note that in this model with some plumbing configurations, effectiveness depends on $v_{p}$ and $v_{p}$ depends of effectiveness. An iterative solution technique is required to find consistent conditions.
When only some shower fixtures within a dwelling unit drain via a DWHR system, savings are assumed to be proportional to the number of included shower fixtures. This is implemented by assigning shower draws in rotation to DWHR or non-DWHR arrangements.

## B5. Hourly Distribution Loss for Central Water Heating Systems

This section is applicable to the DHW system type 3 and 4, as defined in B1. The distribution losses accounted for in the distribution loss multiplier (DLM), Equation 5, reflect distribution heat loss within each individual dwelling unit. Additional distribution losses occur outside dwelling units and they include losses from recirculation loop pipes and branch piping feeding individual dwelling units. The hourly values of these losses, HRDL, shall be calculated according to Equation 17
. Compliance software shall provide input for specifying recirculation system designs and controls according to the following algorithms.

$$
H R D L_{k}=N \operatorname{Loop}_{k} \times H R L L_{k}+H R B L_{k}
$$

Equation 14

## where

HRDL $_{k}=\quad$ Hourly central system distribution loss for $\mathrm{k}^{\text {th }}$ system (Btu).
HRLL $=\quad$ Hourly recirculation loop pipe heat loss (Btu). This component is only applicable to system type 4, see Equation 15
HRBL ${ }_{k}=\quad$ Hourly recirculation branch pipe heat loss (Btu), see Equation 23NLoop $k=$
Number of recirculation loops in water heating system k; this component is only applicable to system type 4, see Section 4.3
A recirculation loop usually includes multiple pipe sections, not necessarily having the same diameter, that are exposed to different ambient conditions. The compliance software shall provide input entries for up to six pipe sections with three sections for supply piping and three sections for return piping for users to describe the configurations of the recirculation loop. For each of the six pipe sections, input entries shall include pipe diameter (inch), pipe length ( ft ), and ambient conditions. Ambient condition input shall include three options: outside air, underground, conditioned or semi-conditioned air. Modeling rules for dealing with recirculation loop designs are provided in Section 4.3.
Outside air includes crawl spaces, unconditioned garages, unconditioned equipment rooms, as well as actual outside air. Solar radiation gains are not included in the calculation because the impact of radiation gains is relatively minimal compared to other effects. Additionally,
the differences in solar gains for the various conditions (e.g., extra insulation vs. minimum insulation) are even less significant.
The ground condition includes any portion of the distribution piping that is underground, including that in or under a slab. Insulation in contact with the ground must meet all the requirements of Section 150.0(j), Part 6, of Title 24.
The losses to conditioned or semi-conditioned air include losses from any distribution system piping that is in an attic space, within walls (interior, exterior or between conditioned and unconditioned spaces), within chases on the interior of the building, or within horizontal spaces between or above conditioned spaces. It does not include the pipes within the residence. The distribution piping stops at the point where it first meets the boundaries of the dwelling unit.

## B5.1. Hourly Recirculation Loop Pipe Heat Loss Calculation

Hourly recirculation loop pipe heat loss (HRLLk) is the hourly heat loss from all six pipe sections. There are two pipe heat loss modes, pipe heat loss with non-zero water flow (PLWF) and pipe heat loss without hot water flow (PLCD). The latter happens when the recirculation pump is turned off by a control system and there are no hot water draw flows, such as in recirculation return pipes.

Compliance software shall provide four options of recirculation system controls listed in Table B-2 or Table B-3. A proposed design shall select a control type from one of the four options. The standard design shall use demand control.
Table B-2. Recirculation Loop Supply Temperature and Pump Operation Schedule (With No Control or Demand Control)

| Hour | No Control <br> Temperature | No Control <br> Input for <br> $\mathbf{S C H}_{\mathbf{k}, \mathbf{m}}$ | Demand <br> Control <br> Temperature | Demand <br> Control <br> Input for $^{\text {SCH }} \mathbf{k} \mathbf{m}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 through 24 | 130 | 1 | 130 | 0.2 |

Source: NORESCO for California Energy Commission
Table B-3. Recirculation Loop Supply Temperature and Pump Operation Schedule (With Temperature Modulation Control)

| Hour | Without <br> Continuous <br> Monitoring <br> Temperature | Without <br> Continuous <br> Monitoring <br> Input for <br> $\mathbf{S C H}_{\mathbf{k}, \mathbf{m}}$ | With <br> Continuous <br> Monitoring <br> Temperature | With <br> Continuous <br> Monitoring <br> Input for <br> $\mathbf{S C H}_{\mathbf{k}, \mathbf{m}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 through 5 | 120 | 1 | 115 | 1 |
| 6 | 125 | 1 | 120 | 1 |
| 7 through | 130 | 1 | 125 | 1 |
| 23 |  |  |  |  |


| Hour | Without <br> Continuous <br> Monitoring <br> Temperature | Without <br> Continuous <br> Monitoring <br> Input for <br> $\mathbf{S C H}_{k, m}$ | With <br> Continuous <br> Monitoring <br> Temperature | With <br> Continuous <br> Monitoring <br> Input for <br> $\mathbf{S C H}_{k, m}$ |
| :---: | :---: | :---: | :---: | :---: |
| 24 | 125 | 1 | 120 | 1 |

Source: NORESCO for California Energy Commission
Pipe heat loss modes are determined by recirculation control schedules and hot water draw schedules. For each pipe section, hourly pipe heat loss is the sum of heat loss from the two heat loss modes.

Hourly heat loss for the whole recirculation loop (HRLL ${ }_{k}$ ) is the heat loss from all six pipe sections, according to the following equation:

$$
H R L L_{k}=\sum_{n}\left[P L W F_{n}+P L C D_{n}\right]
$$

Equation 15
where
PLWF $_{\mathrm{n}}=\quad$ Hourly pipe heat loss with non-zero water flow (Btu/hr), see Equation 16
$P L C D_{n}=\quad$ Hourly pipe heat loss without water flow (Btu/hr), see Equation 21
$\mathrm{n}=\quad$ Recirculation pipe section index, 1 through 6

$$
P L W F_{n}=\text { Flow }_{n} \times\left(1-f_{\text {noflow }, n}\right) \times \rho \times C_{p} \times\left(T_{n, \text { in }}-T_{n, \text { out }}\right)
$$

Equation 16

## where

Flow $_{\mathrm{n}}=$ Flow $_{\text {recirc }}+$ Flow $_{\mathrm{n}, \text { draw }}(\mathrm{gph})$, assuming
Flow $_{\mathrm{n}, \text { draw }}=$ Average hourly hot water draw flow (gph); for supply sections, $\mathrm{n}=1,2$, or 3, Flow $_{n, \text { draw }}=\mathrm{GPH}_{k} /$ NLoop $_{k}$; for return pipes, $n=4,5$, and 6, Flow $_{n \text {,draw }}=0$
Flow recirc $=$ Hourly recirculation flow (gph) is assumed to be 360 gallons based on the assumption that the recirculation flow rate is 6 gpm
$f_{\text {noflow, } n}=\quad$ Fraction of the hour for pipe section $n$ to have zero water flow, see Equation 17
$\rho=\quad$ Density of water, $8.345(\mathrm{lb} / \mathrm{gal})$
$\mathrm{C}_{\mathrm{p}}=\quad$ Specific heat of water, 1 (Btu/lb- ${ }^{\circ} \mathrm{F}$ )
$T_{n, \text { in }}=\quad$ Input temperature of section $n\left({ }^{\circ}\right)$; for the first section ( $n=1$ ), $T_{1, \text { in }}$ shall be determined based on Table B-2. The control schedule of the proposed design shall be based on user input. The standard design is demand control. For other sections, input temperature is the same as the output temperature the proceeding pipe section, $T_{n, \text { in }}=T_{n-1, \text { out }}$
$\mathrm{T}_{\mathrm{n} \text {,out }}=\quad$ Output temperature of section $\mathrm{n}\left({ }^{\circ} \mathrm{F}\right)$, see Equation 18

$$
f_{\text {noflow }, n}=\left(1-S C H_{k, m}\right) \times \text { NoDraw }_{n}
$$

Equation 17

## where

NoDraw $_{n}=$ Fraction of the hour that is assumed to have no hot water draw flow for pipe section n; NoDraw $1=0.2, \mathrm{NoDraw}_{2}=0.4, \mathrm{NoDraw}_{3}=0.6, \mathrm{NoDraw}_{4}=$ NoDraw $_{5}=$ NoDraw $_{6}=1$
$\mathrm{SCH}_{\mathrm{k}, \mathrm{m}}=\quad$ Recirculation pump operation schedule, representing the fraction of the hour that the recirculation pump is turned off, see Table B-2 or Table B-3. $\mathrm{SCH}_{\mathrm{k}, \mathrm{m}}$ for the proposed design shall be based on proposed recirculation system controls. Recirculation system control for the standard design is demand control.

$$
T_{o u t, n}=T_{a m b, n}+\left(T_{i n, n}-T_{a m b, n}\right) \times e^{-\frac{U A_{n}}{\rho C_{p} F l o w_{n}}}
$$

Equation 18

## where

$\mathrm{T}_{\text {Amb, } \mathrm{n}}=\quad$ Ambient temperature of section $\mathrm{n}\left({ }^{\circ} \mathrm{F}\right)$, which can be outside air, underground, conditioned, or semi-conditioned air. Outside air temperatures shall be the drybulb temperature from the weather file. Underground temperatures shall be obtained from Error! Reference source not found. Hourly conditioned air temperatures shall be the same as conditioned space temperature. For the proposed design, $\mathrm{T}_{\text {amb,n }}$ options shall be based on user input. The standard design assumes all pipes are in conditioned air.
$U A_{n}=\quad$ Heat loss rate of section $n\left(B t u / h r-{ }^{\circ} F\right)$, see Equation 19

$$
U A_{n}=\operatorname{Len}_{n} \times \min \left(U_{\text {bare }, n}, f_{U A} \times U_{\text {insul }, n}\right)
$$

where
Len $_{n}=\quad$ Section $n$ pipe length (ft); for the proposed design, use user input; for the standard design, see Equation 30
$U_{\text {bare, } n,} \mathrm{U}_{\text {insul, } \mathrm{n}}=$ Loss rates for bare (uninsulated) and insulated pipe (Btu/hr-ft- ${ }^{\circ} \mathrm{F}$ ), evaluated using Equation 20 with section-specific values, as follows:
$\operatorname{Dia}_{\mathrm{n}}=\quad$ Section n pipe nominal diameter (inch); for the proposed design, use user input; for the standard design, see Equation 31
Thick $_{n}=\quad$ Pipe insulation minimum thickness (inch) as defined in the Title 24 Section 120.3, TABLE 120.3-A for service hot water system

Cond $_{n}=\quad$ Insulation conductivity shall be assumed $=0.26$ (Btu inch/h•sf•F)
$h_{n}=\quad$ Section $n$ combined convective/radiant surface coefficient (Btu/hr-ft2-F) assumed $=1.5$
$f_{U A}=\quad$ Correction factor to reflect imperfect insulation, insulation material degradation over time, and additional heat transfer through connected branch pipes that is not reflected in branch loss calculation. It is assumed to be 2.0.

Equation 20 defines general relationships used to calculate heat loss rates for both loop and branches using appropriate parameters.

$$
\begin{gathered}
\text { Dia }_{o}=\text { Dia }+0.125 \\
U_{\text {bare }}=h \times \pi \times \frac{\text { Dia }_{o}}{12} \\
\text { Dia }_{x}=\text { Dia }_{o}+2 \times \text { Thick } \\
U_{\text {insul }}=\frac{\pi}{\frac{\ln \left(\frac{\text { Diax }_{x}}{\text { Dia }_{o}}\right)}{\frac{2 \times \text { Cond }}{12}}+\frac{12}{h \times \text { Dia }}}
\end{gathered}
$$

Equation 20
where
Dia $=\quad$ Pipe nominal size (in)
Dia $_{0}=\quad$ Pipe outside diameter (in)
Dia $_{\mathrm{x}}=\quad$ Pipe + insulation outside diameter (in)
Thick $=\quad$ Pipe insulation thickness (in)
Cond $=\quad$ Insulation conductivity (Btu in/hr-ft ${ }^{2}$ - ${ }^{\circ}$ F)
$\mathrm{h}=\quad$ Combined convective/radiant surface coefficient (Btu/hr-ft²- of)
Pipe heat loss without water flow shall be calculated according to the following equations:

$$
P L C D_{n}=V o l_{n} \times \rho \times C_{p} \times\left(T_{n, \text { start }}-T_{n, e n d}\right)
$$

## where

Vol $_{n}=\quad$ Volume of section $n($ gal $)$ is calculated as $7.48 \times \pi \times\left(\frac{D i a_{o}}{24}\right)^{2} \times$ Len $_{n}$ where 7.48 is the volumetric unit conversion factor from cubic feet to gallons. Note that the volume of the pipe wall is included to approximate the heat capacity of the pipe material.
$T_{n, \text { start }}=\quad$ Average pipe temperature ( ${ }^{( } \mathrm{F}$ ) of pipe section $n$ at the beginning of the hour. It is the average of $T_{n, \text { in }}$ and $T_{n, \text { out }}$ calculated according to Equation 18 and associated procedures.
$\mathrm{T}_{\mathrm{n}, \text { end }}=\quad$ Average pipe temperature ${ }^{( } \mathrm{F}$ ) of pipe section n at the end of pipe cool down, see Equation 22

$$
T_{n, e n d}=T_{a m b, n}+\left(T_{n, s t a r t}-T_{a m b, n}\right) \times e^{-\frac{U A_{n} \times f_{n o f l o w, n}}{V o l_{n} \times \rho \times C_{p}}}
$$

Equation 22
Equation 22 calculates average pipe temperature after cooling down, so the pipe heat loss calculated by Equation 21 is for pipe with zero flow for fraction $f_{\text {noflow, }}$ of an hour.
Recirculation pumps are usually turned off for less than an hour and there could be hot water draw flows in the pipe. As a result, recirculation pipes usually cool down for less than an hour. The factor $\mathrm{f}_{\text {noflow, } \mathrm{n}}$ calculated according Equation 17 is used to reflect this effect in Equation 22.

## B5.2. Hourly Recirculation Branch Pipe Heat Loss Calculation

The proposed design and standard design shall use the same branch pipe heat loss assumptions. Branch pipe heat loss is made up of two components. First, pipe heat losses occur when hot water is in use (HBUL). Second, there could be losses associated with hot water waste (HBWL) when hot water was used to displace cold water in branch pipes and hot water is left in pipe to cool down after hot water draws and must be dumped down the drain.

The Total Hourly Branch Losses (HRBLk) shall include both components and be calculated as:

$$
H R B L_{k}=\text { Nbranch }_{k} \times(H B U L+H B W L)
$$

Equation 23

## where

HBUL $=\quad$ Hourly pipe loss for one branch when water is in use (Btu/hr), see Equation 24 HBWL $=\quad$ Hourly pipe loss for one branch due to hot water waste (Btu/hr), see

Equation 27
Nbranch $_{k}=$ Number of branches in water heating system k, see Equation 32

The hourly branch pipe loss while water is flowing is calculated in the same way as recirculation pipe heat loss with non-zero water flow (PLWF) using the following equations:

$$
H B U L=\left(\frac{G P H_{k}}{N B r a n c h_{k}}\right) \times \rho \times C_{p} \times\left(T_{b, \text { in }}-T_{b, o u t}\right)
$$

Equation 24

## where

$\mathrm{T}_{\mathrm{b}, \mathrm{in}}=\quad$ Average branch input temperature ( ${ }^{\circ} \mathrm{F}$ ). It is assumed to be equal to the output temperature of the first recirculation loop section, $\mathrm{T}_{1, \text { out }}$
$\mathrm{T}_{\mathrm{b}, \text { out }}=\quad$ Average branch output temperature $\left({ }^{\circ} \mathrm{F}\right)$, see Equation 25

$$
T_{b, o u t}=T_{a m b, b}+\left(T_{b, \text { in }}-T_{a m b, b}\right) \times e^{-\frac{U A_{b}}{\rho \times C_{p} \times F l o w_{b}}}
$$

where
$\mathrm{T}_{\text {amb,b }}=\quad$ Branch pipe ambient temperature ( ${ }^{\circ} \mathrm{F}$ ) Branch pipes are assumed to be located in the conditioned or semi-conditioned air.
$U A_{b}=\quad$ Branch pipe heat loss rate (Btu/hr- ${ }^{\circ}$ F), see Equation 26
Flow $_{\mathrm{b}}=\quad$ Branch hot water flow rate during use (gal/hr). It is assumed to be 2 gpm or $120 \mathrm{gal} / \mathrm{hr}$.
The branch pipe heat loss rate is

$$
U A_{b}=\operatorname{Len}_{b} \times U_{i n s u l, b}
$$

Equation 26

## where

Len $_{b} \quad=$ Branch pipe length ( ft ), see Equation 34
Uinsul,b $\quad=$ Loss rate for insulated pipe (Btu/hr-ft-F), evaluated using Equation 20 with branch-specific values, as follows:
Dia $\quad=\quad$ Branch pipe diameter (inch), see Equation 33
Thick $_{b}=$ Branch pipe insulation minimum thickness (inch) as defined in the Title 24 Section 120.3, TABLE 120.3-A for service hot water system.
Condb $\quad=\quad$ Branch insulation conductivity, assumed $=0.26 \mathrm{Btu} \mathrm{in} / \mathrm{hr}_{\mathrm{b}}-\mathrm{ft}^{2}$ - ${ }^{\circ} \mathrm{F}$
$\mathrm{h}_{\mathrm{b}} \quad=$ Branch combined convective/radiant surface coefficient (Btu/hr-ft²- ${ }^{\circ}$ ) assumed $=1.5$
The hourly pipe loss for one branch due to hot water waste is calculated as follows:

## $H B W L=$

$$
N_{\text {waste }} \times S C H_{\text {waste }, m} \times f_{\text {vol }} \times 7.48 \times \pi \times\left(\frac{D i a_{b}+0.125}{24}\right)^{2} \times \operatorname{Len}_{b} \times \rho \times C_{p} \times\left(T_{b, \text { in }}-T_{\text {inlet }}\right)
$$

Equation 27

## where

$N_{\text {waste }}=\quad$ Number of times in a day for which water is dumped before use. This depends on the number of dwelling units served by a branch. Statistically, the number of times of hot water waste is wasted is inversely proportional to the number of units a branch serves, see Equation 28.
$\mathrm{SCH}_{\text {waste, } \mathrm{m}}=$ Hourly schedule of water waste, see Table B-4
$\mathrm{f}_{\text {vol }}=\quad$ The volume of hot water waste is more than just the volume of branch pipes, due to branch pipe heating, imperfect mixing, and user behaviors. This multiplier is applied to include these effects and is assumed to be 1.4.
$\mathrm{T}_{\mathrm{in}, \mathrm{b}}=\quad$ Average branch input temperature ( ${ }^{\circ} \mathrm{F}$ ) is assumed to equal the output temperature of the first recirculation loop section, Tout,1
$\mathrm{T}_{\text {inlet }}=\quad$ The cold water inlet temperature ( ${ }^{\circ} \mathrm{F}$ ) according to Section 3.3 Cold Water Inlet Temperature

$$
N_{\text {waste }}=19.84 \times e^{-0.544 \times N u n i t_{b}}
$$

Equation 28
Method WHBRWF
where
Nunit $=\quad$ Number of dwelling units served by the branch, calculated using Equation 29 (note that Nunitb is not necessarily integral).
Nunit $_{b}=\frac{\text { Nfloor }}{2}$ Equation 29
Method WH-BRNU
Table B-4. Branch Water Waste Schedule

| Hour | $\mathbf{S C H}_{\text {waste, } \mathbf{m}}$ |
| :---: | :---: |
| 1 | 0.01 |
| 2 | 0.02 |
| 3 | 0.05 |
| 4 | 0.22 |
| 5 | 0.25 |
| 6 | 0.22 |
| 7 | 0.06 |
| 8 | 0.01 |
| 9 | 0.01 |
| 10 | 0.01 |
| 11 | 0.01 |
| 12 | 0.01 |
| 13 | 0.01 |
| 14 | 0.01 |
| 15 | 0.01 |
| 16 | 0.01 |
| 17 | 0.01 |
| 18 | 0.01 |
| 19 | 0.01 |
| 20 | 0.01 |
| 21 | 0.01 |
| 22 | 0.01 |
| 23 | 0.01 |
| 24 | 0.01 |

Source: NORESCO for California Energy Commission

## B5.3. Recirculation System Plumbing Designs

A recirculation system can have one or multiple recirculation loops. Each recirculation loop consists of many pipe sections, which are connected in sequence to form a loop. Each pipe
section could have different pipe diameter, length, and location. The compliance software shall use six pipe sections, with three supply pipe sections and three return pipe sections, to represent a recirculation loop. When multiple recirculation loops exist, all recirculation loops are assumed to be identical. The compliance software shall provide default and standard recirculation system designs based on building geometry according to the procedures described in the following sections. The default design reflects typical recirculation loop design practices. The standards design is based on one or two loops and is used to set recirculation loop heat loss budget.
The first step of establishing recirculation system designs is to determine the number of recirculation loops, Nloop ${ }_{k}$, in water heating system k . The standard design has one recirculation loop, Nloop ${ }_{k}=1$, when Nunit <=8, or two recirculation loops, Nloop $k=2$ for buildings with Nunit > 8 . The proposed design is allowed to specify more than one loop only if the design is verified by a HERS rater. Otherwise, the proposed design can only be specified to have one recirculation loop.
The standard and default recirculation loop designs are based on characteristics of the proposed building. There could be many possibilities of building shapes and dwelling unit configurations, which would determine recirculation loop pipe routings. Without requiring users to provide detailed dwelling unit configuration information, the compliance software shall assume the proposed buildings to have same dwelling units on each floor and each floor to have a corridor with dwelling units on both sides. Recirculation loops start from the mechanical room (located on the top floor), go vertically down to the middle floor, loop horizontally in the corridor ceiling to reach the dwelling units on both ends of the building, then go vertically up back to the mechanical room. At each dwelling unit on the middle floor, vertical branch pipes, connected to the recirculation loop supply pipe, are used to provide hot water connection to dwelling units on other floors above and below.
Both the standard and default recirculation loop designs are assumed to have equal length of supply sections and return sections. The first section is from the mechanical room to the middle floor. The second section serves first half branches connected to the loop and the third section serves the rest of the branches. The first and second sections have the same pipe diameter. Pipe size for the third section is reduced since less dwelling units are served. Return sections match with the corresponding supply pipes in pipe length and location. All return sections have the same diameter. For both the standard and default designs, mechanical room is optimally located so that only vertical piping is needed between the mechanical room and the recirculation pipes located on the middle floor. Pipe sizes are determined based on the number of dwelling units served by the loop, following the 2009 Uniform Plumbing Code (UPC) pipe sizing guidelines. The detailed recirculation loop configurations are calculated as follows:

Pipe Length in the mechanical room ( ft ): $\quad L_{\text {mech }}=8$
Height of each floor (ft): Hfloor=user input floor-to-floor height (ft)
Length of each dwelling unit (ft):

$$
L_{\text {unit }}=\sqrt{C F A U_{k}} \text { ' } \text { (see Equation } 7 \text { ) }
$$

Length of recirculation pipe sections (ft):

$$
\begin{align*}
& \operatorname{Len}_{1}=\text { Len }_{6}=L_{\text {mech }}+H_{\text {floor }} \times \frac{N f l o o r}{2} \\
& \text { Len }_{2}=\text { Len }_{3}=\text { Len }_{4}=\text { Len }_{5}=L_{\text {unit }} \times \frac{\text { Nunit }_{k}}{4 \times \text { Nloop }_{k} \times N \text { floor }} \tag{Equation 30}
\end{align*}
$$

## Method WH-LOOPLEN

Pipe diameters for recirculation loop supply sections depend on the number of dwelling units being served and return section diameters depend only on building type, as follows:

Dia $_{1}$, Dia ${ }_{2}$, and Dia $_{3}$ : derived from Table B-5 based on Nunit ${ }_{1}$, Nunit ${ }_{2}$, and Nunit ${ }_{3}$
$\mathrm{Dia}_{4}=\mathrm{Dia}_{5}=\mathrm{Dia}_{6}=0.75$ in for low-rise multi-family building and hotel/motel less than four stories
Dia $_{4}=$ Dia $_{5}=$ Dia $_{6}=1.0$ in for high-rise multi-family and hotel/motel more than three stories Equation 31

## Method WH-LOOPSZ

## where

Nunit $_{1}=\quad$ Number of dwelling units served by the loop section $1=\frac{N_{n n i t}^{k}}{\text { Nloop }_{k}}$
Nunit $_{2}=$ Nunit $_{1}$
Nunit $_{3}=\frac{\text { Nunit }_{1}}{2}$
Note that Nunit values are not necessarily integers.
Branch pipe parameters include number of branches, branch length, and branch diameter.
The number of branches in water heating system $k$ is calculated as (note: not necessarily an integer):
Nbranch $_{k}=\frac{\text { Nunit }_{k}}{\text { Nunit }_{b}}$
Equation 32
Method WH-BRN

The branch pipe diameter shall be determined as follows:
Diab: derived from Table B-5 based on Nunit ${ }_{b}$
Equation 33
Method WH-BRSZ

The branch length includes the vertical rise based on the number of floors in the building plus four feet of pipe to connect the branch to the recirculation loop.

Proposed designs shall use the same branch configurations as those in the standard design. Therefore, compliance software does not need to collect branch design information.

Table B-5. Pipe Size Schedule
$\left.\begin{array}{|c|c|c|}\hline \begin{array}{c}\text { Number of } \\ \text { dwelling units } \\ \text { served } \\ \text { NUnit }_{\boldsymbol{n}} \text { or NUnit }\end{array} & \begin{array}{c}\text { Loop pipe } \\ \text { nominal size Dia } \\ \mathbf{n}\end{array} \\ \text { in }\end{array} \quad \begin{array}{c}\text { Branch pipe } \\ \text { nominal size Dia } \\ \text { in } \\ \text { in }\end{array}\right]$

## Source: NORESCO for California Energy Commission

## B6. High Rise Residential Buildings, Hotels and Motels

Simulations for high-rise residential buildings, hotels, and motels shall follow all the rules for central or individual water heating with the following exceptions:

- For central systems which do not use recirculation but use electric trace heaters the program shall assume equivalency between the recirculation system and the electric trace heaters.
- For individual water heater systems which use electric trace heating instead of gas, the program shall assume equivalency.


## B7. Energy Use of Individual Water Heaters

Once the hourly adjusted recovery load is determined for each water heater, the energy use for each water heater is calculated as described below and summed.

## B7.1. Consumer or Residential-Duty Commercial Storage Water Heaters

Storage water heaters are rated either by EF (energy factor) or the newer UEF (Uniform Energy Factor). The calculation algorithm for these devices derives a Load Dependent Energy Factor (LDEF) from EF. For water heaters rated with UEF, CBECC-Res calculates an equivalent $E F$.

The hourly energy use of storage gas water heaters is given by the following equation.

$$
W H E U_{j}=\frac{H A R L_{j} \times H P A F_{j}}{L D E F_{j}}
$$

Equation 35

## where

$\mathrm{WHEU}_{\mathrm{j}}=\quad$ Hourly energy use of the water heater (Btu for fuel or kWh for electric);
Equation 35 provides a value in units of Btu. For electric water heaters, the calculation result needs to be converted to the unit of kWh by dividing 3413 Btu/kWh.
$\operatorname{HARL}_{j}=\quad$ Hourly adjusted recovery load (Btu)
$\mathrm{HPAF}_{j}=\quad 1$ for all non-heat pump water heaters
$L D E F_{j}=\quad$ The hourly load dependent energy factor (LDEF) is given by $L D E F_{j}=\min \left[L D E F \max \left(L D E F \ln \left(\frac{A A H A R L_{j} \times 24}{1000}\right)\left(a \times E F_{j}+b\right)\left(c \times E F_{j}+d\right)_{\min }(0)\right)_{\max }\right]$
Equation 36. This equation adjusts the nominal EF rating for storage water heaters for different load conditions.

$$
L D E F_{j}=\min \left[L D E F \max \left(L D E F \ln \left(\frac{A A H A R L_{j} \times 24}{1000}\right)\left(a \times E F_{j}+b\right)\left(c \times E F_{j}+d\right)_{\min }(O)_{\max }\right]_{\text {Equation } 36}\right.
$$

## where

$a, b, c, d=\quad$ Coefficients from the table below based on the water heater type
Table B-6. LDEF Coefficients

| Coefficient | Storage Gas |
| :---: | :---: |
| a | -0.098311 |
| b | 0.240182 |
| c | 1.356491 |
| d | -0.872446 |
| LDEF $_{\min }$ | .1 |
| LDEF $_{\text {max }}$ | .90 |

Source: NORESCO for California Energy Commission
AAHARL $\mathrm{L}_{\mathrm{j}}=$ Annual average hourly adjusted load (Btu) $=\frac{1}{8760} \sum_{1}^{8760} H A R L_{j}$; calculation of AAHARL ${ }_{j}$ requires a preliminary annual simulation that sums HARL ${ }_{j}$ values for each hour.
$E F_{j}=\quad$ Energy factor of the water heater (unitless). This is based on the DOE test procedure. EF for storage gas water heaters with volume less than 20 gallons must be assumed to be 0.58 unless the manufacturer has voluntarily reported an actual EF to the California Energy Commission.

CBECC-Res derives $E F_{j}$ from UEF for water heaters that are rated using updated DOE procedures.

## B7.2. Consumer and Residential-Duty Commercial Water Heaters

UEF-rated consumer and residential-duty commercial instantaneous water heaters (gas and electric) are modeled on a minute-by-minute basis using procedures documented by Lutz (2019).

## B7.3. Small Instantaneous Gas Water Heaters

The hourly energy use for instantaneous gas or oil water heaters is given by Equation 37, where the nominal rating is multiplied by 0.92 to reflect the impacts of heat exchanger cycling under real world load patterns.

$$
W H E U_{j}=\frac{H A R L_{j}}{E F_{j} \times 0.92}
$$

Equation 37

## where

$\mathrm{WHEU}_{\mathrm{j}}=$ Hourly fuel energy use of the water heater (Btu)
HARL $_{j}=$ Hourly adjusted recovery load
$E F_{j}=$ Energy factor from the DOE test procedure (unitless) taken from manufacturers' literature or from the CEC Appliance Database
$0.92=$ Efficiency adjustment factor

## B7.4. Small Instantaneous Electric Water Heaters

The hourly energy use for consumer instantaneous electric water heaters is given by the following equation.

$$
\begin{equation*}
W H E U_{j, \text { elec }}=\frac{H A R L_{j}}{E F_{j} \cdot 0.92 \cdot 3413} \tag{Equation 38}
\end{equation*}
$$

## where

$\mathrm{WHEU}_{\mathrm{j}, \text { elec }}=$ Hourly electric energy use of the water heater $(\mathrm{kWh})$
HARL $_{j}=\quad$ Hourly adjusted recovery load (Btu)
$E F_{j}=$ Energy factor from DOE test procedure (unitless)
$0.92=$ Adjustment factor to adjust for overall performance
$3413=$ Unit conversion factor (Btu/kWh)

## B7.5. Residential-Duty Commercial Instantaneous Electric Water Heater

## B7.6. Mini-Tank Electric Water Heater

Mini-tank electric heaters are occasionally used with gas tankless water heaters to mitigate hot water delivery problems related to temperature fluctuations that may occur between draws. If mini-tank electric heaters are installed, the installed units must be listed in the CEC Appliance Database and their reported standby loss (in Watts) will be modeled to occur each hour of the year. (If the unit is not listed in the CEC Appliance Database, a standby power consumption of 35 W should be assumed.)

$$
W H E U_{j, e l e c}=M T S B L_{j} / 1000
$$

Equation 39
where
$\mathrm{WHEU}_{\mathrm{j}, \mathrm{elec}}=$ Hourly standby electrical energy use of mini-tank electric water heaters (kWh)
MTSBL $_{j}=$ Mini-tank standby power (W) for tank j (if not listed in CEC Appliance directory, assume 35 W )

## B7.7. Large/Commercial Gas Storage Water Heaters

Energy use for large storage gas is determined by the following equations. Note: large storage gas water heaters are defined as any gas storage water heater with a minimum input rate of 75,000 Btu/h.

$$
W H E U_{j}=\frac{H A R L_{j}}{E F F_{j}}+S B L_{j}
$$

Equation 40
where
$\mathrm{WHEU}_{\mathrm{j}}=$ Hourly fuel energy use of the water heater (Btu)
HARL $_{j}=$ Hourly adjusted recovery load (Btu)
$\mathrm{SBL}_{\mathrm{j}}=$ Total Standby Loss (Btu/hr). Obtain from CEC Appliance Database or from AHRI certification database. This value includes tank losses and pilot energy. If standby rating is not available from either of the two databases, it shall be calculated as per Table F-2 of the 2015 Appliance Efficiency Regulations, as follows:
$\mathrm{SBL}=\mathrm{Q} / 800+110(\mathrm{~V})^{1 / 2}$, where Q is the input rating in Btu/hour, and V is the tank volume in gallons.
$E F F_{j}=$ Efficiency (fraction, not \%). Obtained from CEC Appliance Database or from manufacturer's literature. These products may be rated as a recovery efficiency, thermal efficiency or AFUE.

## B7.8. Large/Commercial Instantaneous, Indirect Gas, and Hot Water Supply Boilers

Energy use for these types of water heaters is given as follows:

$$
W H E U_{j}=\frac{H A R L_{j}}{E F F_{j} \times 0.92}+\text { PILOT }_{j}
$$

where
WHEU $_{j}=$ Hourly fuel energy use of the water heater (Btu), adjusted for tank insulation.
HARL $_{j}=$ Hourly adjusted recovery load. For independent hot water storage tank(s) substitute HARL ${ }_{j}$ from Section B3.
$E F F_{j}=$ Efficiency (fraction, not \%) to be taken from CEC Appliance Database or from manufacturers literature. These products may be rated as a recovery efficiency, thermal efficiency or AFUE.
PILOT $_{j}=$ Pilot light energy (Btu/h) for large instantaneous. For large instantaneous water heaters, and hot water supply boilers with efficiency less than 89 percent assume the default is $750 \mathrm{Btu} / \mathrm{hr}$ if no information is provided in manufacturer's literature or CEC Appliance Database.
$0.92=$ Adjustment factor used when system is not supplying a storage system.

## B7.9. Consumer Storage Electric or Heat Pump Water Heaters

Energy use for small electric water heaters is calculated as described in the HPWHsim Project Report (Ecotope, 2016) ) and in documents specified in Section B6 (see also study by NEEA referenced in Appendix F). The HPWH model uses a detailed, physically based, multi-node model that operates on a one-minute time step implemented using a suitable loop at the time-step level within CSE. Tank heat losses and heat pump source temperatures are linked to the CSE zone heat balance as appropriate. Thus, for example, the modeled air temperature of a garage containing a heat pump water heater will reflect the heat extracted.

HPWH can model three classes of equipment:

- Specific air-source heat pump water heaters identified by manufacturer and model. These units have been tested by Ecotope and measured parameters are built into the HPWH code.
- Generic air-source heat pump water heaters, characterized by EF and tank volume. This approach provides compliance flexibility. The performance characteristics of the generic model are tuned to use somewhat more energy than any specific unit across a realistic range of UEF values.
- Electric resistance water heaters, characterized by EF, tank volume, and resistance element power.

Another issue is that the HPWH hot water output temperature varies based on factors like control hysteresis and tank mixing. For compliance applications, it is required that all system alternatives deliver the same energy. To address this, the HPWH tank setup point is modeled at $125^{\circ} \mathrm{F}$ and delivered water is tempered to $t_{s}$. If the HPWH output temperature is above $t_{s}$, it is assumed that inlet water is mixed with it (thus reducing $V_{i, t}$ ). If the output temperature is below $t_{s}$, sufficient electrical resistance heating is supplied to bring the temperature up to $t_{s}$ (preventing under capacity from being exploited as a compliance advantage).

Several issues arise from integration of a detailed, short time step model into an hourly framework. HPWH is driven by water draw quantities, not energy requirements. Thus, to approximate central system distribution and unfired tank losses, fictitious draws are added to the scheduled water uses, as follows:

$$
V_{j, t}=\frac{V S_{k, t}+\frac{H R D L_{k}+\sum_{1}^{N L_{k}} H J L_{l}}{60 \times 8.345 \times\left(t_{s}-t_{\text {inlet }}\right)}}{N W H_{k}}
$$

Equation 42

## where

$\operatorname{HRDL}_{k}=\quad$ Hourly recirculation distribution loss (Btu), see Equation 14; HRDL ${ }_{k}$ is non-zero only for multi-family central water heating systems
$H J L_{I}=\quad$ Tank surface losses of the $t^{\text {th }}$ unfired tank of the $k^{\text {th }}$ system (Btu), see Equation 43
$V S_{k}=\quad$ Hot water draw at the $\mathrm{k}^{\text {th }}$ water heating system's delivery point (gal)
$V_{j, t}=\quad$ Hot water draw (gal) on $\mathrm{j}^{\text {th }}$ water heater for minute t
Another issue is that the HPWH hot water output temperature varies based on factors such as control hysteresis and tank mixing. For compliance applications, it is required that all system alternatives deliver the same energy. To address this, the HPWH tank setup point is modeled at $125^{\circ} \mathrm{F}$ and delivered water is tempered to $t_{s}$. If the HPWH output temperature is above $t_{s, i t}$ is assumed that inlet water is mixed with it (thus reducing $V_{i, t}$ ). If the output temperature is below $t_{s}$, sufficient electrical resistance heating is supplied to bring the temperature up to $t_{s}$ (preventing under sizing from being exploited as a compliance advantage).

## B7.10. Jacket Loss

The hourly jacket loss for the $t^{\text {th }}$ unfired tank or indirectly fired storage tank in the $\mathrm{k}^{\text {th }}$ system is calculated as:

$$
H J L_{l}=\frac{T S A_{l} \times \Delta T S}{R T I_{l}+R E I_{l}}+F T L_{l}
$$

Equation 43
where
HJL $\quad=\quad$ The tank surface losses of the $t^{\text {th }}$ unfired tank of the $\mathrm{k}^{\text {th }}$ system

TSA $=$ Tank surface area (ft²), see Equation 44
$\Delta \mathrm{TS}=$ Temperature difference between ambient surrounding tank and hot water supply temperature ( ${ }^{\circ}$ ). Hot water supply temperature shall be $124{ }^{\circ}$ F. For tanks located inside conditioned space use $75^{\circ} \mathrm{F}$ for the ambient temperature. For tanks located in outside conditions use hourly dry bulb temperature ambient.
$\mathrm{FTL}_{\mathrm{I}}=$ Fitting losses; a constant 61.4 Btu/h
REI $_{\mid} \quad=\quad$ R-value of exterior insulating wrap; no less than $\mathrm{R}-12$ is required
$R T I_{1}=R$-value of insulation internal to water heater; assume 0 without documentation

Tank surface area (TSA) is used to calculate the hourly jacket loss (HJL) for unfired or indirectly fired tanks. TSA is given in the following equation as a function of the tank volume.

$$
T S A_{l}=\left(1.254 \times V O L_{l}^{0.33}+.531\right)^{2}
$$

Equation 44
where
VOL $=$ Tank capacity (gal)

## B7.11. Electricity Use for Circulation Pumping

For single-family recirculation systems, hourly pumping energy is fixed as shown in Table B7.

Multifamily recirculation systems typically have larger pump sizes, and therefore electrical energy use is calculated based on the installed pump size. The hourly recirculation pump electricity use (HEUP) is calculated by the hourly pumping schedule and the power of the pump motor as in the following equation.

$$
H E U P_{k}=\frac{0.746 \times P U M P_{k} \times S C H_{k, m}}{\eta_{k}}
$$

Equation 45

## where

HEUP $_{\mathrm{k}}=$ Hourly electricity use for the circulation pump (kWh)
PUMP $_{\mathrm{k}}=$ Pump brake horsepower (bhp)
$\eta_{\mathrm{k}} \quad=$ Pump motor efficiency
$\mathrm{SCH}_{\mathrm{k}, \mathrm{m}}=\quad$ Operating schedule of the circulation pump, see Table B-2. The operating schedule for the proposed design shall be based on user input control method. The standard design operation schedule is demand control.

Table B-7. Single Family Recirculation Energy Use (kWh) by Hour of Day

| Hour | Non-Demand <br> Controlled <br> Recirculation | Demand- <br> Controlled <br> Recirculation |
| :---: | :---: | :---: |
| 1 | 0.040 | 0.0010 |
| 2 | 0.040 | 0.0005 |
| 3 | 0.040 | 0.0006 |
| 4 | 0.040 | 0.0006 |
| 5 | 0.040 | 0.0012 |
| 6 | 0.040 | 0.0024 |
| 7 | 0.040 | 0.0045 |
| 8 | 0.040 | 0.0057 |
| 9 | 0.040 | 0.0054 |
| 10 | 0.040 | 0.0045 |
| 11 | 0.040 | 0.0037 |
| 12 | 0.040 | 0.0028 |
| 13 | 0.040 | 0.0025 |
| 14 | 0.040 | 0.0023 |
| 15 | 0.040 | 0.0021 |
| 16 | 0.040 | 0.0019 |
| 17 | 0.040 | 0.0028 |
| 18 | 0.040 | 0.0032 |
| 19 | 0.040 | 0.0033 |
| 20 | 0.040 | 0.0031 |
| 21 | 0.040 | 0.0027 |
| 22 | 0.040 | 0.0025 |
| 23 | 0.040 | 0.0023 |
| 24 | 0.040 | 0.0015 |
| Annual Total | 350 | 23 |
|  |  |  |

Source: NORESCO for California Energy Commission

## APPENDIX C - PHOTOVOLTAICS

## Photovoltaics

The compliance software calculates energy generated by photovoltaic systems on an hourly basis using System Advisor Model (SAM) algorithms upon which the PVWatts program is based (see Appendix F). The routines are modified to better represent PV systems with and without sub-array power electronics (i.e., microinverters and DC power optimizers).
Power electronics are used to help minimize efficiency losses when the output of sub-array components (e.g., modules or cells) operate under different conditions. The largest driver of variation in conditions across a PV array is partial shading from nearby obstacles. A small fraction of shaded cells could lead to disproportionate reductions in PV power output. PVWatts, does not explicitly handle this effect. Literature describes a shading impact factor (SIF) which is the ratio of relative power output to fraction shaded:

$$
P_{s h}=P_{s y s} \cdot\left(1-S I F \cdot f_{s h}\right)
$$

Where $P_{\text {sh }}$ is the power output of the shaded system, $\mathrm{P}_{\text {sys }}$ is the power output of the unshaded system, and $\mathrm{f}_{\text {sh }}$ is the fraction shaded.

A value of 1.0 implies that the power output declines proportionally to the fraction shaded. This is a theoretical minimum value of SIF in that it implies there are power electronics that are maintaining output consistent with the level of shading across the module. A value greater than 1.0 implies that shading has a disproportionate effect on system output.
How the individual cells within an array are shaded can have a significant impact on SIF. This is illustrated in a study on a PV module without power electronics (see Figure C-1).

In this study the same module was shaded in different fashions (see Figure C-2). For a given shade ratio, the actual output from the PV system can differ by 30 percent depending on which portions of the system are shaded (Note: the dotted lines in the first figure represent SIF values of $1.0\left[y=1-1.0^{*} x\right]$ and $2.0\left[y=1-2.0^{*} x\right]$ and serve as approximate bounds on the impact). Without cell-level fidelity in our shading model, it is impossible to know which specific cells are shaded at any given time. The compliance software will use a coarse approximation of SIF appropriate for panel and/or array level analysis.
SIF should also change with higher levels of irradiance as shown in this study (see Table C2). However, considering the coarseness of array-wide shading fraction (vs. cell-by-cell), accounting for this effect is not likely to provide a substantial increase in overall accuracy.

Figure C-1


Figure C-2


One problem with applying shading impact factors directly to the power output of the system is that there is a theoretical lower limit to PV production under shaded conditions: diffuse irradiance. Unless a cell is also blocked from diffuse solar (e.g., the shade is very close to the panel and blocking cells from the rest of the sky), all cells will receive a minimum level of incidence. To account for this, we propose introducing an alternative formulation using an "effective" plane-of-array incidence, where only the beam component is affected by shading.
$\mathrm{I}_{\text {poa,eff }}=\mathrm{I}_{\text {poa,diff }}+\mathrm{I}_{\text {poa,beam,eff }}$
$\mathrm{I}_{\text {poa, beam,eff }}=\max \left(\mathrm{I}_{\text {poa,beam }} *\left(1-\mathrm{SIF}^{*} \mathrm{f}_{\text {sh }}\right), 0.0\right)$
The compliance software shall use an SIF value of 2.0 for central inverters (CEC default) and a value of 1.2 for systems with power electronics (based on a 40 percent shade loss recovery as defined in this paper--see Table C-2. SIF for Total Inverter Efficiency).

## C1 System Loss Assumptions

In PVWatts, a single derating factor is used to cover a variety of system inefficiencies. The compliance software uses slightly different assumptions for this derating factor as described in the table below:

Table C-1. Derating Factor

| Loss Type | Value | Differences from PVWatts <br> Default Assumptions |
| :--- | :--- | :--- |
| Soiling | 0.02 | N/A |
| Shading | 0.0 | Modeled explicitly |
| Snow | 0.0 | N/A |
| Mismatch | 0.0 | Mismatch from shading is <br> characterized using SIF |
| Wiring | 0.02 | N/A |
| Connections | 0.005 | N/A |
| Light-induced degradation | 0.015 | N/A |
| Nameplate rating | 0.01 | N/A |
| Age | 0.05 | Estimated 0.5 percent <br> degradation over 20 years <br> based on these references: <br> [1, 2, 3] |
| Availability | 0.03 | N/A |
| Total | 0.14 | N/A |

Source: NORESCO for California Energy Commission

## C2 Inverter Efficiency

The software shall characterize the inverter efficiency corresponding to either a central inverter or microinverters depending on the type of power electronics used in the system.

## C3 Power Electronics

Options for power electronics are described below:
Table C-2. SIF for Total Inverter Efficiency

| Option | SIF | Total Inverter Efficiency |
| :--- | :--- | :--- |
| None | 2.0 | User input |
| Microinverters | 1.2 | User input |
| DC Power Optimizers | 1.2 | Optimizer efficiency * user <br> input |

Source: NORESCO for California Energy Commission
Optimizer efficiencies are assumed to be 0.99 (corresponding to suggestions in this document).

## Battery Storage

See Status of Modeling Batteries for California Residential Code Compliance, Appendix D.

# APPENDIX D - STATUS OF MODELING BATTERIES FOR CALIFORNIA SINGLE-FAMILY AND MULTIFAMILY RESIDENTIAL CODE COMPLIANCE 

## D1 Modeling of Residential Battery/PV Systems for Self-Utilization Compliance Credit

## Overview

The California Energy Commission added a self-utilization credit for residential battery systems to its residential building energy efficiency standards for 2019. Under these standards, a residential battery paired with an on-site photovoltaic (PV) system would receive fair credit toward the building's Energy Design Rating (EDR) score. Starting with the 2022 standards, multifamily residential buildings are treated differently than single-family residential buildings and use a different approach to determining compliance. Instead of EDR, compliance for multifamily residential buildings is based on Time Dependent Valuation and source energy. This document defines how the CBECC-Res compliance software will produce the battery EDR credit for single-family residential buildings, and how the CBECC compliance software will produce the battery calculations withing the compliance framework for multifamily residential buildings.

Whereas most energy upgrades reduce energy use in a house or multifamily building, battery systems actually increase electricity consumption in exchange for some shaping of the load. A 14-kWh battery, with a 90\% round-trip efficiency, that cycles 13 of those kWh 300 times a year, will consume 4.1 MWh of electricity and discharge 3.7 MWh of electricity per annum. But by charging when there is excess PV production and discharging when PV production is low and electricity is expensive, the battery both saves money for the residence and provides value to the electricity system overall. Thus, the single-family EDR self-utilization credit and multifamily compliance credit must account not for energy savings, but for savings in value-of-energy.

Distributed electric storage can provide value to the electricity system overall through load shaping and other behaviors. Bolstering demand during low periods helps to leave efficient power plants running full time and reduces ramping requirements. Reducing peak demand helps in a number of ways, including by allowing expensive peaker plants to remain idle more days of the year. In recent years, a growing electricity contribution from solar generation has resulted in what many in California have termed the duck curve (see Figure 1): a dip in net electric load between the morning and evening peaks. Batteries, by filling in the midday trough and shaving the evening peak, can be part of the remedy for a worsening duck curve.


Figure 1: An illustration of how increasing solar production in recent years has created a trough in net electric load between the morning and evening peaks. (Source: https://www. eia.gov/todayinenergy/detail. php?id=32172.)

Using building energy code compliance incentives to encourage the adoption of residential batteries can mitigate the impact of the duck curve at the source: more batteries to soak up plentiful midday solar production, more batteries to serve the evening peak and to mitigate the steep ramp in early evening hours when solar production is going down and demand is rising as people turn on appliances in their homes. In addition, having the batteries on-site can help reduce wear-and-tear on distribution systems.

## D2Time Dependent Valuation and Source Energy

The California Title 24 Building Energy Efficiency Standards have utilized a Time Dependent Valuation (TDV) methodology since 2005 to account for the time value of energy for load and for self-generation credit. TDV is a composite measure of the actual cost of energy (for each of electricity, natural gas, and propane) to the utility, customers, and society at large. It has been crafted for evaluating energy efficiency savings based on when those savings manifest. The TDV sources and methods have recently been updated for 2019 (Ming, et al., 2017).

The TDV concept allows even-footing comparison of a set of time-series simulations of how different building designs use energy. Accordingly, it is the mechanism by which the CBECCRes (for single-family residential) and CBECC (for multifamily residential) compliance software converts a residential battery's load shaping patterns into a self-utilization credit. If a building charges a battery from on-site PV during midday, the simulation foregoes a small TDV credit for power it would have fed to the grid. When the battery discharges in the
evening, it can earn a much larger credit for reducing load when TDV is high. That net-TDV reduction counts toward reducing the single-family residential building's design EDR or the multifamily residential building's performance with respect to the compliance margin.

Starting with the 2022 standards, hourly source energy factors are also used to determine compliance for multifamily residential buildings.

## D3Calculating Compliance

For single-family residential buildings, CBECC-Res calculate compliance for a proposed design based on its energy design ratings (EDRs). An EDR is generally calculated as the annual integrated time-dependent value (TDV) of energy (across fuels) of a building relative to a common reference design (based on the 2006 International Energy Conservation Code) (Wilcox, 2017). Accordingly, the EDR is a unitless score, generally between 0 and 100, with a lower score being better.

EDR scores are based on the total annual-TDV of a design relative to the total annual-TDV of the reference design. Annual-TDV is the sum across hours of the net energy used in that hour multiplied by that hour's TDV ${ }^{1}$.

$$
\text { AnnualTDV }=\sum_{\text {hour }=1}^{8760} T D V(\text { hour }) * \operatorname{load}(\text { hour })
$$

The EDR for a specific design is (generally) calculated as the ratio of the design's annualTDV and the reference building's annual-TDV:

$$
E D R_{\text {design }}=\frac{\text { AnnualTD } V_{\text {design }}}{\text { AnnualTD } V_{\text {reference }}}
$$

Compliance for a proposed design in CBECC-Res has two requirements:

1. The EDR of the building design, ignoring contributions from renewable generation and battery storage (except for the self-utilization credit described below), must be lower than the EDR of the code prescriptive standard design (also ignoring contributions from renewable generation and battery storage). These EDRs are called the "Efficiency EDR" for the respective proposed and standard designs. The intent of this requirement is to encourage designs that reduce loads in addition to generating energy.

[^0]2. The EDR of the final design (including contributions from renewable generation and battery storage) must be lower than the EDR of the code prescriptive standard design (also including contributions from renewable generation and battery storage). These EDRs are called the "Final EDR" for the respective proposed and standard designs.

A minimum of five annual-TDV calculations are required to evaluate the compliance of a specific proposed design:

1. Proposed design "Efficiency" TDV
2. Proposed design "Final" TDV
3. Standard design "Efficiency" TDV
4. Standard design "Final" TDV
5. Reference design TDV (the common denominator in all EDR calculations)

The specific computations that produce the EDR from annual-TDV totals are described in the Single-Family Residential Alternative Compliance Method (ACM) Reference Manual. Because the reference design annual-TDV is used in the calculation of all EDRs, the compliance of a building design can generally be determined when the TDV of the proposed design is lower than the TDV of the standard design, for both the "Efficiency" and "Final" simulations (though this is not strictly the case due to some small adjustment factors used to determine EDR). The standard design is also described in the ACM.

Beginning with the 2022 Energy Code update, code compliance evaluates EDR using two metrics: TDV, and source energy. Similar to TDV, source energy also accounts for hourly variation in how energy is generated and delivered through the grid.

For multifamily residential buildings, CBECC calculates compliance for a proposed design based on its TDV and source energy performance. Two TDV values are calculated by the software for both the proposed and standard design: one value for Total TDV (which accounts for flexibility measures such as PV and battery storage) and one value for Efficiency TDV (which does not account for flexibility measures). Additionally, a total source energy value is calculated for the proposed and standard design (including flexibility measures).

Compliance for a proposed design in CBECC has three requirements:

1. The TDV, ignoring contributions from renewable generation and battery storage (except for the self-utilization credit described below), must be equal or lower than the TDV of the code prescriptive standard design (also ignoring contributions from renewable generation and battery storage). These values are called the "Efficiency TDV" for the respective proposed and standard designs. The intent of this
requirement is to encourage designs that reduce loads in addition to generating energy.
2. The TDV of the final design (including contributions from renewable generation and battery storage) must be equal or lower than the TDV of the code prescriptive standard design (also including contributions from renewable generation and battery storage). These values are called the "Total TDV" for the respective proposed and standard designs.
3. The source energy of the proposed design must be equal or lower than the source energy of the standard design.

A minimum of six annual calculations are required to evaluate the compliance of a specific proposed design:

1. Proposed design "Efficiency" TDV
2. Proposed design "Total" TDV
3. Proposed design source energy
4. Standard design "Efficiency" TDV
5. Standard design "Total" TDV
6. Standard design source energy

The specific computations that produce these values are described in the Nonresidential and Multifamily Alternative Compliance Method (ACM) Reference Manual. The standard design is also described in the ACM.

## Self-Utilization Credit

Initially implemented in the 2019 energy code, the self-utilization credit for a residential battery system allows proposed designs with PV systems and batteries ( 5 kWh or larger) to subtract additional TDV from the "Efficiency" TDV of the proposed design. The selfutilization credit is capped at a fraction of the PV-related TDV of the standard design. The cap varies by climate zone and is between $7 \%$ and $14 \%$ for a single-family residence and between $2 \%$ and $9 \%$ for a multi-family building (see Table 1 in Section 2.1.5.5).

The actual credit applied to the "Efficiency" TDV of the proposed design is the lesser of the battery related TDV in the "Final" proposed design and the cap defined above. Effectively, the self-utilization credit allows the proposed "Efficiency" design to also get credit for a portion of the TDV savings that would otherwise be seen only in the "Final" design.

## D4 The CBECC-Res, CBECC and CSE Software Packages

Annual building loads used in the annual TDV calculation for single-family buildings in CBECC-Res and for multifamily buildings in CBECC are simulated using the underlying California Simulation Engine (CSE). CSE models the thermal and electrical interactions within a building. CBECC-Res and CBECC generates CSE input files based on the Title 24 rulesets. Separate CSE inputs files are created to simulate the standard design, and proposed design. For single-family buildings the reference design is also created. CBECC-Res then processes the CSE simulation results to determine the Efficiency and Final EDR values as described in the previous section and CBECC processes the CSE simulation results to determine the Efficiency and Final TDV, and source energy values described in the previous section. While the capability of CBECC-Res and CBECC are intentionally constrained by ruleset definitions, CSE has much greater flexibility to simulate a wide range of building components. CSE has the unique capability to define dynamic battery system control strategy using its built-in expression language. CSE predicts the building load and PV generation and operates the battery according to expressions pre-defined by CBECC-Res and CBECC rules.

## D5Battery Representation in CSE

In each simulated timestep, the control strategy sends a charge/discharge request to the battery module. The control strategies themselves are described in the next section. For now, it will suffice to say that the input to the battery module is a charge request (in kW) that can be either positive or negative.

```
charge_request > 0 // charge
charge_request < 0 // discharge
charge_request = 0 // do nothing
```

The battery has maximum charge and discharge rates (kW) with default values set based on the battery's size. CBECC-Res and CBECC define both defaults as the same fixed fraction (kW/kWh) of the battery's user-defined maximum capacity ( $k W h$ ). These default values may be overridden with custom values by the user.

```
max_charge_power = 0.42 * max_capacity
max_discharge_power = 0.42 * max_capacity
```

And both a charge and discharge efficiency (fraction), which are user-defined:
П_charge
П_discharge

The user has the option to input a round-trip efficiency (fraction) as an alternative to inputting both the charge and discharge efficiencies. In this case, the charge and discharge efficiency would be equal to:

```
\eta_charge = sqrt(\eta_rte)
\eta_discharge = sqrt(n_rte)
```

At each timestep, there are also maximum charge and discharge limits (kW) defined by the state of charge on the battery. Charge and discharge power levels are measured at the battery's edge: before efficiency losses in the case of charging and after efficiency losses in the case of discharging. The battery's state-of-charge is metered between the two efficiency multipliers.

```
max_charge_available = (max_capacity - charge_level) /
    \eta_charge * (timestep_minutes / 60)
max_discharge_available = charge_level * \eta_discharge *
    (timestep_minutes / 60)
```

Altogether, that enables the module to determine the amount the battery should charge or discharge in the hour:
if charge_request >0:
charge_power = min(charge_request, max_charge_rate, max_charge_available)
else if charge_request < 0 :
charge_power = max(charge_request, max_discharge_rate, max_discharge_available)
else:
charge_power = 0
At the conclusion of that timestep, the battery's charge level will have been updated:
if charge_power > 0: // charging charge_level = charge_level + charge_power*n_charge
else if charge_power < 0 // discharging
charge_level = charge_level + charge_power/n_discharge
else:
charge_level = charge_level

## D6Battery Control Strategies

There are three battery control strategies enabled in CBECC-Res and CBECC: "Basic", "Time of Use" (TOU), and "Advanced DR Control". These strategies are responsible for the timestep-by-timestep charge requests that are sent to the CSE battery module.

## Basic Strategy

The Basic strategy charges when a) production exceeds demand and b) the battery is not fully charged and discharges when $a$ ) demand exceeds production and b) the battery is not fully drained. That is, the battery both charges and discharges as soon as it can.
charge_request = -load_seen

By charging from any excess production and discharging as soon as it can to serve load, the basic strategy maximizes self-consumption of the on-site PV production. The other strategies account for the time-varying value of electricity (e.g., as measured by TDV) to varying degrees to increase the TDV-savings the battery provides.

If the battery system is standalone (no PV system), then basic control is not an available control option.


Figure 2: Illustrations of the battery control strategies' different responses to a single day. Note that the TOU and Advanced strategies can discharge directly to the grid. Also notice that Advanced charges the battery from PV while serving loads from the grid.

## Time of Use Strategy

The TOU strategy attempts to preferentially discharge during high-value hours during a selected period of months. For a PV-tied system, the default duration for TOU months is July through September. For a standalone battery storage system, the default duration for TOU months is all year. Users can optionally input custom values for the first and last months to apply TOU control.
Charging rules are the same as the basic strategy for battery storage systems paired with a solar PV system. For standalone battery storage systems, the software provides a prescribed input to specify the hour of each day to start charging called "Charge Start Hour". The charging starts at midnight (hour 1) of each day.

Battery discharge follows the same approach for PV-tied and standalone batteries. The discharge period is statically defined (per climate zone) by the first hour of the expected TOU peak, which is a user-input within CBECC-Res and CBECC called "Discharge Start Hour." The default value for "Discharge Start Hour" is 19:00 for Climate Zones 2, 4, 8-15, and 20:00 for all other Climate Zones. The user has the option to change this value within CBECC-Res and CBECC if desired.
Consider a summer day in which the evening peak is defined to start at 20:00 but during which simulation load exceeds PV production during the 19:00 hour. While a simulation utilizing the Basic strategy would discharge to neutralize the net load during the 19:00 hour, a simulation on the TOU strategy would reserve the battery until 20:00 before commencing discharge. Because the TDV at 20:00 is likely to be higher than the TDV at 19:00, this strategy of reserving the battery for higher-value hours results in a lower (better) annual TDV.

A second difference: During the peak window, the battery is permitted to discharge at full power, even exceeding the site's net load. This is in contrast to the Basic strategy, which is limited to the net load.

> if charge_start_hour <= this_hour < discharge_start_hour: charge_request = -min(load_seen, 0) // only charge
else:

$$
\text { charge_request }=-1000 \text { // maximum discharge }
$$

Outside of selected months for the TOU strategy, control reverts to the Basic strategy.


Figure 3: On days with high Electricity TDV, the TOU and Advanced control strategies can accumulate large negative TDV by discharging to the grid during peak hours. The Advanced strategy specifically targets the top three TDV hours each day for discharging.

## Advanced DR Control

The Advanced DR (i.e., Demand Response) strategy uses the current day's TDV schedule to make dynamic time-of-use priorities. This strategy activates on days that have their peak TDV value ranking amongst the highest of all days of the year. The default value is the top 20 days. This value is hardcoded by CBECC-Res and CBECC unless EnableResearchMode is ON, in which case this value may be changed by the user. On all other days, the simulation reverts to the Basic strategy for PV-tied battery storage systems and to the Time of Use strategy for standalone battery storage systems.

The algorithm calculates the duration required to fully discharge the battery. On a peak day (as defined by having a peak TDV value ranking in the top 20 days of the year), the strategy finds the top TDV Values for the number of hours in that day matching the battery discharge duration. The algorithm asks for full discharge during these top hours in order of highest TDV hour first. Charging rules are the same as the Basic strategy for battery storage systems paired with a solar PV system. For standalone battery storage systems, the charging strategy should mimic the standalone battery charging strategy defined for Time of Use control.

As with the TOU strategy, the battery may discharge in excess of net load during peak hours.
The Advanced DR strategy is also allowed to charge from the PV system before production overtakes load. Whereas the Basic and TOU strategies only charge with surplus production, this strategy will-on a peak day-charge from the first PV production available.

```
if day_tdv_peak_ranking <= 20:
    if tdv_rank(this_hour) in [1, 2, ..., discharge_duration-1]:
                            // maximum discharge top TDV hours
                            charge_request \(=-1000\)
    else if tdv_rank(this_hour) == discharge_duration:
                            // in next highest TDV hour, use remaining charge
                    charge_request = max_capacity -
    2*(max_discharge_rate/n_discharge)
    else:
            // charge from PV, without subtracting load
            charge_request \(=p v \_\)production
else if system_type == PV-tied:
    // on non-peak days, revert to basic strategy for PV-tied sys
    charge_request = -load_seen
else if system_type == standalone:
    // on non-peak days, revert to TOU strategy for standalone sys
    if charge_start_hour <= this_hour < discharge_start_hour:
            charge_request = -min(load_seen, 0 ) // only charge
    else:
            charge_request =-1000 // maximum discharge
```


## Battery Parameters Included in CBECC-Res/CBECC/CSE

CBECC-Res and CBECC allow the modeler to adjust several battery parameters (Figure 4):

- Battery capacity (kWh): the CBECC-Res and CBECC software enforces a 5 kW minimum size for the battery to qualify for the Self Utilization Credit.
- A checkbox to indicate if a standalone battery (no PV system) is modeled.
- Control strategy, chosen from the three options described in the Battery Control Strategies section of this appendix.
- Note that "Basic" control is not an available control option for standalone battery systems


Figure 4: The CBECC-Res (top) and CBECC (bottom) battery dialog box allows the modeler to set battery capacity, control strategy, charge/discharge efficiencies, and other control parameters related to charging and discharging hours and TOU months. Example images are shown for the TOU and Advanced DR Control options. (Source: screenshot of CBECC-Res and CBECC software user interface.)

- Charging and discharging efficiency (fraction): CSE allows charging and discharging efficiencies to be defined independently. The CBECC-Res and CBECC default is 0.95 for each, resulting in a default round-trip efficiency of 0.9025.
- A single input for round-trip efficiency may be input by selecting the checkbox "Set Roundtrip Efficiency" (as shown in Figure 4). When checked, the inputs for charge and discharge efficiency are hidden and only the round-trip efficiency input is shown. Round-trip efficiency inputs less than $80 \%$ will result in no battery included in the simulation.
- Charge start hour may be input for standalone battery systems. Discharge start hour may be input for standalone and PV-tied battery systems. Allowable inputs are integers between 1 and 24 (inclusive).
- TOU period start and end months may be input. Allowable inputs are integers between 1 and 12 (inclusive).

CBECC-Res and CBECC also makes a set of assumptions to set CSE battery parameters. These are parameters that can be set in the lower-level CSE but that CBECC-Res and CBECC define itself.

- CBECC-Res and CBECC assume that the input battery capacity is the capacity of a brand new system. To account for aging across the battery's life cycle, the software derates the effective battery capacity to $85 \%$ of the input battery capacity. A nominal 10 kWh battery gets 8.5 kWh of usable capacity in the simulation.
- The $85 \%$-of-input-capacity figure interrelates with the fact that battery systems often have different published values for total and usable capacity. The battery management system prevents complete discharges, so the usable capacity is typically single-digit percentages lower than the total capacity (e.g., the Powerwall 2 has 14 kWh total and 13.5 kWh available energy). CBECC-Res and CBECC should clarify whether the input capacity should be total or useful, and the degradation derate figure should be consistent with the input CBECC-Res and CBECC expect.
- CBECC-Res and CBECC derive the CSE parameters Maximum charge rate and Maximum discharge rate (kW/hr) from the battery capacity. They are each defined to be battery capacity ${ }^{*} 0.42$. That is, the battery is sized to have 2.38 hours of storage at full discharge. $(1 / 0.42=2.38)$. That ratio is likely derived from the 14 kWh capacity and 5 kW discharge power of the Powerwall 2: $14 * 0.85 / 5=2.38$.
- The battery is assumed to start the simulation fully discharged. It is not required to be fully charged at the conclusion of the simulation.
- Battery-PV installations come in a range of electrical configurations, sometimes with independent inverters for each component (AC-coupled), sometimes sharing an inverter (DC-coupled). CSE assumes an AC-battery-module electrical configuration as shown in Figure 5. The modeling implication of that configuration is that the userinput battery charge and discharge efficiencies should include the losses associated with the battery module's onboard inverter. In CSE, the PV system always has a dedicated PV inverter.


Figure 5: General diagram of an AC battery module layout of a residential PV-battery system. (Source https://www.cleanenergyreviews.info/blog/ac-coupling-vs-dc-coupling-solar-battery-storage.)

Simulated battery performance is static in the current CSE implementation. In particular, charge and discharge efficiencies do not vary with either charge rate or temperature. In real-world battery systems, efficiency falls from the benchmark a) with age, b) under rapid charging/discharging, and c) when temperatures are outside an ideal range (e.g., $25^{\circ} \mathrm{C} / 77$ ${ }^{\circ} \mathrm{F}$ ).

A real-world battery's available capacity is also subject to age and external conditions. Low temperatures, especially, reduce a battery's in-the-moment available capacity. CSE neglects those effects as well. The long-term dynamics of how batteries age warrants its own section, below.

The existence of a battery in the CBECC-Res and CBECC models also relaxes size limits on PV systems. Without onsite storage, the PV system is limited by the interconnection rule: solar generation must not exceed the building's electricity consumption over the course of a year. Title 24, Part 6 allows a building with a battery larger than 5 kWh to have a PV system that produces up to 1.6 times the building's annual electricity consumption. The CBECC-Res and CBECC software implement this change.

## APPENDIX E - PLUG LOADS AND LIGHTING MODELING

### 1.1 Appliances, Miscellaneous Energy Use and Internal Gains

Full details of the assumptions for lighting and appliance loads are found in the Codes and Standards Enhancement Initiative (CASE) Plug Loads and Lighting Modeling (Rubin 2016, see Appendix D).

### 1.1.1 Background

Rulesets for all plug loads (including appliances and miscellaneous electric loads (MELs)) and lighting loads were updated in 2016. The CASE report describes the methodology, data sources, and assumptions used to develop the rulesets. The updated methodology replaces the rulesets from the 2019 Residential Alternative Calculation Method (ACM) Reference Manual (ACM Reference Manual), which in turn referenced the 2008 California Home Energy Rating System (HERS) Technical Manual.
The rulesets were modified to reflect efficiency levels assuming 2017 federal code baseline or 2017 projected market average performance, depending on whether or not a product is regulated by federal energy efficiency standards. Miscellaneous loads were disaggregated so that the three largest loads in this group-televisions, set-top boxes, and computers and monitors-are modeled individually. The remaining miscellaneous loads are modeled in aggregate. Garage lighting is also disaggregated from interior lighting. Assumptions about how energy use scales with building size were updated for all plug load and lighting end uses.

Updated load profiles were proposed for the majority of the modeled plug load and lighting end uses. The proposed updates include revisions to both the hourly schedules and seasonal multipliers. The updated load profiles are based on the water heating models described in section 2.9 of the ACM Reference Manual for the applicable end uses and otherwise on recent submetering studies.

### 1.1.2 Approach

Rulesets for all modeled end uses reflect the estimated energy consumption of those devices in new homes built during the 2022 Title 24 Code Cycle. The plug load rulesets estimate annual energy consumption (AEC) as a function of number of bedrooms (BR/Unit) and the lighting rulesets estimate AEC as a function of conditioned floor area (CFA/Unit). The relationship between AEC and BR/Unit for dishwashers, clothes washers, and clothes dryers was based on the usage assumptions in the water heating model. The relationship between all other plug load AEC and BR/Unit was generally derived from the 2009 Residential Appliance Saturation Survey (RASS), through a statistical and engineering analysis that
applied modern efficiency assumptions to estimate what the AEC of plug loads within homes included in the 2009 RASS would be if they were built during the 2016 Title 24 code cycle. The relationship between lighting AEC and CFA/Unit was derived using a similar analysis completed on the RASS data but using data from the 2012 California Lighting and Appliance Saturation Survey.
With additional user inputs, the default AEC equations for primary refrigerators, clothes washers, and clothes dryers can be modified to reflect the efficiency of the devices that are actually installed in the building. That is, the modeled energy use can be adjusted downward if more efficient devices are installed (the software tool can also adjust energy use upward if devices are less efficient).

Updated load profiles are derived from the following data sources:

- Dishwashers, clothes washers, and clothes dryers: updated to be consistent with the usage patterns assumed by water heating models described in section 2.9 of the ACM Reference Manual.
- Ovens, cooktops, and televisions: based on data from the Phased Deep Retrofit (PDR) study conducted by the Florida Solar Energy Center (FSEC), which submetered 60 Florida homes in 2012.
- Set-top boxes, computers, and monitors: based on the Northwest Energy Efficiency Alliance (NEEA) Residential Building Stock Assessment (RBSA), released in 2014. This study monitored 100 homes in the Pacific Northwest over the course of one year, submetering major end uses at 15 minute intervals.
- Exterior lighting: the proposed hourly schedule for exterior lighting is derived from the NEEA RBSA light logging data; the proposed exterior lighting seasonal multipliers are no longer constant, but instead equivalent to the interior and garage seasonal multipliers.
Load profiles for interior lighting, garage lighting, and residual MELs were not updated in 2016. The current hourly schedules for interior lighting are based on the 1999 Heschong Mahone Group (HMG) study "Lighting Efficiency Technology Report: California Baseline." The current hourly schedule for residual MELs is derived from the 2008 Building America House Simulation Protocol, which in turn relied on data from a 1989 Pacific Northwest submetering study conducted by the End-Use Load and Consumer Assessment Program (ELCAP).

Refrigerators and freezers use PDR data to adjust estimated energy use on an hourly basis depending on the modeled indoor temperature (using the Title 24 compliance software) in the space where the refrigerator is installed.

### 1.1.3 Problems

The plug load and lighting rulesets have some limitations. The rulesets generally do not account for differences in energy use patterns between single-family and multi-family housing. For example, they do not account for the energy use of laundry equipment in
multi-family residences that is installed in common areas-only laundry equipment in the dwelling units.
The plug load and lighting rulesets were developed to apply to new homes built during the 2016 Title 24 Code Cycle, and thus should not be used for estimating energy use for existing homes.

### 1.1.4 Inputs

### 1.1.4.1 AEC Inputs and Algorithms

Error! Reference source not found. summarizes the user inputs that determine the plug load and lighting annual energy consumption (AEC) estimates. The variable 'BR/Unit' refers to the number of bedrooms in a single-family home or the number of bedrooms in each dwelling unit of a multi-family building. Similarly, 'CFA/Unit' refers to the conditioned floor area per dwelling unit. AEC equations are to be applied to each dwelling unit within a multifamily building, not the building as a whole. Users also specify the zone where certain major appliances are located; however, this affects the modeled internal gains from equipment and lighting, not their estimated energy use of the plug load or lighting load and is therefore not included in the table below. The Optional inputs are not implemented in CBECC-Res 2016.2, but may be allowed in future releases.

## Table 1: User Inputs Affecting Estimated Plug Load and Lighting Energy Use

| End Use | User Inputs that <br> Determine Estimated <br> Energy Use |  |
| :--- | :--- | :--- |


| End Use | User Inputs that Determine Estimated Energy Use | Notes |
| :---: | :---: | :---: |
| Clothes Washer | - BR/Unit <br> - Presence of device <br> - Single-family or multifamily <br> Optional: whether installed device will comply with the 2015 federal efficiency standards (credit for installing new or nearlynew device) | - Ruleset estimates machine energy use only. <br> - Energy use is only included if user indicates the device will be present. <br> Assumed different usage patterns in single family and multi-family when developing algorithms. <br> Default energy use can be reduced if the user specifies the device will meets the 2015 federal standard, which can be determined by looking up the model on the California Appliance Efficiency Database. |
| Clothes Dryer | - Bedrooms per unit <br> - Presence of device <br> - Fuel type (natural gas, propane, or electric) Single-family or multifamily Optional: percent remaining moisture content (RMC) of the clothes washer | Energy use is only included if user indicates the device will be present. <br> User can select fuel type. If user indicates natural gas is available at the site (see Section 2.2.10 of RACM), then the default fuel type is natural gas. If user indicates that natural gas is not available at the site then the default fuel type is electric. User cannot select natural gas as the fuel type if natural gas is not available at the site. Default energy use can be reduced if the user specifies that the installed clothes washer has a rated RMC of less than 50 percent. |
| Oven | - Bedrooms per unit <br> - Presence of device <br> - Fuel type (natural gas, propane, or electric) | Energy use is only included if user indicates the device will be present. User can select fuel type, but default assumption is natural gas if user indicates that natural gas is available on-site and electric if user indicates natural gas is not available on-site |
| Cooktop | N/A | N/A |
| Televisions, SetTop Boxes, Computers and Monitors, Residual MELs | - Bedrooms per unit | N/A |
| Interior Lighting, Exterior Lighting | - CFA/Unit | N/A |


| End Use | User Inputs that <br> Determine Estimated <br> Energy Use | Notes |
| :---: | :--- | :--- |
| Garage Lighting -CFA/Unit <br> Presence of garage | Energy use is only included if user indicates <br> there is a garage present. <br> Garage lighting is assigned to multi-family <br> buildings if there is at least once garage <br> present. <br> Carport lighting is covered under the exterior <br> lighting ruleset. |  |

Source: California Energy Commission
Error! Reference source not found. summarizes the proposed AEC algorithms for plug load and lighting. These linear equations take the following general form where the homes size metric is bedrooms per unit (BR/Unit) for plug loads and CFA/Unit for lighting:

$$
y=m x+b
$$

Where: $\quad$ y $=$ Estimated AEC measured in kWh/yr or therms/yr
$m=$ how AEC changes with home size
$x=$ home size as measured in BR/Unit for plug loads or CFA/Unit for
lighting

> b = minimum energy use (energy use at y-intercept)

BR-based equations are capped at 7 bedrooms, meaning that units with eight or more bedrooms have the same estimated AEC as a 7-bedroom unit. CFA-based equations are capped at 4,150 square feet. For those end uses that list 'presence of device' as a user input in Error! Reference source not found., the AEC equation is only applied if the device is present. Similarly, for the AEC equations for end uses that can be gas or electric are only applied according to the user-specified fuel type. Gas algorithms apply to devices that use natural gas or propane.

Table 2: Algorithms for Plug Load and Lighting Annual Energy Use

| End Use | Standard Design Fuel Type | kWh or therms | Intercept | Slope | Per-Unit BR or CFA |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Primary <br> Refrigerator/Freezer | Electricity | kWh | 454 | 37.0 | BR |
| Non-Primary Refrigerators and Separate Freezers (Single-Family only) | Electricity | kWh | 0 | 71.0 | BR |
| Oven | Electricity | kWh | 138 | 16 | BR |
| Oven | Gas | therms | 6.0 | 0.95 | BR |
| Oven | Gas | kWh | 41 | 4.79 | BR |
| Cooktop | Electricity | kWh | 84 | 5.68 | BR |
| Cooktop | Gas | therms | 5.0 | 0.30 | BR |
| Cooktop | Gas | kWh | 0 | 0 | BR |
| Televisions | Electricity | kWh | 265 | 31.8 | BR |
| Set-Top Boxes | Electricity | kWh | 76 | 59.4 | BR |
| Computers and Monitors | Electricity | kWh | 79 | 55.4 | BR |
| Residual MELs | Electricity | kWh | 672 | 235 | BR |
| Interior Lighting | Electricity | kWh | 100 | 0.1775 | CFA |
| Exterior Lighting | Electricity | kWh | 8.0 | 0.0532 | CFA |
| Garage Lighting | Electricity | kWh | 20 | 0.0063 | CFA |

Source: California Energy Commission
Error! Reference source not found. and Error! Reference source not found. summarize the AEC algorithms for dishwashers, clothes washers and clothes dryers. These rulesets only include machine energy use from dishwashers and clothes washers. Energy use for water heating is accounted for in the water heating model.

Table 3: Single-Family Residence Algorithms for Dishwasher, Clothes Washer, and Clothes Dryer Annual Energy Use

| BRper <br> Unit | Dishwashers <br> (kWh/yr) | Clothes <br> Washers <br> (kWh/yr) | Electric <br> Clothes <br> Dryers <br> (kWh/yr) | Gas Dryer <br> Natural Gas <br> Use <br> (therms/yr) | Gas Dyer <br> Electricity <br> Use <br> (kWh/yr) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 83 | 84 | 634 | 22 | 32 |
| 1 | 83 | 84 | 634 | 22 | 32 |
| 2 | 91 | 85 | 636 | 22 | 32 |
| 3 | 100 | 99 | 748 | 26 | 37 |
| 4 | 99 | 101 | 758 | 27 | 38 |
| $5+$ | 119 | 227 | 877 | 31 | 44 |

Source: California Energy Commission
Table 4: Multi-Family Dwelling Unit Algorithms for Dishwasher, Clothes Washer, and Clothes Dryer Annual Energy Use

| BRper Unit | Dishwashe rs <br> (kWh/yr) | Clothes Washers (kWh/yr ) | Electric Clothes Dryers (kWh/yr) | Gas Dryer Natural Gas Use (therms/yr) | Gas Dyer Electricity Use (kWh/yr) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 56 | 66 | 496 | 17 | 25 |
| 1 | 68 | 70 | 527 | 19 | 26 |
| 2 | 96 | 99 | 745 | 26 | 37 |
| 3 | 94 | 97 | 733 | 26 | 37 |
| 4 | 121 | 118 | 885 | 31 | 44 |
| 5+ | 114 | 107 | 805 | 28 | 40 |

Source: California Energy Commission

### 1.1.4.2 AEC Algorithms for High-Efficiency Appliances

As indicated in Error! Reference source not found., if allowed in the software, users could override the default AEC rulesets for the primary refrigerator, clothes washer and clothes dryer if the software user has additional information about the device that will be installed.

For the primary refrigerator, the default AEC ruleset could be replaced with the rated AEC listed on the refrigerator's Energy Guide label. If using this option, the user will input AEC measured in kWh per year, and that value will replace the AEC value for the primary refrigerator calculated using the equation below. The default AEC of the primary refrigerator cannot be adjusted below a certain value, which is dependent on BR/Unit as described in the following equation:

$$
\text { MinPrimaryRefrigAEC } \frac{k W h}{y r}=\left(8.4 \frac{k W h}{\text { BRperUnit-yr }} \times \text { BRperUnit }\right)+291 \frac{k W h}{y r}
$$

Users could reduce the estimated primary refrigerator AEC to this value, but no lower.
Table 5: Minimum primary refrigerator AEC that builders may claim by BR/Unit

| BR/Unit | Default Primary <br> Refrigerator AEC <br> (kWh/yr) | Minimum <br> Allowable <br> Primary |
| :---: | :---: | :---: |
| Refrigerator AEC |  |  |
| (kWh/yr) |  |  |$|$| 0 | 470 |
| :---: | :---: |
| 1 | 496 |
| 2 | 523 |
| 3 | 550 |
| 4 | 577 |
| 5 | 603 |
| 6 | 630 |
| $7+$ | 657 |

Source: California Energy Commission
For clothes washers, if allowed in the software, the user could specify that the installed clothes washer meets the 2015 federal standards (as documented on the CEC Appliance Efficiency Database). This effectively provides credit if the clothes washer is new or nearly new. Error! Reference source not found. presents the AEC values used if the washer is compliant with the 2015 federal standards.

Table 6: Minimum allowable high-efficiency AEC for clothes washers

| BR/Unit | Single Family Default AEC (kWh/yr) | Single Family High-Efficiency Clothes Washer AEC ${ }^{1}$ (kWh/yr) | Multifami Iy Default AEC (kWh/yr) | Multifamily High-Efficiency Clothes Washer AEC $^{1}$ (kWh/yr) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 84 | 68 | 66 | 53 |
| 1 | 84 | 68 | 70 | 57 |
| 2 | 85 | 68 | 99 | 80 |
| 3 | 100 | 80 | 98 | 79 |
| 4 | 101 | 81 | 118 | 95 |
| $5+$ | 117 | 94 | 107 | 86 |

${ }^{1}$ Applicable to clothes washers that meet the 2015 federal efficiency standards
Source: California Energy Commission
For clothes dryers, if allowed in the software, the user could specify the percent remaining moisture content (RMC) of the installed clothes washer (as documented on the CEC Appliance Efficiency Database) to override the default clothes dryer AEC ruleset. The RMC-
adjusted clothes dryer AEC should be calculated using the equations provided below. For natural gas dryers the RMC-adjusted AEC modifies natural gas use but does not impact electricity use.
Electric Dryer: RMC-adjusted AEC (kWh/yr)

$$
\begin{aligned}
& \text { RMC-adjusted AEC } \frac{k W h}{y r} \\
& \qquad=12.67 \frac{\mathrm{kWh}}{\mathrm{yr}}+\left[\left(3.80 \frac{\mathrm{kWh}}{\text { cycle }}\left(R M C_{\text {User,Input }}\right)+0.25 \frac{\mathrm{kWh}}{\mathrm{cycle}}\right) \times \frac{\text { cycles }}{\mathrm{yr}}\right]
\end{aligned}
$$

## Gas Dryer: RMC-adjusted AEC (therms/yr)

$$
R M C \text {-adjusted AEC } \frac{\text { therms }}{y r}=\left[0.136 \frac{\text { therms }}{\text { cycle }}\left(R M C_{U \text { ser }, \text { Input }}\right)+0.00853 \frac{\text { therms }}{\text { cycle }}\right] \times \frac{\text { cycles }}{y r}
$$

Table 7: Annual clothes dryer cycles estimated based on BR/Unit

| BR/Unit | Clothes <br> Dryer Cycles <br> Single- <br> Family | Clothes <br> Dryer <br> Cycles <br> Multi- <br> Family |
| :---: | :---: | :---: |
| 0 | 290 | 227 |
| 1 | 290 | 241 |
| 2 | 291 | 341 |
| 3 | 342 | 335 |
| 4 | 346 | 405 |
| $5+$ | 401 | 368 |

Source: California Energy Commission

### 1.1.4.3 Load Profiles

Dishwashers and clothes washers loads are specified in the water heating load profiles. Clothes dryers have the same usage assumptions as clothes washers, but shifted one hour later.

The estimated energy use for refrigerators is adjusted for each hour of the year depending on the simulated indoor temperature in the thermal zone where the refrigerator or freezer is installed (user input).Multi-family housing is assumed to have no energy use for non-primary refrigerators or separate freezers.
The following tables summarize the hourly load profiles and seasonal multipliers for the remaining plug load and lighting end uses.

Table 8: Hourly Multiplier - Weekdays

| Hour | Oven <br> and <br> Cooktop | Televisions | Set-Top <br> Boxes | Computers <br> and <br> Monitors | Residual <br> MELs | Interior <br> and <br> Garage <br> Lighting | Exterior <br> Lighting |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.005 | 0.035 | 0.040 | 0.036 | 0.037 | 0.023 | 0.046 |
| 2 | 0.004 | 0.026 | 0.040 | 0.033 | 0.035 | 0.019 | 0.046 |
| 3 | 0.004 | 0.023 | 0.040 | 0.032 | 0.034 | 0.015 | 0.046 |
| 4 | 0.004 | 0.022 | 0.040 | 0.032 | 0.034 | 0.017 | 0.046 |
| 5 | 0.004 | 0.021 | 0.040 | 0.031 | 0.032 | 0.021 | 0.046 |
| 6 | 0.014 | 0.021 | 0.040 | 0.032 | 0.036 | 0.031 | 0.037 |
| 7 | 0.019 | 0.025 | 0.040 | 0.034 | 0.042 | 0.042 | 0.035 |
| 8 | 0.025 | 0.032 | 0.041 | 0.036 | 0.044 | 0.041 | 0.034 |
| 9 | 0.026 | 0.038 | 0.040 | 0.039 | 0.037 | 0.034 | 0.033 |
| 10 | 0.022 | 0.040 | 0.040 | 0.043 | 0.032 | 0.029 | 0.028 |
| 11 | 0.021 | 0.038 | 0.040 | 0.045 | 0.033 | 0.027 | 0.022 |
| 12 | 0.029 | 0.038 | 0.040 | 0.045 | 0.033 | 0.025 | 0.015 |
| 13 | 0.035 | 0.041 | 0.040 | 0.046 | 0.032 | 0.021 | 0.012 |
| 14 | 0.032 | 0.042 | 0.040 | 0.046 | 0.033 | 0.021 | 0.011 |
| 15 | 0.034 | 0.042 | 0.041 | 0.046 | 0.035 | 0.021 | 0.011 |
| 16 | 0.052 | 0.041 | 0.041 | 0.047 | 0.037 | 0.026 | 0.012 |
| 17 | 0.115 | 0.044 | 0.042 | 0.048 | 0.044 | 0.031 | 0.019 |
| 18 | 0.193 | 0.049 | 0.043 | 0.049 | 0.053 | 0.044 | 0.037 |
| 19 | 0.180 | 0.056 | 0.044 | 0.049 | 0.058 | 0.084 | 0.049 |
| 20 | 0.098 | 0.064 | 0.045 | 0.049 | 0.060 | 0.117 | 0.065 |
| 21 | 0.042 | 0.070 | 0.046 | 0.049 | 0.062 | 0.113 | 0.091 |
| 22 | 0.020 | 0.074 | 0.047 | 0.048 | 0.060 | 0.096 | 0.105 |
| 23 | 0.012 | 0.067 | 0.045 | 0.044 | 0.052 | 0.063 | 0.091 |
| 24 | 0.010 | 0.051 | 0.045 | 0.041 | 0.045 | 0.039 | 0.063 |

Source: California Energy Commission
Table 8: Hourly Multiplier - Weekends

| Hour | Oven <br> and <br> Cooktop | Televisions | Set-Top <br> Boxes | Computers <br> and <br> Monitors | Residual <br> MELs | Interior <br> and <br> Garage | Exterior <br> Lighting |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.005 | .035 | 0.041 | 0.036 | 0.037 | 0.023 | 0.046 |
| 2 | 0.004 | 0.027 | 0.041 | 0.034 | 0.035 | 0.019 | 0.046 |
| 3 | 0.003 | 0.022 | 0.040 | 0.033 | 0.034 | 0.015 | 0.045 |
| 4 | 0.003 | 0.021 | 0.041 | 0.033 | 0.034 | 0.017 | 0.045 |
| 5 | 0.003 | 0.020 | 0.040 | 0.032 | 0.032 | 0.021 | 0.046 |
| 6 | 0.005 | 0.020 | 0.040 | 0.033 | 0.036 | 0.031 | 0.045 |


| Hour | Oven <br> and <br> Cooktop | Televisions | Set-Top <br> Boxes | Computers <br> and <br> Monitors | Residual <br> MELs | Interior <br> and <br> Garage <br> Lighting | Exterior <br> Lighting |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 0.010 | 0.022 | 0.040 | 0.033 | 0.042 | 0.042 | 0.044 |
| 8 | 0.027 | 0.029 | 0.040 | 0.035 | 0.044 | 0.041 | 0.041 |
| 9 | 0.048 | 0.037 | 0.041 | 0.038 | 0.037 | 0.034 | 0.036 |
| 10 | 0.048 | 0.043 | 0.042 | 0.042 | 0.032 | 0.029 | 0.030 |
| 11 | 0.046 | 0.042 | 0.042 | 0.044 | 0.033 | 0.027 | 0.024 |
| 12 | 0.055 | 0.039 | 0.041 | 0.045 | 0.033 | 0.025 | 0.016 |
| 13 | 0.063 | 0.040 | 0.041 | 0.046 | 0.032 | 0.021 | 0.012 |
| 14 | 0.059 | 0.042 | 0.041 | 0.047 | 0.033 | 0.021 | 0.011 |
| 15 | 0.062 | 0.045 | 0.041 | 0.047 | 0.035 | 0.021 | 0.011 |
| 16 | 0.068 | 0.048 | 0.042 | 0.048 | 0.037 | 0.026 | 0.012 |
| 17 | 0.091 | 0.051 | 0.042 | 0.049 | 0.044 | 0.031 | 0.019 |
| 18 | 0.139 | 0.052 | 0.043 | 0.049 | 0.053 | 0.044 | 0.038 |
| 19 | 0.129 | 0.056 | 0.044 | 0.048 | 0.058 | 0.084 | 0.048 |
| 20 | 0.072 | 0.061 | 0.044 | 0.048 | 0.060 | 0.117 | 0.060 |
| 21 | 0.032 | 0.065 | 0.045 | 0.048 | 0.062 | 0.113 | 0.083 |
| 22 | 0.014 | 0.069 | 0.045 | 0.047 | 0.060 | 0.096 | 0.098 |
| 23 | 0.009 | 0.064 | 0.044 | 0.044 | 0.052 | 0.063 | 0.085 |
| 24 | 0.005 | 0.050 | 0.039 | 0.041 | 0.045 | 0.039 | 0.059 |

Source: California Energy Commission
Table 9: Seasonal Multipliers

| Month | Oven and <br> Cooktop | Televisions | Set-Top <br> Boxes | Computers <br> and <br> Monitors | Residual <br> MELs and <br> Lighting |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Jan | 1.094 | 1.032 | 1.02 | 0.98 | 1.19 |
| Feb | 1.065 | 0.991 | 0.84 | 0.87 | 1.11 |
| Mar | 1.074 | 0.986 | 0.92 | 0.89 | 1.02 |
| Apr | 0.889 | 0.990 | 0.98 | 1.11 | 0.93 |
| May | 0.891 | 0.971 | 0.91 | 1.14 | 0.84 |
| Jun | 0.935 | 0.971 | 0.94 | 0.99 | 0.80 |
| Jul | 0.993 | 1.002 | 1.05 | 1.05 | 0.82 |
| Aug | 0.920 | 1.013 | 1.06 | 1.01 | 0.88 |
| Sep | 0.923 | 1.008 | 1.06 | 0.96 | 0.98 |
| Oct | 0.920 | 1.008 | 1.14 | 0.97 | 1.07 |
| Nov | 1.128 | 1.020 | 1.03 | 0.99 | 1.16 |
| Dec | 1.168 | 1.008 | 1.05 | 1.04 | 1.20 |

Source: California Energy Commission

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## APPENDIX G - ALGORITHMS

## TABLE OF CONTENTS

1 CALIFORNIA SIMULATION ENGINE (CSE) ..... 1
1.1 Overview ..... 1
1.1.1 Schematic of Zone Thermal Network ..... 3
1.1.2 Schematic of Reduced Thermal Network ..... 4
1.1.3 Zone Balance Calculation Sequence ..... 5
1.2 Updating Layered Mass Temperatures ..... 7
1.3 Zone Energy Balance ..... 8
1.3.1 Implicit Update of Air Temperature ..... 8
1.3.2 Zone Balance Equations ..... 10
1.3.3 Thermostat Logic ..... 12
1.3.4 Limiting Capacities ..... 13
1.4 Discretization Errors ..... 14
1.4.1 Layer Thickness of a Homogeneous Material ..... 14
1.4.2 Choosing the Time Step ..... 15
1.5 Surface Heat Transfer Coefficients ..... 16
1.5.1 Local Wind Velocity Terrain and Height Correction ..... 16
1.5.2 Convection Coefficient for Inside and Outside Surfaces of Zones ..... 18
1.5.3 Outside Radiation Coefficients ..... 28
1.5.4 Sky Temperature ..... 31
1.6 Distribution of SW and LW Radiation inside the Zone ..... 33
1.6.1 Long Wave Radiation Distribution ..... 33
1.7 Window Model ..... 47
1.7.1 Inputs ..... 47
1.7.2 Outputs ..... 47
1.7.3 Matching ASHWAT to CSE Radiant Network ..... 49
1.8 Slab Model ..... 50
1.8.1 Bajanac Simplified Model ..... 50
1.8.2 Addition of a Layered Slab and Earth ..... 53
1.9 Ventilation and Infiltration Air Network ..... 58
1.9.1 Overview ..... 58
1.9.2 Vertical Pressure Distribution ..... 60
1.9.3 Power Law Flow Equation ..... 63
1.9.4 Large Horizontal Openings ..... 69
1.9.5 Large Vertical Openings ..... 71
1.9.6 Newton-Raphson Solution ..... 72
1.10 Duct System Model ..... 77
1.10.1 Description of Model ..... 77
1.10.2 Duct System Inputs ..... 78
1.10.3 Return Duct Air Temperatures ..... 83
1.10.4 Return Plenum Temperature and Return Duct Conductive Heat Losses 84
1.10.5 Temperature Rise through Air Handler Heating or Cooling Equipment ..... 84
1.10.6 Supply Plenum and Supply Register Temperatures ..... 84
1.10.7 Heating/Cooling Delivered and Supply Duct Conductive Heat Loss ..... 86
1.10.8 Duct System Performance when the Load is Less than the Heat Delivered at Full Capacity ..... 87
1.10.9 Duct System Performance when the Load is Greater than the Heat Delivered at Full Capacity ..... 87
1.11 Variable Insulation Conductivity ..... 89
1.12 Ceiling Bypass Model ..... 89
1.13 Zone Humidity Balance ..... 89
1.13.1 Zone Humidity Balance ..... 89
1.13.2 Stability of Solution ..... 91
1.13.3 Hygric Inertia of Zone ..... 91
1.14 Zone Comfort Algorithm ..... 92
1.15 HVAC Equipment Models ..... 92
1.15.1 Compression Air-Conditioner Model ..... 92
1.15.2 Air-Source Heat Pump Model (Heating mode) ..... 99
1.15.3 Equipment Sizing ..... 101
2 COMPLIANCE MANAGER ..... 102
2.1 Overview ..... 102
2.2 One-dimensional Roof Edge Heat Transfer Model ..... 102
2.2.1 Construction Practice ..... 102
2.2.2 One-Dimensional Model ..... 103
2.2.3 Roof Edge Model Validation ..... 108
2.3 How to Build an Airnet ..... 114
2.3.1 Background ..... 114
2.3.2 Approach ..... 114
2.3.3 Inputs ..... 115
2.4 How to Create CSE Conditioned Zone Internal Mass Inputs ..... 126
2.4.1 Background ..... 126
2.4.2 Approach ..... 126
2.4.3 Inputs ..... 127
2.5 Appliances, Miscellaneous Energy Use, and Internal Gains ..... 129
2.5.1 Background ..... 129
2.5.2 Approach ..... 129
2.5.3 Inputs ..... 129
2.6 Seasonal Algorithm ..... 133
APPENDICES ..... 135
Appendix A. Derivation of Duct Loss Equations Using Heat Exchanger Effectiveness and Y-Delta Transformations ..... 135
Appendix B. Screen Pressure Drop ..... 138
Appendix C. Exact Longwave Radiation Model ..... 146
Appendix D. Determining the Form of the Self-weighting Term Fi ..... 149
REFERENCES ..... 150
FIGURES
Figure 1: Schematic of Zones and Air Handler Systems ..... 1
Figure 2: Schematic of Simulation Network ..... 3
Figure 3: Network after Dissolving Massless Nodes ..... 5
Figure 4: Heat Flow Down Situations ..... 19
Figure 5: Heat Flow Up Situations ..... 19
Figure 6: Plots of Equations for Downward and Upward Heat Flow ..... 26
Figure 7: Outside Convection Coefficients, Natural Up and Down, and Forced ..... 28
Figure 8: Carroll Network for Black Surfaces. ..... 34
Figure 9: Carroll Radiant Network for Grey Surfaces ..... 36
Figure 10: Test Room of Walton (1980) ..... 37
Figure 11: Like Figure 9 but with Convective Network Added ..... 41
Figure 12: Radiation Terminology ..... 42
Figure 13: ASHWAT Inputs and Nomenclature ..... 47
Figure 14: Window System Representation in CSE ..... 48
Figure 15: Equivalent Network between the Radiosity of the Window System, Jw, and the Inside Plate ..... 49
Figure 16: Reduced Figure 15 ..... 49
Figure 17: Network between the Radiosity of a Surface and the Mean Radiant Temperature Node ..... 50
Figure 18: Perimeter Coupling ..... 51
Figure 19: Core Coupling ..... 51
Figure 20: Addition of Film, Rug, Slab, and Earth ..... 54
Figure 21: Room Node X Admittance ..... 56
Figure 22: Room Node Y Admittance ..... 56
Figure 23. Schematic of Flow Network ..... 58
Figure 24: Mass Flow m versus Pressure Drop $\boldsymbol{\Delta P}$ ..... 66
Figure 25: Ratio of Actual R to Rated R ..... 80
Figure 26: Standard-Heel Geometry ..... 103
Figure 27: Raised-Heel Geometry ..... 103
Figure 28: Standard-Heel Simplified Geometry for Insulation Path. ..... 104
Figure 29: Standard-Heel 1-D Geometry for Insulation Path ..... 104
Figure 30: Standard-Heel Simplified Geometry for Framing Path ..... 105
Figure 31: Standard-Heel 1-D Geometry for Framing Path ..... 105
Figure 32: Raised-Heel 1-D Insulation Path Geometry ..... 106
Figure 33: Raised-Heel 1-D Framing Path Geometry ..... 106
Figure 34: Standard Truss, Insulation Path, 2-Dimensional Heat Transfer Model Geometry 112
Figure 35: Standard-Heel, Insulation Path, 2-Dimensional Heat Transfer Isotherms and Heat
Transfer Vectors ..... 112
Figure 36. Standard-Heel, Frame Path, 2-Dimensional Heat Transfer Isotherms and Heat Transfer Vectors ..... 112
Figure 37: 2-D Results for Insulation Path of R-60 Standard-Heel ..... 113
Figure 38: 2-D Results for Framing Path of R-60 Standard-Heel ..... 113
APPENDIX FIGURES
Figure A-1: Electrical Analogy of Heat Transfer through a Duct Wall ..... 135
Figure A-2: Heat Transfer through a Duct Wall with Surface Temperature Removed ..... 136
Figure B-1: Screen Pressure Drop ..... 140
Figure B-2: Pressure vs. Flow Characteristics. ..... 142
Figure B-3: Standard Screen Flow Reduction ..... 143
Figure B-4: For Small $\Delta p$ ..... 145
Figure B-5: For Large $\Delta \mathrm{p}$ ..... 145
Figure C-1: View-Factor Method's Radiant Network for Black-Body Surfaces ..... 147
Figure C-2: View-Factor Method's Network for Grey Surfaces ..... 148
Figure C-3: View-Factor Method's Network for Grey Surfaces Reduced to Star Network ..... 148
TABLES
Table 1: Parameters for Standard Terrain Classifications ..... 17
Table 2: Local Shielding Parameters ..... 17
Table 3: Surface Roughness Parameter $\boldsymbol{R f}$ (Walton 1981) ..... 24
Table 4: $\mathbf{E r r}=$ \% rms Error in qi from Equation 87 and Equation 88 in Parenthesis ..... 38
Table 5: \% rms Error in qi from Equation 90 ..... 40
Table 6: Pressure Coefficients for Wind Normal to One Wall ..... 61
Table 7: Hip Roof Wind Pressure Coefficients ..... 62
Table 8: Comparison of 1-D and 2-D Results ..... 114

## 1 California Simulation Engine (CSE)

### 1.1 Overview

The building modeled can have multiple conditioned and unconditioned zones. Each conditioned zone has an air handler associated with it, and each air handler can have supply and/or return ducts in an unconditioned zone (nominally the attic), and in the conditioned zone itself. Air handlers can operate independently in either a heating, cooling, or off mode. See Figure 1.

Every time step (nominally two minutes), the zone model updates the heat transfers to and from the zones and the zone mass temperatures. Each zone's conditions are updated in succession and independently, based on the conditions in the adjacent zones in the last time step.

The conditioned zone thermostat algorithms determine whether an air handler should be in a heating or cooling mode, or floating, and if heating or cooling, the magnitude of the load that must be met by the air handler to keep the conditioned zone at its current setpoint. If the setpoints cannot be satisfied, the conditioned zone floats with heating, cooling, or ventilation, at full capacity. In the off mode case the zones are modeled during the time step without duct or air handler effects.

Figure 1: Schematic of Zones and Air Handler Systems


Although shown partly outside of the envelope, all ducts are assumed to be in either the conditioned or unconditioned zones only.

The duct system model determines duct losses, their effect on the conditions of the unconditioned and conditioned zones, and their effect on the heating or cooling delivery of the air handler system.

The duct system model allows unequal return and supply duct areas, with optional insulation thicknesses. The ducts can have unequal supply and return leakages, and the influence of unbalanced duct leakage on the unconditioned and conditioned zones infiltration and ventilation is taken into account. Every time step it updates the air handler and duct system heat transfers, and HVAC energy inputs, outputs, and efficiency.

For each window, the ASHWAT window algorithm calculates the window instantaneous shortwave, longwave, and convective heat transfers to the zones.

The AIRNET infiltration and ventilation algorithm calculates the instantaneous air flow throughout the building based on the air temperatures in the zones, and on the outside wind and air temperature. AIRNET also handles fan induced flows.

In the update processes, a zones mass-node temperatures are updated using a forward-difference (Euler) finite difference solution, whereby the temperatures are updated using the driving conditions from the last time step. For accuracy, this forwarddifference approach necessitates a small time-step.

The small time-step facilitates the no-iterations approach we have used to model many of the interactions between the zones and allows the zones to be updated independently.

For example, when the zone energy balance is performed for the conditioned zone, if ventilation is called for, the ventilation capacity, which depends on the zone temperatures (as well as maximum possible ventilation openings and fan flows), is determined from the instantaneous balance done by AIRNET. To avoid iteration, the ventilation flows, and the accompanying heat transfers are based on the most recently available zone temperatures.

To avoid iteration, a similar use of the last time-step data is necessary is dealing with inter-zone wall heat transfer. For example, heat transfer through the ceiling depends on the conditions in both zones, but these conditions are not known simultaneously. Thus, ceiling masses are treated as belonging to the attic zone and updated at the same time as other attic masses, partly based on the heat transfer from the conditioned zone to the ceiling from the last time step. In turn, when the conditioned zone is updated it determines the ceiling heat transfer based on the ceiling temperature determined twominutes ago when the attic balance was done.

Similarly, when the conditioned zone energy balance is performed, if for example heating is called for, then the output capacity of the heating system needs to be known, which requires knowing the duct system efficiency. But the efficiency is only known after the air handler simulation is run. To avoid iteration between the conditioned zone and attic zones, the most recent duct efficiency is used to determine the capacity in the
conditioned zones thermostat calculations. When the attic simulation is next performed, if the conditioned zone was last running at capacity, and if the efficiency now calculated turns out to be higher than was assumed by the thermostat calculations, then the load will have exceeded the limiting capacity by a small amount depending on the assumed vs. actual efficiency. In cases like this, to avoid iteration, the limiting capacity is allowed to exceed the actual limit by a small amount, so that the correct energy demand is determined for the conditioned zone load allowed.

### 1.1.1 Schematic of Zone Thermal Network

Figure 2 shows a schematic of the zone model network. It models a single zone whose envelope consists of any number of walls, ceilings, floors, slabs, and windows, and can be adjacent to other conditioned or unconditioned zones. The envelope constructions can be made of any number of layers of different materials of arbitrary thermal conductivity and heat capacity. Each layer is modeled with one or more "T" networks in series. Each T has the layer heat capacitance, $c a p_{i j}$, centered between by two thermal conductances, where the first subscript corresponds to the wall construction number and the second to the layer number. Framed constructions are treated as two separate surface areas, the surface area of the part between framing, and the surface area of the part containing the framing itself; the heat flow is assumed to follow independent and parallel paths through these two surfaces.

Figure 2: Schematic of Simulation Network


The room air, represented by the mass node $T a$, is assumed to be well-mixed and have heat capacitance $C_{\text {air }}(\mathrm{Btu} / \mathrm{F})$. The air is shown in Figure 2 to interact with all of the building interior construction surfaces via convection coefficients $h c I c_{i}$ for surface $i$. The overall conductance through the window between $T_{a}$ and an effective outdoor
temperature $T_{e o}$ is $c c w_{j}$ for window surface $j$. The conductances $c c w_{j}$ and the corresponding radiant value $c r w_{j}$ are outputs of the ASHWAT windows algorithm applied to window $j$ each time step.

A mean radiant temperature node, $T_{r}$, acts as a clearinghouse for radiant exchange between surfaces. With conductances similar to those of the air node: $h r I c_{i}$ and $c r w_{j}$.

Depending on the size of the zone and the humidity of the air, the air is assumed to absorb a fraction of the long-wave radiation and is represented by the conductance $C_{\text {Xair }}$.

The internal gains, Qint, can be specified in the input as partly convective (fraction fintC), partly long wave (fintLW), and partly shortwave (fintSW). The heating or cooling heat transfers are shown as Qhc (+ for heating, - for cooling). If Qhc is heating, a fraction (frConv) can be convective with the rest long-wave. The convective parts of Qint and Qhc are shown as added to the air node. The long wave fraction of Qint and Qhe are shown added to the $\mathrm{T}_{\mathrm{r}}$ node.
Additional outputs of the ASHWAT algorithm are $F P_{j}$ the fraction of insolation $I_{j}$ incident on window $j$ that ultimately arrives at the air node via convection, and $F M_{j}$, the fraction that arrives at the radiant node as long-wave radiation.
The term $Q s_{i}$ is the the total solar radiation absorbed by each construction surface i , as determined by the solar distribution algorithm. The short wave part of the internal gains, Qint $*$ fintSW, is distributed diffusely, with the same diffuse targeting as the diffusely distributed solar gains.

Solar gains absorbed at the outside surface of constructions are represented by $Q s_{o}$ in Figure 2.

The slab is connected to the Ta and Tr in a similar fashion as the wall surfaces, although the slab/earth layering procedure is different than for walls.

### 1.1.2 Schematic of Reduced Thermal Network

Before a zone energy balance is formulated it is convenient to dissolve all the massless nodes from the network of Figure 2 (represented by the black dots), except for the mean radiant temperature node Tr . Figure 3 shows the resulting reduced network. A massless node is eliminated by first removing the short-wave gains from the node by using the current splitting principle (based on superposition), to put their equivalent gains directly onto adjacent mass nodes and other nodes that have fixed temperatures during a time step. Then the massless node can be dissolved by using $Y-\Delta$ transformations of the circuit.

Figure 3: Network after Dissolving Massless Nodes


For example, to eliminate the massless surface node of layered mass in Figure 2, the gain Qsi absorbed by the surface node is split into three parts: $Q s_{i}^{\prime}$ to the $\mathrm{T}_{\mathrm{i}, 1}$ node, $\mathrm{Qsi}{ }^{\prime \prime}$ to the Ta node and Qsi" to the Tr node. For example, by current splitting,

$$
Q s_{i}^{\prime}=Q s_{i} \frac{U h_{i, 1}}{h c I c_{i}+h r I c_{i}+U h_{i, 1}}
$$

Equation 1
A $Y$ - $\Delta$ transformation of the remaining $Y$ circuit gives the $\operatorname{ccc}_{i}$ and $\mathrm{crc}_{\mathrm{i}}$ conductances, as well as an additional cross conductance CXCi that is added to CXair. For example,

$$
c c c_{i}=\frac{h c I c_{i} * U h_{i, 1}}{h c I c_{i}+h r I c_{i}+U h_{i, 1}}
$$

Equation 2

### 1.1.3 Zone Balance Calculation Sequence

The temperatures in the zone are determined using a thermal balance method. The following procedure is followed each time step.

At the start of the simulation, say time t , assume all temps $\operatorname{Ta}(t), \operatorname{Tr}(t), T_{i, 1}(t), T_{i, 2}(t)$, etc. are known along with all the solar gains, internal gains, etc.
(1) First, the layered mass temperatures are updated using the explicit Euler routine (see Section 1.2), giving $T_{i, 1}(t+d t), T_{i, 2}(t+d t)$, etc. The Euler method determines each of these mass temperatures assuming that all the boundary conditions (temperatures and heat sources) that cause the change in the mass temperatures,
are conditions at time t . Thus the mass node temperatures can be in any order, independently of each other.
(2) Next, a steady-state instantaneous energy balance at the Ta and Tr nodes is made at time $t+d t$. This balance involves the mass temperatures determined for time $t$ +dt in Step-1, as well other heating or cooling sources at time $t+\mathrm{dt}$. The balance in this step involves querying the HVAC control algorithm which allows heating, cooling and ventilation (forced or natural) in response to scheduled setpoints. The idealized control system is assumed to keep the zone at exactly the scheduled setpoint unless Ta is in the deadband or if the HVAC capacity is exceeded, whereupon the system runs at maximum capacity, and Ta floats above or below the relevant setpoint. While the heating, cooling and forced ventilation system capacities are scheduled inputs, the natural ventilation capacity is dependent on the current zone and environment conditions.

Thus, the energy balance at the Ta and $\operatorname{Tr}$ nodes returns either the heating, the cooling or the ventilation required to meet the setpoint, or else returns the floating Ta that results at the capacity limits or when Ta is in the deadband.

At this stage the conditions have been predicted for the end of the time step, and steps 1 and 2 and repeated. The various boundary conditions and temperature or air flow sensitive coefficients can be recalculated as necessary each time step at the beginning of step (1), giving complete flexibility to handle temperature sensitive heat transfer and control changes at a time step level.

Note that step (2) treats the energy balance on Ta as a steady state balance, despite the fact that air mass makes it a transient problem. However, as shown in Section 1.3.1, if the air mass temperature is updated using an implicit-difference method, the effect of the air mass can be duplicated by employing a resistance, $\Delta t /$ Cair, between the Ta node and a fictitious node set at the beginning of the time-step air temperature $\mathrm{TaL}=\mathrm{Ta}(\mathrm{t})$, and shown as such in Figure 3.

The overall CSE Calculation Sequence is summarized below:

## Hour

Determine and distribute internal gains.

## Sub-hour

Determine solar gain on surfaces.
Determine surface heat transfer coefficients.
Update mass layer temperatures.
Find AirNet mass flows for non-venting situation (building leakage + last step HVAC air flows).

Find floating air temp in all zones / determine if vent possibly useful for any zone.

## If vent useful

- Find AirNet mass flows for full venting
- Find largest vent fraction that does not sub-cool any zone; this fraction is then used for all zones.
- If largest vent fraction $>0$, update all floating zone temperatures assuming that vent fraction
Determine HVAC requirements for all zones by comparing floating temp to setpoints (if any)
- System heating / cooling mode is determined by need of $1^{\text {st }}$ zone that requires conditioning
- For each zone, system indicates state ( $t$ and $w$ ) of air that could be delivered at register (includes duct loss effects). Zone then requests air flow rate required to hold set-point temperature
Determine HVAC air flow to zones (may be less than requested); determine zone final zone air temperatures.

Determine system run fraction and thus fuel requirements.
Determine zone humidity ratio for each zone.
Calculate comfort metrics for each zone.

### 1.2 Updating Layered Mass Temperatures

The heat transfer through the layered constructions is assumed to be one dimensional. The heat conduction equation ( $\frac{\partial^{2} T^{2}}{\partial x^{2}}=\frac{1}{\alpha} \frac{\partial T}{\partial t}$ ) is solved by using finite differences ( $\Delta$ t and $\Delta x$ ) to approximate the differential increments in time and distance; $a$ is the thermal diffusivity. The smaller the finite increments, the more accurate the solution. The homogeneous layers are divided into lumps $\Delta x$ thick, and the lumps are represented by the two-conductance/one-capacitance "T" circuits shown for each layer in Figure 2. Frequently the actual layer thicknesses as sufficiently thin that $\Delta x$ can be taken as the layer thickness. However, at times the actual layer of homogeneous material must be divided into smaller thicknesses. See Section 1.4-Discretization Errors for the criterion used to determine $\Delta x$ and $\Delta t$.

The temperatures of the mass nodes are updated every time step using the Euler explicit numerical integration method (see Press et al), whereby the change in temperature of the mass during the time step is based only on the boundary conditions at the beginning of the time step. The boundary conditions are the temperatures of the surrounding nodes and other heat flow sources.

To update $T_{i, 1}$ in Figure 3, for example, if the rate of heat transfer into $T_{i, 1}$ is equated to its rate of change in internal energy, resulting in the differential equation for mass temperature $T_{i, 1}$ :

$$
\frac{d T_{i, 1}}{d t}=\frac{T s s_{i, 1}-T_{i, 1}}{\tau}
$$

Equation 3
where $T_{i, 1}$ is the surface layer mass temperature, and $T s s_{i, 1}$ is the temperature $T_{i, 1}$ would have if steady state were reached:

$$
T s s_{i, 1}=\frac{c c c_{i, 1} T a+c r c_{i, 1} T r+U b n_{i, 1} T_{i, 2}+Q s_{i}^{\prime}}{c c c_{i, 1}+c r c_{i, 1}+U b n_{i, 1}}
$$

Equation 4
$Q s_{i}^{\prime}$ is given by
Equation 1, $c c c_{i, 1}$ by Equation $2, T_{i, 2}$ is the temperature of mass node 2, and $\tau$ is the time constant of mass node 1 given by:

$$
\tau=\frac{c a p_{i, 1}}{c c c_{i, 1}+c r c_{i, 1}+U b n_{i, 1}}
$$

Equation 5
The heat capacity of layer- 1 is $\operatorname{cap}_{i, 1}\left(\mathrm{Btu} / \mathrm{t}^{2}-\mathrm{F}\right) . U b n_{i, 1}$ is the conductance between nodes 1 and 2 , given by:

$$
U b n_{i, 1}=\frac{1}{\frac{1}{U h_{i, 1}}+\frac{1}{U h_{i, 2}}}
$$

Equation 6
To integrate of Equation 3 over a time step, the Euler procedure assumes that the right hand side of the equation remains constant over the time step at its value at the beginning of the time step. In this case the mass temperature at the end of the time step becomes:

$$
T_{-}(i, 1)(t+\Delta t)=T_{-}(i, 1)(t)(1-\Delta t / \tau)+T s s(\Delta t / \tau)
$$

Equation 7
If the capacitance of any layer is zero (a convecting air layer for example) its updated temperature is set equal to Tss. That is, the temperature at the central node is determined by a steady state energy balance.

All of the mass nodes are updated in an analogous fashion each time step. The order in which the masses are updated is irrelevant because they are updated based only on the values of variables at the beginning of the time step, not on the values that may have been updated since.

### 1.3 Zone Energy Balance

### 1.3.1 Implicit Update of Air Temperature

Similar to the energy balance on the construction mass nodes, an energy balance on the air node gives the differential equation:

$$
\frac{d T_{a}}{d t}+\frac{T_{a}}{\tau_{a}}=\frac{T s s}{\tau_{a}}
$$

Equation 8
where Tss, the asymptotic steady state temperature of $T a$, includes all the sources connected to Ta. For simplicity, if the zone only contained the one construction ( $i=1$ ) and one window $(j=1)$, like in Figure 3, then from a steady state energy balance Tss is given by:

$$
\begin{gathered}
T_{s s}=\left[T_{\text {out }}\left(U_{\text {inf }}+U_{v}\right)+c c w_{1} A w i n_{1} T_{\text {out }}+\text { ccc }_{1} \text { Acon }_{1} T_{1,1}+\text { Acon }_{1} Q s_{1}^{\prime \prime}+\text { Awin }_{1} Q s w_{1}^{\prime \prime}\right. \\
+ \text { Qint } * \text { fintC }+Q h c * \text { frConv }+C X * T r] / \text { Usum }
\end{gathered}
$$

Equation 9
where

$$
\begin{aligned}
& C X=\text { CXair }+ \text { CXW } 1+C X C_{1} \\
& U s u m=U_{i n f}+U_{v}+c c w_{1} \text { Awin }_{1}+c c c_{1} \text { Acon }_{1}+C X
\end{aligned}
$$

Equation 10

Equation 11
and the air time constant is:

$$
\tau_{a}=\frac{\text { Cair }}{\text { Usum }}
$$

Equation 12
Equation 8 is solved using an full implicit (or backward time) difference, similar to the Euler explicit method except here the right hand side of the equation remains constant over the time step at its value at the end of the time step, not its value at the beginning as in the Euler method. Thus,

Equation 8 then becomes:

$$
T_{a}(t+\Delta t)=\frac{\frac{T_{a}(t) \tau_{a}}{\Delta T}+\operatorname{Tss}(t+\Delta t)}{\frac{\tau_{a}}{\Delta t}+1}
$$

Equation 13
Where the times $t$ and $t+\Delta t$ in parenthesis indicate the terms are evaluated at the beginning and end of the time step, respectively. Substituting Equation 12 for $\tau_{a}$, Equation 13 can be put in the convenient form:

$$
T_{a}(t+\Delta t)=\frac{\frac{T_{a}(t) \text { Cair }}{\Delta T}+U \operatorname{sum} * T \operatorname{ss}(t+\Delta t)}{\frac{\text { Cair }}{\Delta t}+U \operatorname{sum}}
$$

Equation 14
As this equation shows, with the implicit difference the effect of the air mass can be thought of as a resistance, $\Delta t /$ Cair, between the Ta node and a fictitious node set at the air temperature at the value it was at the beginning of the time step, $T a L=T a(t)$.

This alternative is known as an 'associated discrete circuit'. Leaving out the explicit time references, Equation 14 can be written:

$$
T a=\frac{\frac{T a L * \text { Cair }}{\Delta T}+U s u m * T s s}{\frac{\text { Cair }}{\Delta t}+U s u m}
$$

Equation 15
where $T a$ and Tss are evaluated at the end of the time step, and TaL stands for $T_{a}(t)$ at the beginning of the time step. Note that Equation 15 still contains the variable Tr (hidden in Tss) which is unknown. $\operatorname{Tr}$ can be eliminated by making an energy balance on the $\operatorname{Tr}$ node and substituting the expression for $\operatorname{Tr}$ into Equation 15. This is done for the complete set of equations that follow.

### 1.3.2 Zone Balance Equations

The complete set of zone energy balance equations for multiple windows and constructions are given below. Terms containing $Q v$ and $Q h c$ are kept separate so that the resulting equations can be solved for $Q v$ or $Q h c$ when $T a$ is fixed at a setpoint.

### 1.3.2.1 Air Node Balance

The energy balance equation on the Ta node, comparable to Equation 15 above is:

$$
T a=\frac{Q v+Q h c \cdot f r C o n v+N a i r+C X \cdot T r}{\text { Dair }+C X}
$$

Equation 16
The Equation 16 form, using $\mathrm{Q}_{\mathrm{v}}$, is used when heat is transferred to a conditioned zone with ventilation or infiltration air. When heat is transferred to an unconditioned zone due to ventilation or infiltration, $Q_{V}$ is replaced by the essentially equivalent form given by Equation 17, wherein $Q_{v}$ is replaced by $Q_{v}=m C_{p} \Delta T$ such that mdot $* c_{p} * T$ is added to the numerator and mdot $* \mathrm{c}_{\mathrm{p}}$ is added to the denominator. This was implemented to eliminate oscillations in Ta.

$$
T a=\frac{S T+Q h c \cdot f r C o n v+N a i r+C X \cdot T r}{S B+\text { Dair }+C X}
$$

Equation 17
where,

$$
S T=\sum m \operatorname{dot} * c_{p} * T
$$

Equation 18
where T is the temperature of the air in the zone supplying the infiltration or ventilation air.

$$
S B=\sum m \operatorname{dot} * c_{p}
$$

Equation 19

$$
C X=C X a i r+\sum^{c o n} A \operatorname{con}_{i} \cdot c x c_{i}+\sum^{\text {win }} A w i n_{i} \cdot c x w_{i}
$$

Equation 20
with the sum's for all constructions and all windows respectively.

$$
\begin{aligned}
& \text { Nair } \\
& =\text { TaL }\left(\frac{\text { Cair }}{d t}\right)+\text { Qint } \cdot \text { fintC } \\
& +\sum^{\text {con }} A c o n_{i} \cdot\left(c c c_{i} \cdot T_{i 1}+\frac{\left.Q_{s i} \cdot{h c I c_{i}}^{h c I c_{i}+h r I c_{i}+U h_{i 1}}\right)+\sum^{\text {win }}\left[A w i n_{i}\left(c c w_{i} \cdot T e o_{i}+I_{j} \cdot F P_{j}\right)\right]}{}\right.
\end{aligned}
$$

Equation 21

$$
\text { Dair }=\frac{\text { Cair }}{d t}+\sum^{\text {con }} A c o n_{i} \cdot c c c_{i}+\sum^{\text {win }} A \text { win }_{i} \cdot c c w_{i}
$$

Equation 22
Qv is the heat transfer to the air node due to infiltration and forced or natural ventilation.

### 1.3.2.2 Radiant Node Balance

An energy balance on the $\operatorname{Tr}$ node gives Equation 23.

$$
\operatorname{Tr}=\frac{Q h c \cdot(1-f r \operatorname{Conv})+N_{\text {rad }}+C X \cdot T a}{\operatorname{Drad}+C X}
$$

Equation 23
where,

$$
\begin{aligned}
& \text { Nrad }=\text { Qint } \cdot \text { fintLW }+\sum^{\text {con }} \text { Acon }_{i}\left[\operatorname{crc}_{i} \cdot T_{i 1}+Q s i_{i} \cdot \frac{h r I c_{i}}{h c I c_{i}+h r I c_{i}+U h_{i 1}}\right] \\
& +\sum^{\text {win }} \text { Awin }_{i}\left[\mathrm{crw}_{i} \cdot \mathrm{Teo}_{i}+I_{j} \cdot F M_{j}\right] \\
& \text { Drad }=\sum^{\text {con }} \text { Acon }_{i} \cdot \mathrm{crc}_{i}+\sum^{\text {win }} \text { Awin }_{i} \cdot \mathrm{crw}_{i}
\end{aligned}
$$

### 1.3.2.3 Simultaneous Solution of Ta and Tr Equations

Equation 16 and Equation 23 can be solved simultaneously to eliminate $\operatorname{Tr}$ and give $T a$ explicitly:

$$
T a=\frac{(Q v+Q h c \cdot f r C o n v+\text { Nair })(\text { Drad }+C X)+C X(\text { Nrad }+Q h c(1-f r C o n v)}{(\text { Dair }+ \text { Drad }) C X+\text { Dair } \cdot \text { Drad }}
$$

Equation 26
Similar to Equation 16 and Equation 17), the alternate form of Equation 26 is given by Equation 27.

$$
T a=\frac{(S T+Q h c \cdot f r C o n v+\text { Nair })(\text { Drad }+C X)+C X(\text { Nrad }+Q h c(1-f r C o n v)}{(S B+\text { Dair }+ \text { Drad }) C X+(S B+\text { Dair }) \text { Drad }}
$$

Equation 27
Substituting Ta from Equation 26 into Equation 23 gives $\operatorname{Tr}$.

### 1.3.2.4 Qhc and Qv Equations

When Ta is at either the heating or cooling setpoints, Equation 26 is solved to determine the required Qhc. In this case Qv is set to QvInf.

$$
\begin{aligned}
& \text { Qhc } \\
& =\frac{T a(\text { Dair } * \operatorname{Drad}+C X(\text { Dair }+ \text { Drad }))-(\text { Nair }+Q v)(\text { Drad }+C X)-\text { Nrad } * C X)}{\text { frConv } * \text { Drad }+C X}
\end{aligned}
$$

Equation 28
Similarly, when Ta is at the ventilation setpoint, Equation 26 can be solved for Qv to give:

$$
Q v=\frac{(\text { Dair } * \text { Drad }+C X(\text { Dair }+ \text { Drad })) T a-C X(N r a d+Q h c(1-f r C o n v))}{\text { Drad }+C X}
$$

Equation 29
With Qhc $=0$ this becomes:

$$
\mathrm{Qv}=\frac{(\text { Dair } * \operatorname{Drad}+C X(\text { Dair }+ \text { Drad })) T a-N a i r(\operatorname{Drad}+C X)-C X * N r a d}{\text { Drad }+C X}
$$

The zone balance is essentially an instantaneous balance, so all the temp inputs are simultaneous values from the end of the time step (with the exception of TaL; see Section 1.3.1). Although the balance is with contemporary temperatures, many of the heat flows in Nair etc., are based on last time step conditions.

### 1.3.3 Thermostat Logic

At the end of each time step the program finds the floating temperature of the zone without HVAC $(Q h c=0)$ and with venting $\mathrm{Qv}=\mathrm{QvInf}$. This floating temperature found
from Equation 26 is defined as TS1. Next, the venting capacity is determined (see Section 1.9.3.10, Heat Flow), and Equation 26 is solved for Ta at the full venting capacity. This Ta is defined as TS2.
TS1 will satisfy one of the four cases:

- TS1>TC
- TC > TS1 > TD
- TD > TS1 > TH
- TH > TS1

Similarly, TS2 will satisfy one of the four cases:

- TS2 > TC,
- TC > TS2 > TD
- TD > TS2 > TH
- TH > TS2
where TC, TD, and TH are the scheduled cooling, ventilation, and heating setpoints, with TC > TD > TH.

Based on the cases that TS1 and TS2 satisfy, nested logic statements determine the appropriate value of heating, cooling, venting, or floating .
For example, if TS1 and TS2 are both > TC, then $Q_{v}$ is set QvInf and Ta is set to TC, and Equation 28 is solved for the required cooling, Qhc. If Qhc is smaller than the cooling capacity at this time step then Qhc is taken as the current cooling rate and the zone balance is finished and the routine is exited. If Qhc is larger than the cooling capacity then Qhc is set to the cooling capacity, and Equation 26 is solved for Ta, floating above TC due to the limited cooling capacity. If Ta < TS2 then Ta and Qhc are correct and the zone balance routine is exited. If this $\mathrm{Ta}>\mathrm{TS} 2$ then Ta is set equal to TS2, Qhc is set to zero, and Equation 29 is solved for the ventilation rate $Q v$, and the Zone Balance routine is complete.

Similar logic applies to all other logically possible combinations of the TS1 and TS2 cases above.

### 1.3.4 Limiting Capacities

The limiting capacity of the heating and cooling system is determined each time step by multiplying the scheduled nominal air handler input energy capacity by the duct system efficiency. To avoid iteration between the conditioned zone and unconditioned zone simulations, the duct system efficiency is taken from the last time-step's unconditioned zone simulation, or unity if the system mode (heating, cooling, venting, or floating) has changed.

### 1.4 Discretization Errors

The temperatures predicted by Equation 7, which updates the layered mass temperatures, is subject to errors due to the finite lump size chosen to represent real wall homogeneous layers. It is also subject to errors due to the finite time step $\Delta \mathrm{t}$. Similarly Equation 14 for updating the air mass temperature is subject to error due to the finite time step chosen.

Discretization errors can be made negligible by reducing the layer thicknesses and time step to very small values. However for practical run time minimization purposes it is useful to have large $\Delta t$ and $\Delta x$ layers, insofar as accuracy allows. The range of choices of $\Delta t$ and $\Delta x$ is narrowed if accurate results are only required for a limited range of frequencies of the driving boundary conditions. Only extremely thin lumped layers have the correct frequency response at high frequencies. To model environmental influences, 3 cycles/day (8-hr period sinusoid) is likely the highest frequency necessary to consider when determining the frequency response of buildings (Goldstein, Anderson and Subbarao). Higher frequencies may be desirable for accurately modeling things like control step changes. During the program development, accuracy was measured by analyzing the frequency response at 3 cycles/day.

The exact frequency response of a layered wall can be obtained using the matrix method (Section 3.7 of Carslaw \& Jaeger) which gives the inside driving point admittance (from the inside air node), the outside driving point admittance, and the transfer admittance, for any frequency. The magnitude of the inside driving point admittance is the principle parameter used to assess algorithm accuracy.

At the frequency chosen, 3 cycles/day say, the exact driving point admittance of the real wall (with homogeneous layers) can be obtained from the matrix method. Similarly the exact driving point admittance of the lumped wall which the user has chosen to represent the real wall, can also be determined by the matrix method. Comparing these two results shows the accuracy of the lumping assumptions, independent of time step considerations.

The time discretization error associated with Equation 7 at the frequency chosen can be assessed by comparing the driving point admittance predicted by the CSE code, when the air node is driven with a sinusoidal temperature at the chosen frequency, to the theoretical admittance of the lumped wall. Note that this procedure measures the global discretization error, larger potentially than the per time-step error.

Using this procedure for typical lightweight residential construction, we have confirmed that the errors in the temperature predictions made by the CSE finite difference algorithms indeed tend toward zero as $\Delta t$ and $\Delta x$ are reduced toward zero.

### 1.4.1 Layer Thickness of a Homogeneous Material

The lumped layer thickness, $\Delta x$, should be is chosen thin enough that the single temperature of the lumped layer is a good measure of the average temperature over a
width $\Delta x$ of the sinusoidal temperature distribution in the material. That is, the temperature of the sinusoidal wave should not vary much over the layer width. This criterion is similar to that used by Chirlian (1973) to determine the appropriate lump sizes in electrical circuits.

The wave length of the temperature distribution in a particular material is given by

$$
\lambda=2 \pi d_{p}
$$

Equation 31
where $d_{p}$, the penetration depth, an intrinsic characteristic of the material, is given by

$$
d_{p}=\sqrt{2 \alpha / \omega}
$$

Equation 32
where the angular frequency $\omega=\frac{2 \pi}{\text { period }}, \mathrm{a}$ is the thermal diffusivity of the layer material, and $\omega$ is the highest angular frequency of the environmental boundary conditions for which good frequency response is desired. As a general guideline it is suggested that the lumped layer thicknesses, $\Delta x$, be chosen to be thinner than the penetration depth for the layer. That is, select

$$
\Delta x \lesssim d_{p}
$$

Equation 33
Substituting Equation 32 into Equation 33 shows that the rule of Equation 33 limits the lump size $\Delta x$ to about $16 \%$ of the wavelength:

$$
\Delta x \lesssim d p=\lambda / 2 \pi=0.16 \lambda
$$

Equation 34
The Equation 33 rule is more important for the modeling of layers on the inner side of the wall, where the layers are subjected to the higher frequency harmonics of inside driving conditions. Deeper into the wall the high frequency harmonics begin to be damped (by about a factor of $e^{-\frac{\Delta x}{d_{p}}}$ ), so accurate modeling is of less significance.

### 1.4.2 Choosing the Time Step

The time step used in the code is input by the user. For high accuracy Equation 7 and Equation 14 should be applied using a time step that is a small fraction of the smallest time constant of any layer.

$$
\Delta t \ll \tau
$$

Equation 35
Thin layers of a material have a smaller time constant $\tau$ than thick layers. The time constant of a layer scales as $\sim \beta^{2}$, where $\beta$ is the a layers dimensionless thickness defined as $\beta=\frac{\Delta x}{d_{p}}$. Thus, if a layers dimensionless thickness is reduced by a factor of
two, the time constant is reduced by a factor of four. Therefore the time to run an annual simulation can increase rapidly for small $\beta$ 's. Small tau layers have cv increased such that tau $=\mathrm{dt}$.

Note that the Euler mass layer update algorithm of Equation 7becomes unstable when $\Delta t>\tau$. . The predicted temperatures will oscillate with increasing amplitude each time step. The code outputs warnings whenever a mass node update is performed for which $\Delta t>\tau$.

Like the explicit Euler method, the implicit differencing used at the air node is most accurate for small time steps relative to the air's time constant (Equation 12). The implicit difference method is never unstable, and time steps larger than the air time constant give useful, if somewhat inaccurate predictions. The air balance could have been solved using an Euler difference, but since the air time constant is likely the smallest in the zone, it would dictate smaller time steps than is afforded using the implicit method

### 1.5 Surface Heat Transfer Coefficients

The radiation coefficients for surfaces inside the conditioned zone are given in Section 1.6.1 where the long-wave radiant network model is discussed.

### 1.5.1 Local Wind Velocity Terrain and Height Correction

The wind velocity as a function of height at the house site is obtained from the meteorological station wind measurement by making adjustments for terrain and height differences between the meteorological station and the house site.

### 1.5.1.1 Sherman-Grimsrud method

This method uses Equation 36 which determines the wind velocity $V(z)$, in $\mathrm{ft} / \mathrm{sec}$, at any height $z$ (ft) based on the wind velocity, $V_{m e t}$ in $\mathrm{ft} / \mathrm{sec}$, measured at a location with a Class II terrain (see Table 1) and at a height of 10-meters ( 32.8 ft ):

$$
V(z)=S C * V_{m e t} * \alpha *\left(\frac{z}{32.8}\right)^{\gamma}
$$

Equation 36
where,
$\alpha$ and $\gamma$ are obtained from Table 1 for the terrain class at the building location.
$S C=$ shielding coefficient from Table 2 for the building location.
$V(z)=$ wind velocity at height $z$ at the building location ( $\mathrm{ft} / \mathrm{sec}$ ).
$V_{\text {met }}=$ wind velocity ( $\mathrm{ft} / \mathrm{sec}$ ) measured at 10-meters height in a Class II location.
The terrain factor of Table 1 is a general factor describing the influence of the surroundings on a scale on the order of several miles. The shielding factor of Table 2 is
a local factor describing the influence of the surroundings on a scale of a few hundred yards.

Table 1: Parameters for Standard Terrain Classifications

| Class | Y | A | Description |
| :---: | :---: | :---: | :--- |
| I | 0.10 | 1.30 | Ocean or other body of water with at <br> least 5 km of unrestricted expanse |
| II | 0.15 | 1.00 | Flat terrain with some isolated obstacles <br> (buildings or trees well separated) |
| III | 0.20 | 0.85 | Rural areas with low buildings, trees, etc. |
| IV | 0.25 | 0.67 | Urban, industrial, or forest areas |
| V | 0.35 | 0.47 | Center of large city |

Source: NORESCO for California Energy Commission
Table 2: Local Shielding Parameters

| Class | $\mathbf{C}^{\prime}$ | SC | Description |
| :--- | :---: | :---: | :--- |
| I | 0.324 | 1.000 | No obstructions or local shielding |
| II | 0.285 | 0.880 | Light local shielding and few obstructions |
| III | 0.240 | 0.741 | Moderate local shielding, some <br> obstructions within two house heights |
| IV | 0.185 | 0.571 | Heavy shielding, obstructions around <br> most of the perimeter |
| V | 0.102 | 0.315 | Very heavy shielding, large obstructions <br> surrounding the perimeter within two <br> house heights |

Source: NORESCO for California Energy Commission

### 1.5.1.2 Implementation

If it is assumed that the default value of the terrain classification at the building location is Class IV terrain of Table 1, and the default local shielding coefficient is $\mathrm{SC}=0.571$ of Class IV of Table 2, then the wind velocity at the building site at height $\boldsymbol{z}$ is given by:

$$
\mathrm{V}(\mathrm{z})=\mathrm{SC} * \mathrm{~V}_{\mathrm{met}} * \alpha *\left(\frac{\mathrm{ze}}{32.8}\right)^{\gamma}=0.571 * V_{\text {met }} * 0.67 *\left(\frac{z}{32.8}\right)^{0.25}
$$

or,

$$
V(z)=0.16 * z^{0.25} * V_{m e t}
$$

For example, for 1,2 , and 3 story buildings, of $9.8 \mathrm{ft}(3-\mathrm{m}), 19.7 \mathrm{ft}(6-\mathrm{m})$, and 29.5 ft ( $9-m$ ), respectively, then the local eave height wind velocities are:
$\mathrm{V}(9.8)=0.16 * 9.8^{0.25} * V_{\text {met }}=0.28 \mathrm{~V}_{\text {met }}$ for a 1-story building.

V(19.7)
$=0.34 \mathrm{~V}_{\text {met }}$ for a 2-story building.
$\mathrm{V}(29.5) \quad=0.38 \mathrm{~V}_{\text {met }}$ for a 3-story building.
(References: Sherman \& Grimsrud (1980), Deru \& Burns (2003), Burch \& Casey (2009), European Convention for Constructional Steelwork (1978).)

### 1.5.2 Convection Coefficient for Inside and Outside Surfaces of Zones

The schematic buildings in Figure 4 and Figure 5 show all of the possible interior heat transfer situations for which the convection heat transfer coefficients are determined. The figures symbolically show the nature of the heat transfer boundary layer, and the heat flow direction. The symbols used are explained at the end of this document. Similar schematics have not been done for the outside surfaces.

The equations are developed that give the heat transfer coefficient for each of the Figure 4 and Figure 5 situations, and for the building outside surfaces. The heat transfer coefficients depend on the surface tilt angle $\theta(0 \leq \theta \leq 90)$, the surface and air temperatures, and on whether the heat flow of the surface has an upward or downward facing component.

The results, which apply to both the UZ and CZ zones, can be summarized as follows:

### 1.5.2.1 Inside surfaces

For floors, and either vertical walls, or walls pulled-in-at-the-bottom:
If Tair > Tsurf use Equation 53. (heat flow down)
If Tair < Tsurf use Equation 52. (heat flow up)
For ceilings (horiz or tilted), and walls pulled-in-at-the-top:
If Tair > Tsurf use Equation 52. (heat flow up)
If Tair < Tsurf use Equation 53. (heat flow down)

### 1.5.2.2 Outside surfaces

For all vertical walls, and walls with moderate tilts use Equation 54.
For horizontal or tilted roof, use Equation 57.

Figure 4: Heat Flow Down Situations


Figure 5: Heat Flow Up Situations


## Explanation of Symbols


symbolizes the air is stratified next to the surface and the direction the heat flow would be if there is any--the figure equations give zero heat flow in this case.
na
symbolizes the equations are not applicable to the adjacent surface, because the surface doesn't have an upward or downward facing component, as the case requires.
symbolizes plumes of air buoyantly leaving the adjacent surface, and the direction of heat flow, due to Rayleigh-Benard instability.

### 1.5.2.3 Natural convection equations

Equation 37, from Churchill and Chu (see Eq. 4.86, Mills (1992)), is used to determine the natural convection coefficients for tilted surfaces. The choice of this equation is partly informed by the work of Wallenten (2001), which compares the Churchill and Chu equation with other correlations and experimental data.

Equation 37 is for turbulent convection $\left(10^{9}<\mathrm{Ra}<10^{12}\right)$, expected to be the dominant case in room heat transfer.

Equation 37 applies to either side of a tilted surface for angles ( $0 \leq \theta \leq 88^{\circ}$ ) if the heat flow has a downward component, or the heat flow is horizontal.

Equation 37 also applies to either side of a tilted surface for angles $\theta<60^{\circ}$ if the heat flow has an upward component, or the heat flow is horizontal.

$$
N u=0.68+0.67(R a \cdot \psi)^{0.25}\left(1+1.6 * 10^{-8} R a \cdot \psi\right)^{\frac{1}{12}} \quad \text { Equation } 37
$$

where,
$R a=$ the Rayleigh number.
$N u=$ the Nusselt number.
$\operatorname{Pr}=$ the Prandtl number.

$$
\psi=\left[1+\left(\frac{0.492}{P r}\right)^{\frac{9}{16}}\right]^{-\frac{16}{9}}
$$

Using $\psi=0.349$ for $\operatorname{Pr}=0.72$, Equation 37 reduces to:

$$
N u=0.68+0.515 R a^{0.25}\left(1+5.58 * 10^{-9} R a\right)^{\frac{1}{12}}
$$

Equation 38
For high Ra $\left[R a \approx>10^{9}\right]$, neglecting the additive terms " 1 " and " 0.68 " in Equation 37 gives:

$$
N u=0.1057 R a^{\frac{1}{3}}
$$

Equation 39
By the definition of the Nusselt number, the natural convection heat transfer coef, $h_{n}$ is:

$$
\begin{aligned}
& h_{n} \equiv N u * \frac{k(\text { air })}{L_{\text {char }}} \\
& \text { At } 70 F, R a=1.66 \times 10^{6} L^{3}|\Delta T| \cos (\theta) \text {, and } k=0.0148, \\
& \text { reduces to: }
\end{aligned}
$$

$$
h_{n}=0.185(|\Delta T| \cos (\theta))^{\frac{1}{3}}
$$

Equation 40
Note that $h_{n}$ is independent of characteristic length $\mathrm{L}_{\mathrm{char}}$.

## Downward heat flow

According to Mill's(1992) Equation 37 doesn't apply to downward heat flow for $\theta>88^{\circ}$. At 70 F, for a 20 ft characteristic length, the 0.68 term predicted by Equation 38 for $\theta=$ $90^{\circ}$ corresponds to $h_{n}=\frac{0.68 k}{L}=0.0005$; essentially zero. Although the downward heat flow is ideally stably stratified (three cases shown in Figure 4), most measurements and modeling practice indicate $h$ may be larger than zero. We use the equation of Clear et al. for the minimum, for heat flow down:

$$
N u=0.27 R a^{0.25} \quad \text { Equation } 41
$$

Clear's equation reduces to:

$$
h_{n}=0.27(0.0148)\left(\frac{4}{L}\right)\left(1.642 E 6 \Delta T\left(\frac{L}{4}\right)^{3}\right)^{0.25}
$$

or

$$
h_{n}=0.202\left|\frac{\Delta \mathrm{~T}}{\mathrm{~L}_{\text {char }}}\right|^{\frac{1}{4}}
$$

where, $L_{\text {char }}$ is the wall characteristic length; see Equation 51 definitions.
Adding this h to Equation 40 gives:

$$
h_{\text {down }}=M A X\left[0.185(|\Delta T| \cos (\theta))^{\frac{1}{3}}, 0.202\left|\frac{\Delta \mathrm{~T}}{\mathrm{~L}_{\text {char }}}\right|^{\frac{1}{4}}\right] 0 \leq \theta \leq 90
$$

Equation 43
The following simplification is made, where the exponent of the second term is changed to $1 / 3$, so that $|\Delta T|^{\frac{1}{3}}$ can be factored out:

$$
h_{\text {down }}=|\Delta T|^{\frac{1}{3}} M A X\left[0.185(\cos (\theta))^{\frac{1}{3}}, 0.202 \mathrm{~L}_{\text {char }}^{-\frac{1}{3}}\right] \quad 0 \leq \theta \leq 90
$$

## Equation 44

Changing the exponent means Equation 44 gives same answer as Equation 43 only when $\frac{\Delta T}{L_{\text {char }}}=1$. But Equation 44 would have acceptable error for other $\frac{\Delta T}{L_{\text {char }}}$ ratios, and gives more or less the right dependence on $\Delta T$. If in addition, one assumes a typical $L_{\text {char }}=15$, say, then the minimum term becomes: $0.202 \mathrm{~L}_{\text {char }}^{-\frac{1}{3}}=0.08$, giving the final reasonable form:

$$
h_{\text {down }}=|\Delta T|^{\frac{1}{3}} M A X\left[0.185(\cos (\theta))^{\frac{1}{3}}, 0.08\right] \quad 0 \leq \theta \leq 90
$$

Equation 45

## Upward heat flow for $\boldsymbol{\theta} \leq \mathbf{6 0}^{\boldsymbol{o}}$

For the inside \& outside of walls where the heat flow has an upward (or horizontal heat flow at the limit $\theta=0^{\circ}$ ), and the outside of roofs, Equation 40 applies:

$$
h_{n}=0.185(|\Delta T| \cos (\theta))^{\frac{1}{3}}
$$

## Upward heat flow for $\boldsymbol{\theta}>\mathbf{6 0}^{\boldsymbol{o}}$

To handle cases of upward heat flow for $\theta>60^{\circ}$, $h_{u p}$ is found by interpolating between Equation 40, evaluated at $\theta=60^{\circ}$, and Equation 47 at $90^{\circ}$. Equation 46, for heat transfer from a horizontal surface $(\theta=90)$, is from Clear et al. (Eq. 11a). It is close to the much used McAdams equation suggested by both the Mills(1992) and Incropera-Dewitt textbooks.

$$
N u=0.15 R a^{\frac{1}{3}}
$$

Equation 46
At 70-F, Equation 46 reduces to

$$
h_{n}=0.26(\Delta T)^{\frac{1}{3}}
$$

## Equation 47

Interpolating, for upward heat flow cases with $\theta \geq 60^{\circ}$ :

$$
h_{u p}=0.185(\Delta T \cos (60))^{\frac{1}{3}}+\frac{\left(0.26(\Delta T)^{\frac{1}{3}}-0.185(\Delta T \cos (60))^{\frac{1}{3}}\right)(\theta-60)}{30}
$$

which reduces to:

$$
h_{u p}=(0.00377 * \theta-0.079)|\Delta T|^{\frac{1}{3}} \quad \text { for } 60^{\circ} \leq \theta \leq 90 \quad \text { Equation } 48
$$

where $\theta$ is in degrees.

### 1.5.2.4 Inside forced convection equation

Measured forced convection heat transfer coefficients are frequently correlated using an equation of the form

$$
h_{\text {ach }}=h_{\text {forcedIN }}=C_{\text {ach }} * A C H^{0.8}
$$

## Equation 49

The RBH model (Barnaby et al. (2004) suggests using $\mathrm{h}_{\mathrm{f}}=0.88 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}{ }^{2} \mathrm{~F}$ at ACH $=8$. This gives $C_{\text {ach }}=0.167$. Walton (1983) assumes $\mathrm{h}=1.08$ when the "air handler system is moving air through the zone." If this was at 8 ach, then this implies $C_{a c h}=0.205$.
1.5.2.5 Outside forced convection equation for all walls and all roofs

From Clear et al. (2001, Eq. (11a)),

$$
N u=W_{f} R_{f} 0.037 \operatorname{Re}^{0.8} \operatorname{Pr}^{\frac{1}{3}}
$$

Equation 50
Clear et al. used the Reynolds number based on a free-stream wind velocity 26.2 ft ( 8 m ) above the ground.

At 70F, Equation 50 reduces to:

$$
h_{V}=k * \frac{N u}{L}=0.527 W_{f} R_{f} \frac{V^{0.8}}{L^{0.2}}
$$

Equation 51
where for walls,
$L=$ wall $L_{\text {char }}=4 \frac{\text { Wall Area }}{\text { Wall Perimeter }}\left(=4 \frac{\text { Heightwidth }}{2(\text { Height }+ \text { width })} \approx\right.$ height of square wall $\left.=Z_{\text {eave }}\right)$
$V=$ wind velocity at eave height at building location, in $\mathrm{ft} / \mathrm{sec},=0.16 * \mathrm{Z}_{\text {eave }}^{0.25} *$
$V_{\text {met }}$ from Section 1.5.1.2.
$V_{m e t}=$ freestream wind velocity, in $\mathrm{ft} / \mathrm{sec}, 10 \mathrm{~m}(32.8 \mathrm{ft})$ above the ground at the meteorological station site.
$R_{f}=$ Table 3 value.
$W_{f}=0.63$
The wind direction multiplier, $W_{f}$, is defined as the average $h$ of all of the vertical walls, divided by the $h$ of the windward wall, with this ratio averaged over all wind directions. We estimated $W_{f}$ using the CFD and wind tunnel data of Blocken et al. (2009) for a cubical house. Blocken's Table 6 gives a windward surface convection coefficient of $\mathrm{h}_{\mathrm{c}} \approx 4.7 \mathrm{~V}^{0.84}$ (SI units), averaged over wind direction. Blocken's

Figure 9 gives $h_{c} \approx 7.5$ averaged over all vertical surfaces, for wind speed $V_{\text {met }}=3-$ $\mathrm{m} / \mathrm{s}$. Thus, we estimate $\mathrm{W}_{\mathrm{f}}=\frac{7.5}{4.7 \mathrm{~V}^{0.84}}=0.63$.
and for roofs,

$$
\mathrm{L}=\text { roof } \mathrm{L}_{\text {char }}=4 \frac{\text { Roof Plan Area }}{\text { Roof Perimeter }}\left(=4 \frac{\text { Length } w \text { Width }}{2(\text { Length }+ \text { width })} \approx \sqrt{\text { Roof Area }} \text { for square roof } \approx Z_{\text {eave }}\right)
$$

$\mathrm{V}=$ wind velocity $9.8 \mathrm{ft}(3 \mathrm{~m})$ above the eave height at building location, in $\mathrm{ft} / \mathrm{sec}$.

$$
\begin{aligned}
& \quad=0.16 *\left(\mathrm{Z}_{\text {eave }}+9.8\right)^{0.25} * \mathrm{~V}_{\mathrm{met}} \\
& W_{f}=1 \\
& R_{f}=\text { Table } 3 \text { value } .
\end{aligned}
$$

Walton (1983) assumed that the ASHRAE roughness factors of Table 3 apply to the convection coefficient correlations. The Clear et al. (2001) experiments tend to confirm the validity of these factors. Blocken et al. (2009) says, "The building facade has been assumed to be perfectly smooth. Earlier experimental studies have shown the importance of small-scale surface roughness on convective heat transfer. For example, Rowley et al. found that the forced convection coefficient for stucco was almost twice that for glass. Other studies showed the important influence of larger-scale surface roughness, such as the presence of mullions in glazed areas or architectural details on the facade, on the convection coefficient."

Table 3: Surface Roughness Parameter $R_{f}$ (Walton 1981)

| Roughness <br> Index | Rf | Example |
| :--- | :--- | :--- |
| 1 (very rough) | 2.1 | Stucco |
| 2 (rough) | 1.67 | Brick |
| 3 (medium rough) | 1.52 | Concrete |
| 4 (Medium <br> smooth) | 1.13 | Clear pine |
| 5 (Smooth) | 1.11 | Smooth plaster |
| 6 (Very Smooth) | 1 | Glass |

Source: NORESCO for California Energy Commission
1.5.2.6 Inside combined natural and forced convection

The combined convection coefficient is assumed to be the direct sum of the natural and forced convection coefficients:

For upward and horizontal heat flow:

$$
h_{\text {combined }}=h_{u p}+h_{\text {ach }}
$$

Equation 52
where,

$$
\begin{aligned}
& h_{u p}=\text { Equation } 40 \text { or Equation } 48 \text { depending on whether } \theta \text { is }<\text { or }>60^{\circ} . \\
& h_{a c h}=\text { Equation } 49
\end{aligned}
$$

For downward heat flow:

$$
h_{\text {combined }}=h_{\text {down }}+h_{\text {ach }}
$$

Equation 53
where,

$$
\begin{aligned}
& h_{\text {down }}= \\
& h_{\text {ach }}=\text { Equation } 49
\end{aligned}
$$

Equation 45
1.5.2.7 Outside combined natural and forced convection

The conclusion of Clear et al. (2001) is that the combined convection coefficient is best correlated by assuming it to be the sum of the natural and forced coefficients. For roofs, Clear et al. (2001) assumes that the natural and forced convection are additive, but that natural convection is suppressed by the factor $\eta$ given by Equation 56 when forced convection is large ( $\eta \rightarrow 0$ as the Reynolds number becomes large). We also assume this attenuation of the natural convection applies to the outside of the walls.

For all vertical walls, and walls with moderate tilts:

$$
h_{\text {combined }}=\eta h_{n}+h_{V}
$$

Equation 54
where,

$$
\begin{aligned}
& h_{n}=\text { Equation } 40 \\
& h_{V}=\text { Equation } 51 \\
& \eta=\frac{1}{1+\frac{1}{\ln \left(1+\frac{0.06 L|\Delta T|}{V^{2}}\right)}} \text { (to avoid divide by zero, if } \mathrm{V}=0 \text {, could set to } \mathrm{V}=0.001 \text { ) }
\end{aligned}
$$

where $L \& V$ are the same as used in Equation 51 for walls.
For roofs, Clear et al. (2001) assumes that the natural and forced convection are additive, but that natural convection is suppressed by the factor $\eta$ when forced convection is large ( $\eta \rightarrow 0$ as the Reynolds number becomes large). Clear gives $\eta$ as:

$$
\eta=\frac{1}{1+\frac{1}{\ln \left(1+\frac{G r_{L}}{R e_{L}^{2}}\right)}}
$$

At 70F, with $G r=2.28 \times 10^{6} L^{3}|\Delta T|, \mathrm{Re}^{2}=(6140 \mathrm{VL})^{2}$, and $\mathrm{L}=\mathrm{L}_{\text {char }}$ for surface , Equation 55 reduces to:

$$
\eta=\frac{1}{1+\frac{1}{\ln \left(1+\frac{0.06 L|\Delta T|}{V^{2}}\right)}}
$$

Equation 56
For roofs:

$$
h_{\text {combined }}=\eta h_{n}+h_{V}
$$

Equation 57
where,
$h_{n}=$ Equation 45 for downward heat flow.
$h_{n}=$ Equation 47 for upward heat flow.
$h_{V}=$ Equation 51 for upward or downward heat flow.
$\eta$ is from Equation 56
$L \& V$ are the same as used in Equation 51 for roofs.

### 1.5.2.8 Plots of equations

In Figure 6, the left hand column of plots are of Equation 53, for the downward heat flow cases shown in Figure 4. The right hand side plots are of Equation 52, for upward heat flow cases of Figure 5. All of the plots assume $T_{\text {film }}=70 F$, and $L_{\text {char }}=Z_{\text {eave }}=20$ ft.

Figure 6: Plots of Equations for Downward and Upward Heat Flow

DOWNWARD HEAT FLOW (Equation 53):


UPWARD HEAT FLOW (Equation 52):





## OUTSIDE surface convection coefficient Plot for a horizontal roof:

For $\Delta T=50 F, L_{\text {char }}=20 \mathrm{ft}, R_{f}=1.67:$
Figure 7: Outside Convection Coefficients, Natural Up and Down, and Forced


### 1.5.3 Outside Radiation Coefficients

### 1.5.3.1 Wall surfaces

The net long wave radiation heat exchange between the outside surface and the environment is dependent on surface temperature, the spatial relationship between the surface and the surroundings, and the properties of the surface. The relevant material properties of the surface, emissivity $\varepsilon$ and absorptivity $\alpha$, are complex functions of temperature, angle, and wavelength. However, it is generally assumed in building energy calculations that the surface emits or reflects diffusely and is gray and opaque ( $\alpha=\varepsilon, \tau=0, \rho=1-\varepsilon$ ).

The net radiant heat loss from a unit area of the outside of a construction surface to the outside environment is given by:

$$
q_{\text {rad }}=\varepsilon \epsilon_{g} \sigma F_{\text {gnd }}\left(T_{s}^{4}-T_{g}^{4}\right)+\varepsilon \sigma F_{s k y} \beta\left(T_{s}^{4}-T_{s k y}^{4}\right)+\varepsilon \sigma F_{s k y}(1-\beta)\left(T_{s}^{4}-T_{a}^{4}\right)
$$

Equation 58
where,

$$
\begin{aligned}
& \varepsilon=\text { surface emissivity. } \\
& \epsilon_{g}=\text { ground emissivity is assumed to be } 1 . \\
& \sigma=\text { Stephan-Boltzmann constant. }
\end{aligned}
$$

$$
\begin{aligned}
& T_{s}=\text { outside surface temperature. } \\
& T_{a}=\text { outside dry bulb temperature. } \\
& T_{g}=\text { ground surface temperature. } \\
& T_{s k y}=\text { effective temperature of sky. } \\
& F_{g n d}=\text { view factor from surface to ground. } \\
& F_{s k y}=\text { view factor from surface to sky. } \\
& \beta=\cos \left(\frac{\phi}{2}\right)
\end{aligned}
$$

The sky irradiance is taken as a $\beta$ weighted average of that from $\mathrm{T}_{\text {sky }}$ and that from $T_{a}$.

### 1.5.3.2 Fsky, Fgnd, and $\beta$

Howell (1982, \#C-8, p.94), gives the fraction of the radiation leaving the window surface and reaching the sky by:

$$
F_{s k y}=\frac{1+\cos \phi}{2}
$$

The fraction leaving the window incident on the ground is:

$$
F_{g n d}=\frac{1-\cos \phi}{2}
$$

where, $\phi=$ surface tilt angle, the angle between ground upward normal and window outward normal ( $0^{\circ}$ corresponds to a horizontal skylight, $90^{\circ}$ to a vertical surface).

The parameter $\beta$ accounts for the sky temperature's approach to the air temp near the horizon. $\beta$ is the fraction of the sky effectively at Tsky; (1- $\beta$ ) is the fraction of the sky effectively at Ta. $\beta$ is used by Walton (1983), and Energy Plus (2009), but appears to have little theoretical or experimental basis.

Walton (1983) give $\beta$ as:

$$
\beta=\cos \left(\frac{\phi}{2}\right)
$$

Since ${ }^{\cos \left(\frac{\varphi}{2}\right)=\sqrt{\frac{1+\cos \varphi}{2}}}$, it is noted that $F_{s k y}=\beta^{2}$, and $F_{s k y} \beta=\beta^{3}$.
1.5.3.3 Net radiant heat loss from a unit area

Equation 58 can be written as

$$
q_{r a d}=h_{r g}\left(T_{s}-T_{g}\right)+h_{r s k y}\left(T_{s}-T_{s k y}\right)+h_{r a i r}\left(T_{s}-T_{a}\right)
$$

where,

$$
\begin{aligned}
& h_{r g}=\varepsilon \sigma F_{g n d}\left(T_{s}^{2}+T_{g}^{2}\right)\left(T_{s}+T_{g}\right) \\
& h_{r s k y}=\epsilon \sigma F_{s k y} \beta\left(T_{s}^{2}+T_{s k y}^{2}\right)\left(T_{s}+T_{s k y}\right) \\
& h_{\text {rair }}=\varepsilon \sigma F_{s k y}(1-\beta)\left(T_{s}^{2}+T_{a}^{2}\right)\left(T_{s}+T_{a}\right) .
\end{aligned}
$$

$T_{g}$ is assumed to be equal to Ta, so Equation 59 becomes

$$
q_{r a d}=h_{r s k y}\left(T_{s}-T_{s k y}\right)+h_{r a}\left(T_{s}-T_{a}\right)
$$

Equation 60
where,

$$
\begin{aligned}
& h_{r s k y}=\epsilon \sigma F_{s k y} \beta\left(T_{s}^{2}+T_{s k y}^{2}\right)\left(T_{s}+T_{s k y}\right) \\
& h_{\text {rair }}=\varepsilon \sigma\left(F_{s k y}(1-\beta)+F_{g n d}\right)\left(T_{s}^{2}+T_{a}^{2}\right)\left(T_{s}+T_{a}\right)
\end{aligned}
$$

Equation 61

Equation 62
For a vertical surface, $F_{s k y} \beta=0.354$, and $F_{s k y}(1-\beta)+F_{g n d}=0.646$, so

$$
\begin{aligned}
& h_{r s k y}=0.354 \epsilon_{s} \sigma\left(T_{s}^{2}+T_{s k y}^{2}\right)\left(T_{s}+T_{s k y}\right) \approx 4(0.354) \varepsilon_{s} \sigma \bar{T}^{3} \\
& h_{\text {rair }}=(0.146+0.5) \varepsilon_{s} \sigma\left(T_{s}^{2}+T_{a}^{2}\right)\left(T_{s}+T_{a}\right) \approx 4(0.646) \varepsilon_{s} \sigma \bar{T}^{3}
\end{aligned}
$$

1.5.3.4 Total effective conductance and outside effective temperature, $T_{\text {env }}$ for walls Adding the exterior convection coefficient, hco, of Equation 40 to Equation 60 gives the total net heat transfer from the outside surface :

$$
q_{\text {rad }+ \text { conv }}=h_{r s k y}\left(T_{s}-T_{s k y}\right)+\left(h_{\text {rair }}+h_{c o}\right)\left(T_{s}-T_{a}\right)
$$

Equation 63
This can be written as,

$$
q_{r a d+c o n v}=h_{o}\left(T_{s}-T_{e n v}\right)
$$

Equation 64
where $h_{o}$ is the effective exterior conductance to the conductance weighted average temperature, $T_{\text {env }}$.

$$
\begin{aligned}
& h_{o}=h_{r s k y}+h_{r a i r}+h_{c o} \\
& T_{e n v}=\frac{h_{r s k y} T_{s k y}+\left(h_{\text {rair }}+h_{c o}\right) T_{a}}{h_{r s k y}+h_{r a i r}+h_{c o}}
\end{aligned}
$$

Equation 65
Equation 66

### 1.5.3.5 Outside window surfaces

The ASHWAT window algorithm of Section 1.7 utilizes the irradiation intercepted by the window. From

Equation 58 this can be deduced to be:

$$
G=F_{g n d} \sigma T_{g n d}^{4}+F_{s k y} \beta \sigma T_{s k y}^{4}+F_{s k y}(1-\beta) \sigma T_{\text {air }}^{4}
$$

## Equation 67

### 1.5.4 Sky Temperature

It is possible to approximate the long wave radiation emission from the sky as a fraction of blackbody radiation corresponding to the temperature of the air near the ground.
The sky emittance $\varepsilon_{s k y}$ is defined such that the sky irradiation on a horizontal surface is $\sigma \varepsilon_{s k y} T_{a}^{4}$.

The effective temperature of the sky is obtained by equating the blackbody emissive power of the sky at $T_{s k y}$, to the sky irradiation:

$$
\sigma T_{s k y}^{4}=\sigma \varepsilon_{s k y} T_{a}^{4}
$$

or,

$$
T_{s k y}=\varepsilon_{s k y}^{0.25} T_{a}
$$

Equation 68
where $T_{s k y}$ and $T_{a}$ are in degrees Rankine.
The value of $\varepsilon_{s k y}$ depends on the dewpoint temperature, cloud cover, and cloud height data. Martin and Berdahl (1984) give the $\varepsilon_{s k y}$ for clear skies as $\varepsilon_{o}$ :

$$
\varepsilon_{o}=0.711+0.56 \frac{T_{\text {dew }}}{100}+0.73\left(\frac{T_{\text {dew }}}{100}\right)^{2}+0.013 \cos \left(\pi \frac{h r}{12}\right)+0.00012\left(P_{\text {atm }}-1000\right)
$$

Equation 69
where,

$$
T_{\text {dew }}=\text { the dewpoint temperature in Celsius. }
$$

$h r=$ hour of day (1 to 24).
$P_{a t m}=$ atmospheric pressure in millibars.

### 1.5.4.1 Palmiter version of Martin-Berdahl model

The clear sky emissivity is corrected to account for cloud cover by the following algorithm, developed by Larry Palmiter (with Berdahl's imprimatur), that represents the Martin and Berdahl model when weather tape values of cloud ceiling height, and total and opaque cloud fractions are available.

$$
\epsilon_{s k y}=\varepsilon_{0}+\left(1-\varepsilon_{0}\right)\left(n_{o p} \varepsilon_{o p} \Gamma_{o p}+n_{t h} \varepsilon_{t h} \Gamma_{t h}\right)
$$

where,

$$
n_{o p}=\text { the opaque cloud fraction }
$$

$$
n_{t h}=\text { the thin cloud fraction: } n_{t h}=n-n_{o p}
$$

$$
\begin{aligned}
& \mathrm{n}=\text { the total sky cover fraction } \\
& \varepsilon_{o p}=\text { the opaque cloud emittance is assumed to be } 1 . \\
& \varepsilon_{t h}=\text { the thin cloud emittance; assumed to be } 0.4
\end{aligned}
$$

The cloud factor $\Gamma$ is used to adjust the emissivity when the sky is cloudy due to the increasing cloud base temperature for decreasing cloud altitudes. The cloud base temperature is not available on the weather tapes, so assuming a standard lapse rate of $5.6^{\circ} \mathrm{C} / \mathrm{km}$, 「is correlated with the more commonly measured cloud ceiling height, h (in meters), giving by the general expression:

$$
\Gamma=e^{-\frac{h^{h}}{8200}}
$$

For thin clouds, $\Gamma_{t h}$ is determined using an assumed cloud height of $8000-\mathrm{m}$, so,

$$
\Gamma_{t h}=e^{-\frac{8000}{8200}}=0.377
$$

## Equation 71

For opaque clouds,

$$
\Gamma_{o p}=e^{-\frac{h}{8200}}
$$

Equation 72
If ceiling height data is missing (coded 99999 on TMY2), the Palmiter model assumes that the opaque cloud base is at $h=2000 \mathrm{~m}$. If ceiling height is unlimited (coded as 77777) or cirroform (coded 88888), it is assumed that the opaque cloud base is at $h=$ 8000 m.

Using the assumed cloud cover and emissivity factors, Equation 70 becomes:

$$
\epsilon_{s k y}=\epsilon_{0}+\left(1-\epsilon_{0}\right)\left[n_{o} \Gamma_{o p}+\left(n-n_{o}\right) * 0.4 * 0.377\right]
$$

or,

$$
\epsilon_{s k y}=\epsilon_{0}+\left(1-\epsilon_{0}\right)\left[n_{o} e^{-\left(\frac{h}{8200}\right)}+0.151\left(n-n_{o}\right)\right]
$$

Equation 73

### 1.5.4.2 When opaque cloud cover data, $\mathrm{n}_{\mathrm{o}}$, is missing

In this case it is assumed that the cloud cover is opaque, $n_{o}=n$, when the ceiling height is less than $8000-\mathrm{m}$, and half opaque, $n_{o}=\frac{n}{2}$, when the ceiling height is equal or greater than 8000 . That is,
for $h<8000 m$ (from Equation 73 with $n_{o}=n$ ):

$$
\epsilon_{s k y}=\epsilon_{0}+\left(1-\epsilon_{0}\right) n e^{-\left(\frac{h}{s 200}\right)}
$$

Equation 74
For $h \geq 8000 m$ (from Equation 73 with $n_{o p}=n_{t h}=\frac{n}{2}$ ):

$$
\epsilon_{s k y}=\epsilon_{0}+\left(1-\epsilon_{0}\right) n\left[\frac{1}{2} e^{-\left(\frac{h}{s 200}\right)}+0.0754\right]
$$

1.5.4.3 When both opaque cloud cover and ceiling height data is missing When only total sky cover is available using an h of 2000-m reduces Equation 74 to:

$$
\epsilon_{s k y}=\epsilon_{0}+0.784\left(1-\epsilon_{0}\right) n
$$

Equation 76

### 1.6 Distribution of SW and LW Radiation inside the Zone

### 1.6.1 Long Wave Radiation Distribution

### 1.6.1.1 Carroll model

The radiant model used in CSE is based on the "MRT Network Method" developed by Joe Carroll (see Carroll $1980 \& 1981$, and Carroll \& Clinton $1980 \& 1982$ ). It was chosen because it doesn't require standard engineering view factors to be calculated, and yet gives a relatively accurate radiant heat distribution for typical building enclosures (see Carroll 1981).

It is an approximate model that simplifies the "exact" network (seeC) by using a mean radiant temperature node, Tr , that act as a clearinghouse for the radiation heat exchange between surfaces, much as does the single air temperature node for the simple convective heat transfer models. For $n$ surfaces this reduces the number of circuit elements from ( $n-1$ )! in the exact case, to $n$ with the Carroll model.

For black surfaces the radiant network is shown in Figure 8. For n surfaces, $T_{r}$ floats at the conductance, $A_{i} F_{i}$, weighted average surface temperature:

$$
T_{r}=\frac{\sum_{1}^{n} A_{i} F_{i} T_{i}}{\sum_{1}^{n} A_{i} F_{i}}
$$

Equation 77
The actual areas, $A_{i}$, need not be equal, nor limited to three.

Figure 8: Carroll Network for Black Surfaces


The factor $F_{i}$, in the radiant conductance between the $T_{i}$ surface node and the $T_{r}$ node is Carroll's "MRT view factor", that corrects for the self-weighting (seeD) of $T_{I}$ in the temperature $T_{r}$. The $F_{I}$ factors are obtained from the set of $n$ nonlinear equations for n surfaces:

$$
F_{i}=\frac{1}{1-\frac{A_{i} F_{i}}{\sum_{1}^{n} A_{j} F_{j}}}
$$

Given the surface areas, these equations are easily solved at the beginning of the simulation by successive substitution, starting with all $F_{i}=1$. This converges for realistic enclosures, but won't necessarily converge for enclosures having only two or three surfaces, particularly if there are large area disparities.
$F_{i}$ is always larger than 1 because it's role is to raise the conductance between $T_{\mathrm{r}}$ and $T_{i}$ to compensate for the potential difference $\left|\mathrm{T}_{\mathrm{r}}-\mathrm{T}_{\mathrm{I}}\right|$ being smaller than it would be had $T_{\mathrm{I}}$ not been part of the conductance weighted average $T_{r}$. The $\mathrm{F}_{\mathrm{I}}$ values can be seen to be close to 1 , since

Equation 78 is roughly approximated by $F_{i} \approx 1+\left(A_{i} / A_{\text {all surfaces }}\right)$.
The net radiant heat transfer [Btu/hr] from surface $i$ is:

$$
q_{i}=h_{b} A_{i} F_{i}\left(T_{i}-T_{r}\right)
$$

Equation 79
Using a $Y-\Delta$ transformation, the Figure 8 circuit can put in the form of the exact solution network of Figure $\mathrm{C}-1$ in C , showing the implicit view factors $F_{i j}$ to be:

$$
F_{i j}=\frac{F_{i} A_{j} F_{j}}{\sum_{k=1}^{n} A_{k} F_{k}}
$$

Equation 80
Thus the implicit view factors are independent of the relative spacial disposition of the surfaces, and almost directly proportional to the surface area $A j$ of the viewed by surface $i$. Also, without special adjustments (see Carroll (1980a)), all surfaces see each other, so coplanar surfaces (a window and the wall it is in) radiate to each other.

Equation 79 is exact (i.e., gives same answers as the C model) for cubical rooms; for which

Equation 78 gives $F_{i}=1.20$. Substituting this into Equation 80 gives the implied $F_{i j}=$ 0.2. This is the correct $F_{i j}$ for cubes using view-factor equations Howell(1982). It is likely accurate for all of the regular polyhedra.

## Grey surfaces

Carroll's model handles gray surfaces, with emissivities $\varepsilon_{i}$, by adding the Oppenheim surface conductance $\frac{A_{i} \varepsilon_{i}}{1-\varepsilon_{i}}$ in series with the conductances $h_{b} A_{i} F_{i}$. As shown in Figure 9, the conductance between $T_{i}$ and $T_{r}$ becomes $h_{b} A_{i} F_{i}^{\prime}$, where the $F_{i}^{\prime}$ terms are:

$$
F_{i}^{\prime}=\frac{1}{\frac{1}{F_{i}}+\frac{1-\varepsilon_{i}}{\varepsilon_{i}}}
$$

Equation 81
The net radiant heat transfer [Btu/hr] from surface $i$ is given by:

$$
q_{i}=h_{b} A_{i} F_{i}^{\prime}\left(T_{i}-T_{r}\right)
$$

Equation 82
where for grey surfaces $T_{r}$ is the " $h_{b} A_{i} F_{i}^{\prime \text { " }}$ weighted average surface temperature given by:

$$
T_{r}=\frac{\sum_{1}^{n} A_{i} F_{i}^{\prime} T_{i}}{\sum_{1}^{n} A_{i} F_{i}^{\prime}}
$$

Equation 83
Similar to Equation 77 for a black enclosure, Equation 83 shows that $T_{r}$ for grey surfaces is the conductance, $A_{i} F_{i}^{\prime}$, weighted average surface temperatures.

The role of $F_{i}$ hasn't changed, but since the conductance $A_{i} F_{i}$ is now connected to the radiosity node rather than the surface node, $E_{r}\left(=\sigma T_{r}^{4}\right)$ can be thought of as the $A_{i} F_{i}-$ weighted average radiosity of the surfaces, rather than the $A_{i} F_{i}$-weighted average emissive power of the surfaces as in the black enclosure case.

Figure 9: Carroll Radiant Network for Grey Surfaces


This completes the description of the basic Carroll model. The principle inputs are the interior surface areas in the zone, the emissivities of these surfaces, and the typical volume to surface area ratio of the zone (see Section 1.6.1.3). All of the interior surfaces, including ducts, windows, and interior walls, are assumed to exchange heat between each other as diffusely radiating gray body surfaces.

Longwave radiant internal gains can be added, in Btu/hr, to the radiant node Tr. This distributes the gains in proportion to the conductance $A_{i} F_{i}^{\prime}$.

## Conversion to delta

Using a $Y-\Delta$ transformation, the radiant network of Figure 9 can be converted to the $C$, Figure C - 3 circuit form, eliciting the $F_{i j}^{\prime}$ interchange factors implicit in Carroll's algorithm. Similar in form to Equation 80,

$$
A_{i} F_{i j}^{\prime}=\frac{A_{i} F_{i}^{\prime} A_{j} F_{j}^{\prime}}{\sum_{k=1}^{n} A_{k} F_{k}^{\prime}}
$$

Equation 84
Using these $A_{i} F_{i j}^{\prime}$ values, $q_{i j}$ can be obtained from

$$
q_{i j}=h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right)
$$

Equation 85
The total net heat transfer from surface $i$ (i.e., the radiosity minus the irradiation for the un-linearized circuit) is given by summing Equation 85 for all the surfaces seen by surface i:

$$
q_{i}=\sum_{j=1}^{n} h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right)
$$

which will agree with the result of Equation 82.

### 1.6.1.2 Accuracy of Carroll model

The Carroll model of Figure 9 is exact for cubical enclosures with arbitrary surface emissivities. It is surprisingly accurate for a wide variety of shapes, such as hip roof attics and geodesic domes.

Carroll (1981) compared his model, and other simplified models, with the exact solution for the enclosure shown in Figure 10. Half the south wall and half of the west wall are glass with $\varepsilon=0.84$, and the rest of the interior surfaces have $\varepsilon=0.9$.

Figure 10: Test Room of Walton (1980)


Comparisons were made primarily regarding three types of errors:

## Heat balance errors

The first law requires that the sum of the net radiation emitted by each of the surfaces, plus any internal gain source of long-wave radiation, must equal zero. That is, $\mathrm{q}_{\text {int }}+$ $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{q}_{\mathrm{i}}=0$.

Due to their fixed conductance circuits, both the Carroll method and the Walton(1983) method are inherently free of heat balance errors. Carroll found BLAST and NBSLD algorithms to have rms heat balance errors of $9.8 \%$ (12\%) and $1.7 \%$ (3.4\%) for the Figure 10 enclosure.

## Individual surface net heat transfer errors

For a given enclosure, these are errors in an individual surfaces net heat flow, $q_{i}$, compared to the exact method. For Carroll's method, this finds the error in $q_{i}$ determined from the $A_{i} F_{i j}^{\prime}$ values of Equation, compared to the $q_{i}$ values found using the exact $A_{i} F_{i j}^{\prime}$ values (obtainable from Figure C-3 of C).
Carroll found the \% rms error in the $q_{i}$ values for a given enclosure in two different ways.

The first method, Equation 87, gives the rms error of $q_{i}$ for each surface divided by the rms of the $n$ net heat transfers from each surface:

$$
\operatorname{Err}=\left(\frac{\frac{1}{n} \sum_{i=1}^{n} \Delta q_{i}^{2}}{\frac{1}{n} \sum_{i=1}^{n} \bar{q}_{i}^{2}}\right)^{\frac{1}{2}} * 100
$$

Equation 87
where

$$
\Delta q_{i}=q_{i}-\bar{q}_{i}, \text { is the error in } q_{i}
$$

$q_{i}=\sum_{j=1}^{n} h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right)$, using $F_{i j}^{\prime}$ values from Carroll's model, Equation 87.
$\bar{q}_{i}=\sum_{j=1}^{n} h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right)$, using the exact $F_{i j}^{\prime}$ values of Figure $\mathrm{C}-3$ of C
$T_{i}-T_{j}=1^{0} F$ assumed in all cases.
$n=$ the number of surfaces
The second method, Equation 88, gives the rms of the percentage error in $\mathrm{q}_{\mathrm{I}}$ of each surfaces. This method increases the weight of smaller surfaces such as windows.

$$
E r r=\frac{1}{n} \sum_{i=1}^{n}\left(\frac{\Delta q_{i}}{q_{i}}\right)^{2} * 100
$$

Equation 88

## Results

For the enclosure of Figure 10, Carroll found his method gives Err $=0.11 \%$ for the first method and $0.19 \%$ for the second method.

These results are shown in Table 4, along with the results for other shape enclosures, and the errors determined by Carroll using the radiant interchange algorithms of Walton (1980) and NBSLD and BLAST simplified models.

Table 4: $E r r=\%$ rms Error in $\boldsymbol{q}_{i}$ from Equation 87 and Equation 88 in Parenthesis

|  | Figure 10 room <br> $\mathbf{2 . 5 : 4 : 6}$ <br> $\boldsymbol{\varepsilon = 0 . 9 ( . 8 4}$ <br> wdws) | Corridor <br> $\mathbf{1 0 : 1 : 1}$ <br> $\boldsymbol{\varepsilon}=\mathbf{0 . 9}$ | Warehouse <br> $\mathbf{1 0 : 1 0 : 1}$ <br> $\boldsymbol{\varepsilon}=\mathbf{0 . 9}$ |
| :--- | :--- | :--- | :--- |
| Carroll | $0.11(0.19)$ | $0.06(0.05)$ | $0.07(0.04)$ |
| Walton(1980) | $1.9(1.30)$ | $0.6(0.6)$ | $4.4(3.0)$ |
| NBSLD, BLAST | $3.2(2.1)$ | $3.2(2.6)$ | $7.5(4.4)$ |

## Errors in an individual surface's distribution of heat transfer to other surfaces

These are errors in $q_{i j}$, the heat exchanged between surfaces i and j (both directly and by reflections from other surfaces), relative to the exact total net heat transfer from surface igiven by Figure C-3 of C.

Carroll gives two percentage error results.
By the first method, for each surface $i$, the rms of the error, $\Delta q_{i j}$, in heat exchange to each of the $n-1$ other $j$ surfaces is obtained. Then the rms of these $n$ rms error values is obtained, giving a representative distribution error for the enclosure. Dividing this by the rms value of the exact net surface heat transfers, $q_{i}$, of all the surfaces gives the final distribution error in percent:

$$
E r r=\frac{\sqrt{\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{n} \Delta q_{i j}^{2}}{n(n-1)}\right)}}{\sqrt{\sum_{i=1}^{n}\left(\frac{\bar{q}_{i}^{2}}{n}\right)}} * 100
$$

Equation 89
where

$$
\begin{aligned}
& \Delta q_{i j}=q_{i j}-\bar{q}_{i j}, \\
& q_{i j}=h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right) \text { with } F_{i j}^{\prime} \text { values from Equation } 84 . \\
& \bar{q}_{i j}=h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right) \text { with the exact } F_{i j}^{\prime} \text { values of Figure C-3 of C. }
\end{aligned}
$$

By the second method, for each surface $i$, the rms of the percentage error in heat exchange $q_{i j}$, relative to the exact net heat transfer from that surface, $q_{i}$, is obtained.

$$
E r r=100 * \sqrt{\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{n}\left(\frac{\Delta q_{i j}}{\bar{q}_{i}}\right)^{2}}{n(n-1)}\right)}
$$

## Distribution error results

For the Figure 10 room, Carroll's model gives errors of $2.1 \%$ and $3.9 \%$ for methods 1 and 2 respectively. Walton's model has corresponding errors of $2.4 \%$ and $3.7 \%$.

Equation 91 was used for the results in parenthesis.

Table 5: \% rms Error in $q_{i}$ from Equation 90

|  | Figure 10 room <br> $\mathbf{2 . 5 : 4 : 6}$ <br> $\boldsymbol{\varepsilon}=\mathbf{0 . 9}(.84$ <br> wdws | Corridor <br> $\mathbf{1 : 1 0 : 1}$ <br> $\boldsymbol{\varepsilon}=\mathbf{0 . 9}$ | Warehouse <br> $\mathbf{1 : 1 0 : 1 0}$ <br> $\boldsymbol{\varepsilon}=\mathbf{0 . 9}$ |
| :--- | :--- | :--- | :--- |
| Carroll | $2.1(3.9)$ | $3.3(2.8)$ | $0.6(1.9)$ |
| Walton(1980) | $2.4(3.7)$ | $3.3(2.8)$ | $2.8(3.0)$ |
| BLAST | $2.8(4.4)$ | $3.4(4.4)$ | $3.4(15)$ |
| NBSLD | $1.7(3.5)$ | $1(0.83)$ | $3.3(1.9)$ |

(Equation 91 was used for the results in parenthesis.)

Source: NORESCO for California Energy Commission
Carroll's model is seen to give very respectable results, despite giving no special treatment to coplanar surfaces.

### 1.6.1.3 Air absorption

The Carroll model also accounts for the absorption of long-wave radiation in the air, so that the air and mrt nodes are thermally coupled to each other as well as to the interior surfaces. Carroll (1980a) gives an air emissivity by the following dimensional empirical equation that is based on Hottel data from McAdams(1954):

$$
\varepsilon_{a}=0.08 \varepsilon_{s} \ln \left(1+\left(\frac{4 V}{\varepsilon_{s} A} R P_{a t m}\right) e^{\frac{T a F-32}{30.6}}\right)
$$

Equation 91
The logarithm is natural, and,
$\varepsilon_{s}=$ the area-weighted average long-wave emissivity for room surfaces, excluding air.
$\mathrm{V} / \mathrm{A}=$ the room volume to surface area ratio, in meters.
$R=$ the relative humidity in the zone. ( $0 \cdot R \cdot 1$ ).
Patm = atmospheric pressure in atmospheres.

- a = zone air temperature, in • F.

Following a heuristic argument Carroll assigns an effective area $A_{a}$ to the air that is the product of • $a$ and the sum of all of the zone surface areas, as if the absorbing part of the air were consolidated into a surface of area $A_{a}$.

$$
A_{a}=\varepsilon_{a} \sum A_{i}
$$

Equation 92
Using this area, the value of $F_{a}$ for this 'surface' can be calculated along with the other $F_{i}$ by Equation 80 . The value of the conductance between the air and radiant nodes in

Figure 11 is given by:

$$
C_{x}=h_{b} A_{a} F_{a}
$$

Figure 11: Like Figure 9 but with Convective Network Added


## Facets

Suppose one of the interior surfaces of total area $A_{i}$ is composed of $N_{i}$ identical flat subsurfaces, each at the same temperature, and similar views to each other, like the facets of a geodesic dome. The $F_{i}$ values would be the same if each facet is treated as a separate surface. To avoid redundant solutions to Equation 80, it is easy to show that $A_{i}$ can be treated as one surface in Equation 6-4 if $N_{i}$ is introduced into Equation 80 as follows:

$$
F_{i}=\frac{1}{1-\frac{A_{i} \cdot F_{i} / N_{i}}{S\left(A_{i} \cdot F_{i}\right)}}
$$

Equation 94

The facet feature is utilized in the simulation to represent attic truss surfaces.
Short Wave Radiation Distribution
This routine was used in the development code for this program. It is not currently implemented in CSE, being replaced by a simplified but similar routine.

The short wave radiation (solar insolation from hourly input) transmitted by each window can, at the users discretion, be all distributed diffusely inside the zone, or some of the insolation from each window can be specifically targeted to be incident on any number of surfaces, with the remaining untargeted radiation, if any, from that window, distributed diffusely. The insolation incident on any surface can be absorbed, reflected, and/or transmitted, depending on the surface properties inputted for that surface. The radiation that is reflected from the surfaces is distributed diffusely, to be reflected and absorbed by other surfaces ad infinitum.

Since some of the inside surfaces will be the inside surface of exterior windows, then some of the solar radiation admitted to the building will be either lost out the windows or absorbed or reflected by the widows.

### 1.6.1.4 Radiation removed at each surface of a zone by a single source of targeted insolation

Assume a spherical zone with total insolation $\mathrm{S}(\mathrm{Btu} / \mathrm{hr})$ admitted into the zone through one window. Assume that the portion $a_{i} Q_{i}\left(\mathrm{ft}^{2} * \mathrm{Btu} /\left(\mathrm{hr}-\mathrm{ft}^{2}\right)\right)$ of $S(\mathrm{Btu} / \mathrm{hr})$ is targeted to surface i with area $a_{i}$ such that,

$$
\sum_{i} a_{i} Q_{i}=S
$$

## Equation 95

where the sum is over all surfaces $i$. The total spherical area is $a_{S}=\sum_{i} a_{i}$. Also incident on surface i will be the irradiation $\mathrm{G}_{\mathrm{i}}\left(\mathrm{Btu} / \mathrm{hr}-\mathrm{ft}^{2}\right)$ from other surfaces that have reflected a portion of the radiation they have received. We distinguish between the $\mathrm{Q}_{i}$ incident on the surface directly from the window, and the irradiation $\mathrm{G}_{\mathrm{i}}$ which is composed of radiation reflected to i from all the surfaces, and that reflected by windows. All radiation (including Incident beam) is assumed to be reflected diffusely.

Each surface i will also reflect short-wave radiation, with a radiosity $\mathrm{J}_{\mathrm{i}}[\mathrm{Btu} / \mathrm{hr}-\mathrm{sf}]$.
Figure 12: Radiation Terminology

surface i

The derivation below determines the equations to obtain $J_{i}$ and $G_{i}$ for known $Q_{i}$ values, for all the surfaces of the sphere, $\mathrm{i}=1$ to n .

First a relationship between $\mathbf{G i}$ and Ji is developed:
Since $\mathrm{G}_{\mathrm{i}}$ is composed only of reflected radiation,

$$
a_{i} G_{i}=\sum_{k} J_{k} a_{k} F_{i k}
$$

where the sum is over all surfaces $n$ of the sphere of area $\mathrm{a}_{\mathrm{s}}$.
Using the view-factor reciprocity principle,

$$
a_{k} F_{k i}=a_{i} F_{i k}
$$

$G_{i}$ becomes

$$
G_{i}=\sum_{k} J_{k} F_{i k}
$$

For spherical geometry, the view factor is $F_{i k}=\frac{a_{k}}{a_{S}}$, where $a_{S}=\sum a_{k}$, so $\mathrm{G}_{i}$ can be written

$$
G_{i}=\frac{1}{a_{S}} \sum_{k} J_{k} a_{k}
$$

Equation 97
The right hand side is the area-weighted average radiosity, showing that $\mathrm{G}_{\mathrm{i}}$ is independent of $i$,

$$
G_{i}=\bar{J} \quad \text { Equation } 98
$$

## Next a separate relationship between $J_{i}$ and $G_{i}$ is obtained, $\mathbf{G}_{i}$ eliminated and $\mathrm{J}_{\mathrm{i}}$ solved for explicitly:

The radiosity of surface $i$ is composed of the reflected part of both the irradiation and the targeted solar

$$
J_{i}=\rho_{i}\left(G_{i}+Q_{i}\right) \quad \quad \text { Equation } 99
$$

Substituting Equation 97 for $G_{i}$ gives

$$
\frac{J_{i}}{\rho_{i}}=\frac{1}{a_{S}} \sum_{k} J_{k} a_{k}+Q_{i}
$$

Equation 100
Since by Equation $98 G_{i}$ is independent of $i$, then Equation 99 shows that the radiosity of any surface $i$ is related to the radiosity of any surface $k$ by the relationship

$$
\frac{J_{i}}{\rho_{i}}-Q_{i}=\frac{J_{k}}{\rho_{k}}-Q_{k}
$$

Substituting this into Equation 100 gives

$$
\frac{J_{i}}{\rho_{i}}=\frac{1}{a_{S}} \sum_{k}\left(a_{k} \rho_{k}\left(\frac{J_{i}}{\rho_{i}}+Q_{k}-Q_{i}\right)\right)+Q_{i}
$$

This can be solved explicitly for $J_{i}$ :

$$
J_{i}=\frac{\frac{1}{a_{S}} \rho_{i}}{1-\bar{\rho}}\left(\sum_{k} a_{k} \rho_{k} Q_{k}\right)+Q_{i} \rho_{i}
$$

From Equation 101, the area weighted average $J$ is

$$
\bar{J}=\frac{\frac{1}{a_{S}} \bar{\rho}}{1-\bar{\rho}}\left(\sum_{k} a_{k} \rho_{k} Q_{k}\right)+\frac{1}{a_{S}} \sum_{i} a_{i} Q_{i} \rho_{i}
$$

Equation 102
where $\bar{\rho}$ is the area weighed average reflectivity.

## Now that $J_{i}$ and $G_{i}$ are known an energy balance will give the net heat transfer:

The net energy rate (Btu/hr) absorbed and/or transmitted by surface i, is:

$$
\text { Qnet }_{i}=\left(G_{i}+Q_{i}-J_{i}\right) a_{i}=\left(\bar{J}-J_{i}+Q_{i}\right) a_{i}
$$

Equation 103
Substituting Equation 101 and Equation 102 into this gives

$$
\text { Qnet }_{i}=\frac{a_{i}}{a_{S}}\left(\frac{1-\rho_{i}}{1-\bar{\rho}}\right) \sum_{k} a_{k} \rho_{k} Q_{k}+a_{i} Q_{i}\left(1-\rho_{i}\right)
$$

Equation 104
The first term in Equation 104 is from the absorption and/or transmission of radiation that reached and is absorbed by surface i after having been reflected, ad infinitum, by the interior surfaces. The second term is from the absorption of the "initially" incident insolation $Q_{i}$ on surface $i$.

If none of the insolation is specifically targeted, and instead $S$ is assumed to be distributed isotropically then $Q_{i}$ is the same for each surface:

$$
Q_{i}=\frac{S}{a_{S}}
$$

Equation 105
Substituting this into Equation 104 gives Qnet $_{i}$ for isotropically distributed insolation:

$$
\text { Qnet }_{i}=\frac{a_{i}}{a_{S}}\left(\frac{1-\rho_{i}}{1-\bar{\rho}}\right) S
$$

Equation 106
1.6.1.5 Radiation removed at each surface of a zone by multiple window sources of targeted insolation

The targeting can be different for each window. Adding an additional subscript "j" to Equation 104 allows it to represent the energy removal for each surface separately for each window j. That is, Equation 104 becomes Equation 107, the rate of energy
removal at each surface due to insolation $S_{j \text {, }}$ that is distributed according to the assigned targeted values $Q_{j i}$.

$$
\text { Qnet }_{j i}=\frac{a_{i}}{a_{s}}\left(\frac{1-\rho_{i}}{1-\bar{\rho}}\right) \sum_{k} a_{k} \rho_{k} Q_{j k}+a_{i} Q_{j i}\left(1-\rho_{i}\right)
$$

Equation 107
The targeting fractions $H_{j k}$, to be user input, are defined as the fraction of insolation from window $j$ that is incident on surface $k$ :

$$
H_{j k}=\frac{a_{k} Q_{j k}}{S_{j}}
$$

Equation 108
With this definition, Equation 107 can be written as

$$
\text { Qnet }_{j i}=a_{i} S_{j}\left(1-\rho_{i}\right)\left(\frac{H_{j i}}{a_{i}}+\frac{1}{a_{S}(1-\bar{\rho})} \sum_{k} \rho_{k} H_{j k}\right)
$$

Equation 109
The effective absorptivity of the targeted surfaces is defined as

$$
\alpha e f f T_{j i}=\frac{\text { Qnet }_{j i}}{S_{j}}
$$

Equation 110
Replacing the spherical surfaces $\mathrm{a}_{\mathrm{i}}$ in Equation 109 by $a_{i}=A_{i} F_{i}$, and substituting Equation 109 into Equation 110 gives the targeted gain equation used in the CZM code:

$$
\alpha e f f T_{j i}=A_{i} F_{i}\left(1-\rho_{i}\right)\left(\frac{H_{j i}}{A_{i} F_{i}}+\frac{1}{(1-\bar{\rho}) \sum_{i} A_{i} F_{i}} \sum_{k} \rho_{k} H_{j k}\right)
$$

Equation 111
If $\sum_{\mathrm{k}} \mathrm{H}_{\mathrm{jk}}<1$ then it is assumed that the remaining insolation $\mathrm{S}_{\mathrm{j}}\left(1-\sum_{\mathrm{k}} \mathrm{H}_{\mathrm{jk}}\right)$ is distributed isotropically. From Equation 105 it is

Equation 112
The definition of the effective absorptivity for isotropic insolation is:

$$
\alpha e f f I_{j i}=\frac{\text { Qnet }_{j i}}{s_{j}}
$$

Equation 113
Changing Equation 112 to utilize zone areas, $\mathrm{a}_{\mathrm{i}}=\mathrm{A}_{\mathrm{i}} \mathrm{F}_{\mathrm{I}}$, and substituting Equation 112 into Equation 113 gives the amount of the diffuse part of the insolation from each window j that is absorbed in each surface i . This is used in the CZM code.

$$
\alpha e f f I_{j i}=\frac{A_{i} F_{i}}{\sum_{k} A_{k} F_{k}}\left(\frac{1-\rho_{i}}{1-\bar{\rho}}\right)\left(1-\sum_{k} H_{j k}\right)
$$

Equation 114
Note that no distinction has been made between surfaces that are opaque like walls, and partially transparent window surfaces. They are treated equally. The difference is that the energy removed by an opaque wall is absorbed into the wall, whereas that removed by the window surfaces is partly transmitted back out the window, and partly absorbed at the window inside surface. The CZM development code lets the user specify a fraction of the radiation that is absorbed in the room-side surface of the window, which slightly heats the window and thus the zone.

Adding Equation 111 and Equation 114 gives the total effective absorptivity of surface i from the insolation admitted through window $j$ :

$$
\alpha e f f_{j i}=A_{i} F_{i}\left(1-\rho_{i}\right)\left(\frac{1}{(1-\bar{\rho}) \sum_{k} A_{k} F_{k}}\left[1-\sum_{k}\left(1-\rho_{k}\right) H_{j k}\right]+\frac{H_{j i}}{A_{i} F_{i}}\right)
$$

Equation 115
The net radiation absorbed in surface $i$ from window $j$ is thus

$$
\text { Qnet }_{j i}=A_{i} F_{i}\left(1-\rho_{i}\right) S_{j}\left(\frac{1}{(1-\bar{\rho}) \sum_{k} A_{k} F_{k}}\left[1-\sum_{k}\left(1-\rho_{k}\right) H_{j k}\right]+\frac{H_{j i}}{A_{i} F_{i}}\right)
$$

Equation 116
Summing this over all windows gives the total SW radiation absorbed and/or transmitted by surface i as:

$$
\text { Qnet }_{i}=A_{i} F_{i}\left(1-\rho_{i}\right) \sum_{j}\left[S_{j}\left(\frac{1}{(1-\bar{\rho}) \sum_{k} A_{k} F_{k}}\left[1-\sum_{k}\left(1-\rho_{k}\right) H_{j k}\right]+\frac{H_{j i}}{A_{i} F_{i}}\right)\right]
$$

### 1.7 Window Model

The ASHWAT algorithm is used to model complex windows with diatherminous layers and curtains, etc. (Wright and Kotey 2006, Wright, J.L. 2008). Given the environmental conditions on each side of the window, ASHWAT determines the long wave, short wave and convection heat transfers to the conditioned space.

For the following input and output discussion, ASHWAT is treated as a black box.

### 1.7.1 Inputs

Each time step, for each window, ASHWAT is given the environmental inputs:
$I=$ insolation incident on window system.
$I_{\text {refl }}=$ insolation reflected diffusely from the other room surfaces.
$T_{a, o u t}=$ outside dry bulb air temperature.
$T_{a, i n}=$ inside dry bulb air temperature.
$T_{r, i n}=$ the temperature of the indoor plate.
$T_{r, \text { out }}=$ the temperature of the outdoor plate.

Figure 13: ASHWAT Inputs and Nomenclature


### 1.7.2 Outputs

ASHWAT's output gives heat transfer rates and circuit elements of Figure 14. The circuit of Figure 14 is part of the conditioned zone radiant network of Figure 2 and Figure 3 (with some different nomenclature).

Figure 14: Window System Representation in CSE


In Figure 14,
$F P=$ fraction of the heat from Insolation absorbed in the various window layers that ends up being transferred to the inside radiant node.
$F M=$ fraction of the heat from Insolation absorbed in the various window layers that ends up being convected to the inside air node.
frin $=$ fraction of total non-solar heat transfer that goes to the inside radiant node; dimensionless.
frout $=$ fraction of non-solar heat transfer to the outside that goes to the outside radiant node.
$U=$ conductance between the inside and outside effective temperatures $T_{e i}$ and $T_{e o} ; \mathrm{Btu} /(\mathrm{hr}-\mathrm{sf}-\mathrm{F})$, where $T_{e i}=T_{a, i n}(1-$ frin $)+T_{r, i n}$ frin.
$T_{e o}=T_{a, \text { out }} *(1-$ frout $)+T_{r, \text { out }} *$ frout $=$ the effective outdoor temperature; F.
$C_{x w}=$ the cross coupling term; Btu/(hr-sf-F).
$\tau=$ the short wave transmissivity of the window system.
Note that the solar heat gain coefficient is: $S H G C=\tau+F P+F M$.
Net energy into zone via window, per unit COG area $=+I(\tau+F P+F M)-I_{r e f l}+$ $U\left(T_{e o}-T_{e i}\right)$

### 1.7.3 Matching ASHWAT to CSE Radiant Network

1.7.3.1 Outside boundary conditions

ASHWAT models the irradiation on the outside of the window system as if it were emitted by a black plate parallel to the window at temperature $T_{r, \text { out }}$, as shown in Figure 13. The irradiation on the window system from the outside plate is thus $G_{o u t}=$ $\sigma T_{r, \text { out }}^{4}$, so

$$
T_{r, \text { out }}=\left(\frac{G_{\text {out }}}{\sigma}\right)^{0.25}=\left[F_{\text {sky }} \beta T_{\text {sky }}^{4}+\left[F_{\text {gnd }}+F_{\text {sky }}(1-\beta)\right] T_{\text {air }}^{4}\right]^{0.25}
$$

where $G_{\text {out }}$ has been replaced by Equation 67.

### 1.7.3.2 Inside boundary conditions

From Figure 13, the equivalent network between the radiosity of the window system, $J_{w}$, and the inside plate is shown in Figure 15. The circuit parameters are in the conductance form. The " 1 " is the view factor between the plate and the window.

Figure 15: Equivalent Network between the Radiosity of the Window System, $J_{w}$, and the Inside Plate


Figure 15 reduces to:
Figure 16: Reduced Figure 15


Thus the heat transfer rate per unit area, with $Q$ positive from window to room, is:

$$
Q=\varepsilon_{r}\left(J_{w}-E_{r}\right)
$$

Equation 119
From Figure 9 the network between the radiosity of a surface and the mean radiant temperature node is shown in Figure 17. This corresponds to Figure 16 for the ASHWAT algorithm:

Figure 17: Network between the Radiosity of a Surface and the Mean Radiant Temperature Node

with the corresponding heat transfer rate:

$$
Q=F_{w}\left(J_{w}-E_{r}\right)
$$

Equation 120
Comparing Equation 119 and Equation 120 shows that to obtain the heat flow consistent with the Carroll network ASHWAT must model the window by setting inside plate's emissivity to the value of $F_{w}$.

$$
\varepsilon_{r}=F_{w}
$$

Equation 121
$F_{w}$ is the Carroll MRT view factor defined in Section 1.6.1. $F_{w}$ is slightly larger than 1 , and serves to increase the heat transfer between $J_{w}$ and $E_{r}$ to compensate for the fact that $\left|J_{w}-E_{r}\right|$ is smaller than it would if $T_{r, i n}$ had not included the window temperature in its average. This MRT view factor effect cannot be simulated by a parallel plate model without the trick of artificially raising the emissivity of the inside plate to the value $F_{w}$.

### 1.8 Slab Model

### 1.8.1 Bajanac Simplified Model

The CZM slab model is partly based on the Simplified Method for the calculation of heat flow through foundations, presented by Bazjanac et al. (2000). They divide a slab into two regions.

### 1.8.1.1 Perimeter region

The perimeter area of the slab is defined as a 2 ft wide strip along external walls. Through this perimeter path, the interior air is assumed to be coupled via conductances $\alpha_{1}, \alpha_{2}$, and $\alpha_{3}$ to three environmental temperatures: $\mathrm{T}_{\text {week } /}, \mathrm{T}_{\text {ground }}$ and $\mathrm{T}_{\text {year }}$ :

Figure 18: Perimeter Coupling


Thus the instantaneous heat flow from the room Temp node to perimeter slab, in $\mathrm{Btu} / \mathrm{hr}-\mathrm{sf}-\mathrm{F}$, is given by:

$$
\text { Qperim }=\left[\alpha_{1}\left(T_{\text {air }}-T_{\text {week }}\right)+\alpha_{2}\left(T_{\text {air }}-T_{\text {ground }}\right)+\alpha_{3}\left(T_{\text {air }}-T_{\text {year }}\right)\right]
$$

Equation 122
where,
$\mathrm{T}_{\text {air }}=$ the current interior-space effective temperature (involving both Ta and Tr).
$\mathrm{T}_{\text {week }}=$ the average outside air temperature of the preceding two-weeks.
$\mathrm{T}_{\text {ground }}=$ the current average temperature of the earth from the surface to a 10 ft depth.
$\mathrm{T}_{\text {year }}=$ the average yearly dry bulb temperature.
a's = conductances from Table 3 of Bazjanac et al; Btu/sf-hr-F.

### 1.8.1.2 Core region

The core region couples $T$ to $T_{\text {ground }}$ and $T_{\text {year }}$, via conductances $\alpha_{4}$ and $\alpha_{5}$.
Figure 19: Core Coupling


$$
\text { Qcore }=\left[\alpha_{4}\left(T_{\text {air }}-T_{\text {ground }}\right)+\alpha_{5}\left(T_{\text {air }}-T_{\text {year }}\right)\right]
$$

Bazjanac et al. determined the conductances $\alpha_{1}$, through $\alpha_{5}$ by multi-linear regression analysis of the numerical results from a two-dimensional finite-difference slab-earth model. The conductances were determined for 52 slab foundation conditions and given in their Table 3.

### 1.8.1.3 Properties

The Bozjanac Table 3 conductances were obtained assuming the following properties:

1. Properties of earth:
-conductivity = 1 Btuh/ft-F. (The k chosen was justified by assuming that lawns and other vegetation around California houses was watered during the dry season).

- density $=115 \mathrm{lbm} / \mathrm{ft}^{3}$
- specific heat $=0.2 \mathrm{Btu} / \mathrm{lbm}-\mathrm{F}$
-thermal diffusivity $=0.0435 \mathrm{ft}^{2} / \mathrm{hr}$.

2. Slab: "heavy construction grade concrete"
-thickness $=4$-inches

- conductivity $=0.8$
- density $=144$
- specific heat $=0.139$

3. Rrug $=2.08 \mathrm{hr}-\mathrm{ft}$ ²-F/Btu (ASHRAE 2005HF, p. 25.5 'carpet fibrous pad').
4. Rfilm $=0.77 \mathrm{Btu} / \mathrm{hr}^{-\mathrm{ft}^{2} \mathrm{~F}}$, the inside surface-to-room-temperature combined convective and radiative conductance.

### 1.8.1.4 Ground temperature

The above model uses the ground temperature determined by Kusuda and Achenbach (1965). Using the classical semi-infinite medium conduction equations for periodic surface temperature variation (Carslaw and Jaeger), they found the average ground temperature from the surface to a depth of 10 ft to be given by:

$$
\mathrm{T}_{\text {ground }}=\mathrm{T}_{\mathrm{yrAve}}-\mathrm{GM}\left(\frac{\mathrm{TyrMax}-\mathrm{TyrMin}}{2}\right) \cos \left(\left(\frac{2 \pi}{8760}\right) \theta-P O-\phi\right)
$$

where,
$\mathrm{T}_{\mathrm{yrAve}}=$ average outdoor temperature over year; F .
$\mathrm{T}_{\mathrm{yrMax}}=$ highest average monthly outdoor temperature for the year; F .
$\mathrm{T}_{\mathrm{yrMin}}=$ lowest average monthly outdoor temperature for the year; F .
$\mathrm{GM}=\sqrt{\frac{\mathrm{e}^{-2 \beta-2 e^{-\beta} \cos \beta+1}}{2 \beta^{2}}}$ = dimensionless amplitude for integrated depth average.
$\beta=L \sqrt{\frac{\pi}{\mathrm{D}+\mathrm{PY}}}=$ dimensionless depth.
$\mathrm{L}=10 \mathrm{ft}$, the depth over which average is taken.
$\mathrm{D}=$ thermal diffusivity of soil, $\mathrm{ft}^{2} / \mathrm{hr}$.
$\mathrm{PY}=8760 \mathrm{hr}=$ period of 1 year.
$\theta=24\left(\frac{365 \mathrm{M}}{12}-15\right) \approx$ elapsed time from Jan-1 to middle of month $M$; hours.
$M=$ month, 1 à 12.
$\phi=\operatorname{atan}\left(\frac{1-e^{-\beta}(\cos \beta+\sin \beta)}{1-e^{-\beta}(\cos \beta-\sin \beta)}\right)=$ phase angle for depth averaged $\mathrm{T}_{\text {ground }} ;$ radians.
$\mathrm{PO}=0.6$ radians $=$ phase lag of ground surface temperature (assumed equal to air temperature) relative to January 1. From measured data, see Fig. 7 in Kusuda and Achenbach.

### 1.8.2 Addition of a Layered Slab and Earth

The Bazjanac model assumes a constant indoor temperature, so cannot be applied directly to a whole building thermal-balance simulation model that allow changing indoor temperatures. To apply this model to CZM, with changing indoor temperatures, requires incorporating the dynamic effects of the slab and earth due to changing inside conditions.

This is done by putting a one-dimensional layered construction, representing the slab and some amount of earth mass, into the steady-state Bazjanac model circuit--replacing part of its resistance by a thermal impedance (which is equal to the resistance for steady state conditions). In this way the correct internal temperature swing dynamics can be approximated.

First, the circuit of Figure 18 is alternately expressed as shown in Figure 20(a), with Equation 122 taking the form:

$$
\begin{equation*}
Q=A * U_{g}\left(T_{\text {air }}-T_{\text {geff }}\right) \tag{Equation 125}
\end{equation*}
$$

where $T_{g e f f}$ is the a-weighted average ground temperature:

$$
T_{\text {geff }}=\frac{\alpha_{1} T_{\text {week }}+\alpha_{2} T_{\text {ground }}+\alpha_{3} T_{\text {yrAve }}}{\alpha_{1}+\alpha_{2}+\alpha_{3}}
$$

Equation 126
and

$$
R_{g}=\frac{1}{\alpha_{1}+\alpha_{2}+\alpha_{3}}
$$

Equation 127
Similarly for the core region,

$$
\begin{aligned}
& T_{\text {geff }}=\frac{\alpha_{4} T_{\text {ground }}+\alpha_{5} T_{\text {yrAve }}}{\alpha_{4}+\alpha_{4}} \\
& R_{g}=\frac{1}{\alpha_{4}+\alpha_{5}}
\end{aligned}
$$

Equation 128

Now a one-dimensional layered construction is added into the circuit as shown in Figure 20(b), consisting of a surface film layer, a carpet (if any), the concrete slab, and earth layer. The bottom of the earth layer is then connected to $T_{g e f f}$ through the what's left of $R_{g}$.

A one-dimensional representation of the mass is appropriate for the core region. It is a bit of a stretch for the perimeter slab modeling, because the real perimeter heat flow is decidedly 2 -dimensional, with the heat flow vectors evermore diverging along the path of heat flow.

Figure 20: Addition of Film, Rug, Slab, and Earth


The earth thicknesses required to adequately model the dynamic interaction between the room driving forces (sun and temperature) and the slab/earth model was determined by considering the frequency response of the slab earth model of Figure 20(b). In the frequency domain, the periodic heat flow from the Tair node is given by Equation 130.

$$
\tilde{Q}_{a i r}=\tilde{T}_{a i r} X-\tilde{T}_{g e f f} Y
$$

Equation 130
where,

X = the driving point admittance at the air (or combined air/radiant effective temp) node, in the units of Btu/hr-sf-F. It is the contribution to $Q_{\text {air }}$ per degree amplitude of $T_{a i \text { is }} . \mathrm{X}$ and Y are complex numbers determined from the layer properties (conductivity, heat capacity, density) of the circuit layers in Figure 20(b). See Carslaw and Jaeger; Subbarao and Anderson.
$\mathrm{Y}=$ the transfer admittance at the air node. It is the contribution to $Q_{\text {air }}$ per degreee amplitude of $T_{\text {geff. }}$ [The same value of transfer admittance applies to the $T_{\text {geff }}$ node, even if the circuit is not symmetrical, being the contribution to the $T_{\text {geff }}$ node per degree amplitude of $T_{\text {air }}$ ]
$Q_{\text {air }}=$ the amplitude (Btu/hr-ft²-F) and phase of the heat transfer rate leaving $T_{\text {airr }}$ and is composed of the contribution from all of the frequencies that may be extant in the driving temperatures $T_{\text {air }}$ and $T_{\text {geff. }}$
Note that the layers shown, when modeled as a mass construction, may need to be subdivided into thinner layers, particularly the earth, in order to satisfy the discretization procedure discussed in Section 1.4; but this subdivision is irrelevant to the slab model discussion in this section.

The maximum possible thickness of the earth layer is limited by the need for $R$ to be positive. The limiting maximum possible thickness value, dmax, occurs in the perimeter case, when the foundation is uninsulated (i.e., the foundation insulation value R-0 in Bazjanac et al), and the slab is uncarpeted. In this case, dmax $=2.9 \mathrm{ft}$. The corresponding numbers for an uncarpeted core slab case is 11.8 ft
A depth of 2 ft is implemented in the code, for both the perimeter and core slab earth layers.

The 2 ft value was chosen primarily because, for the frequencies of concern, the magnitude of the X admittance from the Tair node was almost independent of earth layer depth for earth layer depths greater than 2 ft . See Figure 21. The phase shift is similarly essentially independent of depth after 2 ft . This was also the case for the core region.

This is the case for all indoor driving frequencies periods of up to at least $384-\mathrm{hr}=16-$ days. Thus 2 ft of earth is able to portray the dynamics resulting from a cycle of 8 cloudy days followed by 8 sunny days.

Figure 21: Room Node X Admittance


The transfer admittance $Y$ shown in Figure 22 also contributes to $Q_{\text {air }}$ according to the frequencies extant in the driving temperature $T_{\text {geff }}$

Figure 22: Room Node Y Admittance


As seen in Equation 128, for the slab core region, $T_{\text {geff }}$ has the same frequency content as $T_{\text {ground }}$ and $T_{\text {year }}$. $T_{\text {year }}$ is a constant, i.e., zero frequency, steady-state.

As seen in Equation 124, $T_{\text {ground }}$ contains only the annual $8760-\mathrm{hr}$ period. Figure 22 shows that the 8760 -hr waves are transmitted unaffected by the mass layer. That is, $Y$
becomes essentially equal to the steady state transfer admittance, which is the U factor of the assembly, the reciprocal of the $\mathrm{R}_{\mathrm{g}}$ value. Thus, for the core region, the magnitude of the slab loss rates produced by Equation 125 are preserved and unaffected by the added earth layers.
However, although the mass layers don't affect the magnitude of the Bazjanac model slab losses, they do introduce a time lag that is in addition to that already implicit in the $T_{g e f f}$ values. For a 2 ft earth layer the lag is $\sim 40$-hours. A 22 -day lag is already included by $\varphi$ of Equation 124. To eliminate double-counting, the 40 -hrs could be subtracted from phi, but this has not been done since $40-\mathrm{hr}$ is inconsequential compared to 22 days.
For the perimeter region, $T_{\text {geff }}$ has the additional frequency content of the $T_{\text {week }}$, the two-week running average outdoor temperature. $T_{\text {week }}$ is dominated by the annual period, but has small amplitude 6-month period component, and a bit of signal at higher frequencies. Like the annual cycle, the 6 -month period component is transmitted through the layered construction without damping, but again with a small but inconsequential phase lag.

Thus it was concluded that 2 ft of earth thicknesses below a 4-inch concrete slab adequately models changes in room side conditions, and at the same time adequately preserves the same average "deep earth" slab losses and phase lags of the Bazjanac model.

The validity of the response of the core slab construction is expected to be better than for the perimeter slab construction since the perimeter layers added do not properly account for the perimeter two-dimensional effects.

### 1.8.2.1 Warm-up time

The longest pre-run warm-up time is expected to be for a carpeted core slab with the 2 ft earth layer. Using the classical unsteady heat flow charts for convectively heated or cooled slabs (Mills) , the time to warm the slab construction $90 \%$ (of its final energy change) was found to be about 20 -days. Most of the heat-up heat transfer is via the low resistance rug and air film, with less through the higher ground resistance ( R in Figure 20(b)), so the 20-day estimate is fairly valid for the complete range of foundation insulation options given in Bazjanac's Table 3.

### 1.8.2.2 Input properties

Strictly speaking, the same properties assumed in the Bazjanac model in Section 1.8.1.3 should also be used in describing the rug, the concrete slab, and the earth in the layered constructions inputs.
This is particularly true for the carpet, if a carpet is specified, because the regression coefficients (the conductances a1, a2...) obtained for the carpeted slabs were sensitive to the Rrug value used. While inputting a different value than Rrug $=2.08$ may give the
desired carpeted room admittance response, the heat conducted from the deep ground will still give the heat flow based on Rrug $=2.08$.

Small differences between the inputted and above properties is less important for the other layers, and is violated in the code with regard to Rfilm; its value is calculated each time-step and is used instead of 0.77 , even though 0.77 is still the value subtracted from Rg in the code. This allows the correct modeling of the admittance of the slab floor, at the expense of a slight error in the overall resistance of the slab earth circuit.

### 1.9 Ventilation and Infiltration Air Network

### 1.9.1 Overview

This section describes the flow network algorithm used to model infiltration and ventilation air flows between conditioned zones, unconditioned zones, and the outdoors based on pressure and density differences and leakage areas between the zones.

Figure 23 shows the flow network interconnecting two conditioned zones (Z1 and Z2), the unconditioned attic and crawl space zones ( $\mathrm{Z3}$ and $\mathrm{Z4}$ ), and the outside zone ( $\mathrm{Z5}$ ).

Figure 23. Schematic of Flow Network


The black lines represent one or more pressure difference driven and/or buoyancy driven flows between zones.

The blue lines in Figure 23 represent scheduled fan flows not directly dependent on zone-to-zone pressure differences. These include individual house fans (circled x's) and
fan driven duct system supply, return, and leakages flows. The fans will affect the zone pressures, but the pressures won't affect the fan flow. The duct flows, determined by the load and air handler capacity, are assumed to not constitute leakage paths when the air handler is not operating.

Small leakage or ventilation openings will be modeled as orifices using a power law equations of Section 1.9 .3 with an exponent of 0.5 . Infiltration leaks are modeled with the power law equation exponent of 0.65 .

Large vertical holes or infiltration surfaces, large enough that the vertical pressure difference distribution allows two-way flow, are modeled as two vertically separated small holes using the Wolozyn method (see Section 1.9.5- Large Vertical Openings).

The following kind of elements are modeled using the power law equations:
-Wall infiltration for vertical envelope walls, vertical interzone walls, and roof decks.
-Ceiling, floor, and wall base infiltration.

- Interzone doors, door undercuts, jump ducts, relief vents.
-Openable window flow.
-Attic soffit vents, gable vents, roof deck vents, ridge vents.
-Crawl space vents.
-Trickle vents.
-Fire place leakage.
-Infiltration to garage.
Additional equations are used to model large horizontal openings, like stairwells; Section 1.9.4. This type of opening would typically be between zones Z1 and Z2 in Figure 23 when the zones are stacked vertically. The algorithm used is based on that implemented in Energy Plus (2009). In addition to using the power law equation above, the algorithm calculates buoyancy induced flows that can occur when the density of the air above the opening is larger than the density of the air below the opening, causing Rayleigh-Taylor instability.

To determine the flow rates at each time step, the flow through each flow element in the building is determined for an assumed set of zone reference pressures. If the flow into each zones does not match the flow out of the zone, the pressures are adjusted by the Newton-Raphson iterative method until the flows balance in all the zones within specified tolerances.

### 1.9.1.1 Wind direction independent air-network solution

For energy standards application, the air network for the four zone building model is designed to give results that are wind direction independent. In computing ventilation or infiltration air flows from holes in vertical walls exposed to outdoors, the program automatically calculates the sum of the flows through 4 holes each $1 / 4$ the area, one with each cardinal compass orientation, or an offset thereof. Thus, there will be wind induced flows through the envelope leakages that approximate the average flow expected over long periods, and they will be independent of wind direction. This
approach is applied to all zone ventilation or infiltration flow elements connected to the outdoor conditions.

### 1.9.2 Vertical Pressure Distribution

The pressure at a given elevation in a zone, including outdoors, is a combination of stack and wind effects added to the zones reference pressure. The difference in pressure in the zones on each side of a leakage element connecting the zones determines the flow rate through the element.

The pressure on the zone $i$ side of a flow element is given by:

$$
p_{i}=P z_{i}-\rho_{i} g z_{i}
$$

Equation 131
$z_{i}$ is the height of the element above some datum $z=0$. The datum is arbitrary but is nominally taken as ground level. $P z_{i}$ is zone $i$ 's reference pressure. This is the pressure zone $i$ would have at elevation $z=0$, regardless of whether the zone actually extends to this level. For the interior zones the $P z_{i}$ reference pressures are the unknowns that are solved for using the Newton-Raphson method. This method determines what values of zone pressures simultaneously result in a balanced flow in each zone. The value of $P z_{i}$ for the outdoor side of a flow element is given by ( $i$ is 5 if there are 4 conditioned and unconditioned zones):

$$
\mathrm{Pz}_{\mathrm{i}}=\mathrm{patm}+\mathrm{CP} * \mathrm{Pu}
$$

Equation 132
The weather tape atmospheric pressure, patm, is assumed to exist at the elevation $\mathrm{z}=$ 0 far from the building. Patm is taken as zero so that the unknown zones pressures will be found relative to the weather tape atmospheric pressure. Of course the actual weather tape atmospheric pressure is used in determining inside and outside zone air densities.

The wind velocity pressures, Pu , is:

$$
\mathrm{Pu}=\frac{\rho_{\mathrm{out}}(\mathrm{~S} * \mathrm{U})^{2}}{2}
$$

Equation 133
where,
$U$ is the wind velocity at eave height.
S is the shelter coefficient equal to SC of Table 2: Local Shielding Parameters.
$\rho_{\text {out }}$ is the outside air density.
$C P$ is the orientation sensitive pressure coefficient.

### 1.9.2.1 Pressure coefficients used

The wall pressure coefficients in Table 6 are those used by Walker et al. (2005). They are for the four vertical walls of an isolated rectangular house, with the wind perpendicular to the long wall (short wall $=1 / 2$ long wall). As discussed in regard to hip roofs below, only data for the normal wind direction is used. These coefficients are used for all ventilation and infiltration holes in the walls. Soffit vents also use these values since they are assumed to have the same pressure coefficient as the walls under them. This assumption is roughly corroborated by the data of Sharples (1997).

Table 6: Pressure Coefficients for Wind Normal to One Wall

| Pressure <br> Coefficient <br> Upwind Wall | Pressure <br> Coefficient Side <br> walls | Pressure <br> Coefficient <br> Downwind Wall |
| :---: | :---: | :---: |
| +0.6 | -0.65 | -0.3 |

Source: NORESCO for California Energy Commission
Table 7 gives hip roof's pressure coefficients for a range of roof angles. These are used to determine the outside pressure on ridge vents and roof deck vents.

There is little data available for hip roof surface pressure coefficients, or for ridge pressure coefficients (needed to model ridge vents) for any roof type. The data in Table 7 is a simplified synthesis of the data given by Xu (1998) and Holmes (1993, 2003, etc.), informed by a review of ASHRAE, EU AIVC, and other data sets and research papers.
Xu used a wind tunnel to measure pressure coefficients for a hip roofed building which was otherwise identical to the gable roof building wind tunnel data obtained by Holmes. The building had an aspect ratio of $2: 1$, with $0^{\circ}$ wind direction normal to the long side (and normal to the gable ridge and hip roof top ridge). The building eave height was 0.4 the length of the short side. The building had a relatively large eave overhang of about $35 \%$ of the eave height. Xu and Holmes presented data for this building for roof pitch angles of 15,20 , and $30^{\circ}$. Other Holmes data, for both larger and smaller roof angles was used to estimate the pressure coefficients beyond the 15 to 30 degree range. Neither Xu nor Holmes presented average surface pressures, so the average surface data and average ridge pressures given in the table are based on estimates from their surface pressure contour data.

The table is for wind normal to the long side of the building. Similar tables were obtained from Xu's data for the 45 and 90 degree wind angles. Table 7 would ideally be wind direction independent, implying some kind of average pressure coefficient; for example, for each surface take the pressure coefficient that is the average for the 0,45 , and 90 degree angles. However, infiltration flows depend on pressure differences, and the average of the pressure differences is not necessarily indicative of the difference of the average pressures. The soffit vents flows, driven by the pressure difference
between the adjacent wall and the various roof vents complicate any averaging schemes.

Comparison of the pressure coefficients for the three wind directions, while showing plausible differences, arguably does not show a discernable pattern that would obviate just using the normal wind direction data. Given the variety of roofs and building shapes that will be represented by these coefficients, the variety of vent locations and areas, and the deficiencies of the data, using a consistent set of data for only one wind direction is deemed appropriate.

Table 7: Hip Roof Wind Pressure Coefficients

| Roof Pitch $\psi$ | Upwind Roof | Side Hip <br> Roof | Downwind <br> Roof | Ridge |
| :---: | :---: | :---: | :---: | :---: |
| $\psi<10^{\circ}$ | -0.8 | -0.5 | -0.3 | -0.5 |
| $10 \leq \psi<15$ | -0.5 | -0.5 | -0.5 | -0.8 |
| $15 \leq \psi<25$ | -0.3 | -0.5 | -0.5 | -0.5 |
| $25 \leq \psi<35$ | +0.1 (pos) | -0.5 | -0.5 | -0.3 |
| $35 \leq \psi<50$ | +0.3 (pos) | -0.5 | -0.5 | -0.2 |

Source: NORESCO for California Energy Commission

### 1.9.2.2 Density

Zone is air density $\rho_{i}$ is assumed to be only a function of zone temperature $T_{i}$. That is, assuming the air is an ideal gas, at standard atmospheric conditions, the pressure change required to change the density by the same amount as a change in temperature of $1^{\circ} \mathrm{F}$ is $\frac{\partial \rho}{\partial T} / \frac{\partial \rho}{\partial p}=-\rho R_{\text {air }}$, which is approximately - 200 Pascals/F. Since zone pressure changes are much smaller than 200 Pa , they are in the range of producing the same effect as only a fraction of a degree F change in zone temperature; thus the density is assumed to always be based on patm. (This has been changed in code so that $\rho_{i}$ depends on both $T_{i}$ and $P z_{i}$ ).

Using the ideal gas approximation, with absolute temperature units,

$$
\rho_{i}=\frac{P_{a t m}}{R_{a i r} T_{i}}
$$

Equation 134
The pressure difference across the flow element is given by

$$
\Delta p_{i j}=p_{i}-p_{j}=P z_{i}-P z_{j}-g z_{i}\left(\rho_{i}-\rho_{j}\right)
$$

### 1.9.3 Power Law Flow Equation

### 1.9.3.1 Orifice flow power law

For an orifice, with fixed density of air along the flow path (from inlet to vena contracta), Bernoulli's equation gives:

$$
m=C_{D} A \sqrt{2 \rho_{\text {in }} g_{c}}(\Delta p)^{\frac{1}{2}}
$$

where
$C_{D}$ is the dimensionless orifice contraction coefficient.
$C_{D}=\frac{\pi}{\pi+2}=$ Kirchoff's irrotational flow value for a sharp edge orifice.
$C_{D}=0.6$ default for CSE windows
$C_{D}=1$ for rounded inlet orifice as used in ELA definition, and consistent with no vena contracta due to rounded inlet.
$A=$ Orifice throat area, $f t^{2}$.
$\rho_{\text {in }}=$ density of air entering the orifice; $\frac{l b_{m}}{f t^{3}}$.
$g_{c}=32.2 \frac{l b_{m} f t}{l b_{f} \text { sec }^{2}}$

### 1.9.3.2 Infiltration flow power law

The following is based on Sherman (1998). English units are used herein. Measured blower door infiltration data is expressed empirically as a power law:

$$
Q=\kappa \Delta P^{n}
$$

Equation 137
or

$$
m=\rho_{i n} \kappa \Delta P^{n}
$$

where
$Q=$ volume flow in $\mathrm{ft}^{3} / \mathrm{sec}$.
$m=$ mass flow in $l b_{m} / \mathrm{sec}$.
$\rho_{i n}=$ entering air density, $\frac{l b_{m}}{f t^{3}}$
$\Delta P=$ pressure difference in $\frac{l b_{f}}{f t^{2}}=p s f$.
$n=$ measured exponent, assumed to be $\mathrm{n}=0.65$ if measured value is unavailable.

$$
\kappa=\text { measured proportionality constant. }
$$

Equation 137 and Equation 138 are dimensional equations. Thus $\kappa$ is not a dimensionless number but implicitly has the dimensions $\mathrm{ft}^{3+2 n} /\left(\sec * \mathrm{bb}_{\mathrm{f}}\right)$. See Section 1.9.3.8-Converting Units of $\kappa$.

Sherman defines equivalent leakage area, ELA, as the area of a rounded-entrance orifice that gives the same flow as the infiltration of Equation 137 when the pressure difference $\Delta P$ is equal to the reference pressure $\mathrm{P}_{\mathrm{r}}=0.08354 \mathrm{psf}(=4 \mathrm{~Pa})$ By Equation 136, a rounded-entrance nozzle with throat area $E L A$ and $\Delta P=P_{r}$ has a flow rate:

$$
m=E L A \sqrt{2 \rho_{i n} g_{c}}\left(P_{\mathrm{r}}\right)^{\frac{1}{2}}
$$

Equation 139
Equation 137 and Equation 139 with $\Delta P=P_{r}$ gives the ELA as:

$$
E L A=\kappa P_{r}^{n-\frac{1}{2}} \sqrt{\frac{\rho_{i n}}{2 g_{c}}}
$$

Equation 140
Solving Equation 140 for $\kappa$, gives

$$
\kappa=E L A \sqrt{\frac{2 g_{c}}{\rho_{i n}}} P_{r}^{\frac{1}{2}-n}
$$

Equation 141
Substituting Equation 140 into Equation 137 gives the general equation, equivalent to Equation 137, that is the infiltration flow at any pressure difference $\Delta P$ :

$$
m=E L A \sqrt{2 \rho_{i n} g_{c}}{ }_{r}^{\frac{1}{P_{r}^{2-n}}} \Delta P^{n}
$$

Equation 142
(Note that substituting Equation 140 into Equation 142 recovers the empirical Equation 137).

### 1.9.3.3 General power law flow equation

CSE uses Equation 136 to model flow through elements such as windows, doors, and vents. Equation 142 is used for infiltration flows for elements with a defined ELA.

Both equations are special cases of the generalized flow power law Equation 143. For flow from zone $i$ to zone $j$.

$$
\begin{equation*}
m_{i, j}=S P * A_{e} \sqrt{2 \rho_{i n} g_{c}}\left|\Delta p_{i, j}\right|^{n_{g}} \tag{Equation 143}
\end{equation*}
$$

SP is the sign of the pressure difference $\Delta p_{i, j}=p_{i}-p_{j}$, utilized to determine the sign of the flow, defined as + from $i$ to $j$. The exponent is $n_{g}$, " $g$ " for generalized.

Equation 143 reduces to the orifice Equation 136 if:
$-A_{e}=A * C_{D}$ with $C_{D}=0.6$.

- $n_{g}=\frac{1}{2}$

Equation 143 reduces to the infiltration Equation 142 if:

$$
\begin{aligned}
& A_{e}=\left(P_{r}^{\frac{1}{2}-n}\right) E L A \\
& \text { - } n_{g}=n \\
& \text { - } P_{r}=0.08354 \frac{l b_{f}}{f t^{2}}
\end{aligned} \text { where } \mathrm{n} \text { here is the measured exponent. }
$$

[Although $C_{D}$ is dimensionless in Equation 136, the generalization to Equation 143 requires $C_{D}$ to implicitly have the units of $\left.\left(l b_{m}\right)^{\frac{1}{2}-n_{g}}(f t)^{2 n_{g}-1}\right]$.

### 1.9.3.4 Dealing with unbounded derivative at $\Delta \mathrm{P}=0$

The partial derivative of the mass flow of Equation 143 with respect to the pressure in zone $i$ is given by:

$$
\frac{\partial \mathrm{m}_{\mathrm{i}, \mathrm{j}}}{\partial \mathrm{p}_{\mathrm{i}}}=\mathrm{A}_{\mathrm{e}} \mathrm{n}_{\mathrm{g}} \sqrt{2 \rho_{\mathrm{in}}}\left|\Delta \mathrm{p}_{\mathrm{i}, \mathrm{j}}\right|^{\mathrm{n}_{\mathrm{g}}-1}
$$

Equation 144
Since $n_{g}<1$ this derivative à as as $\Delta$ àzero, potentially causing problems with the Newton-Raphson convergence. In order to make the derivative finite for small pressure drops, whenever the pressure difference is below a fixed value, $\Delta P_{L}$, the power law is extended to the origin by a linear power law ( $n_{g}=1$ ),

$$
m_{i, j}=S P * A_{\text {elinear }} \sqrt{2 \rho_{\text {in }} g_{c}}\left|\Delta p_{i, j}\right|^{1}
$$

Equation 145
as shown in Figure 24.
So that the flow rates match when $\Delta P=\Delta P_{L}, A_{\text {elinear }}$ is determined by equating Equation 145 to Equation 143 with $\Delta p=\Delta P_{L}$, giving:

$$
A_{\text {elinear }}=A_{e} \Delta P_{L}{ }^{n g-1}
$$

Equation 146
Note that the derivative of $m$ will be discontinuous when $\Delta p=\Delta P_{L}$, which conceivably could also cause Newton-Raphson problems, but during extensive code testing, none have occurred.

Figure 24: Mass Flow m versus Pressure Drop $\Delta P$


### 1.9.3.5 Summary of inputs to the generalized flow equation

The generalized flow equation, Equation 143,

$$
m_{i, j}=S P * A_{e} \sqrt{2 \rho_{i n} g_{c}}\left|\Delta p_{i, j}\right|^{n g}
$$

is used with the following parameter values, depending on element type and pressure drop $\Delta P$.
1.9.3.6 For windows, doors, and vents

If $\Delta P>\Delta P_{L}$ :

- $A_{e}=A * C_{D}$
- $A=$ area of flow element; $f t^{2}$.
- $C_{D}=0.6$.
- $n_{g}=\frac{1}{2}$

If $\Delta P<\Delta P_{L}$ :

- $A e=C_{D} A * \Delta P_{L}{ }^{0.5-1}=C_{D} \frac{A}{\sqrt{\Delta P_{L}}}$
$\bullet A=$ area of flow element; $f t^{2}$.
- $C_{D}=0.6$
$\bullet n_{g}=1$
$-\Delta \mathrm{P}_{\mathrm{L}}=$ determined by computational experiment.
1.9.3.7 For infiltration leakage elements

If $\Delta P>\Delta P_{L}$ :
$A_{e}=\left(P_{r}^{\frac{1}{2}-n}\right) E L A$
if $n=0.65, A_{e}=1.45 * E L A$, used in CEC ACM manual).
$\bullet n_{g}=n$

- $P_{r}=0.08354 \frac{l b_{f}}{f t^{2}}$
$\bullet E L A$ is determined from either:
- the measured parameters $\kappa$ and $n$ using Equation 140. See Section 1.9.3.8-Converting units of $\kappa$.
- code regulations, in which case $n=0.65$ is assumed.

If $\Delta P<\Delta P_{L}$ :

$$
\begin{aligned}
& A_{e}=\Delta P_{L}^{n-1}\left(P_{r}^{\frac{1}{2}-n}\right) E L A, \text { where } \mathrm{n} \text { here is the measured value, or } 0.65 . \\
& \bullet n_{g}=1 \\
& \bullet P_{r}=0.08354 \frac{l b_{f}}{f t^{2}}
\end{aligned}
$$

### 1.9.3.8 Converting units of $\kappa$

The $\kappa$ in Equation 137 is not dimensionless, so $\kappa$ changes value depending on the units of $Q$ and $\Delta P$ in Equation 137. The analysis herein (Section 1.9) assumes $Q$ in $\frac{f t^{3}}{s e c} \mathrm{~d} \Delta P$ in $\frac{l b_{f}}{f t^{2}}$.

However, conventionally $\mathrm{\kappa}$ is obtained from measured data with $Q$ in $\left(\frac{f t^{3}}{\min }\right)$ and $\Delta P$ in Pascals. With these units Equation 137 takes the form:

$$
Q[c f m]=\kappa^{\prime}(\Delta P(P a))^{n}
$$

Equation 147
Using dimensional analysis the value of $\kappa$ to be used in Equation 137 with Q in $\left(\mathrm{ft}^{3} / \mathrm{sec}\right)$ and $\Delta \mathrm{P}$ in $\left(\frac{\mathrm{lb}_{\mathrm{f}}}{\mathrm{ft}^{2}}\right)$ is:

$$
\kappa=\left(\frac{47.88^{n}}{60}\right) \kappa^{\prime}
$$

Equation 148
where the numbers are from the conversion factors $47.88 \frac{\mathrm{~Pa}}{\frac{\mathrm{laf}}{\mathrm{ft}^{3}}}$ and $60 \mathrm{sec} / \mathrm{min} . \kappa=$ $0.206 \kappa^{\prime}$ for $\mathrm{n}=0.65$.

$$
\kappa^{\prime}=\text { the measured value from data with } Q \text { in }\left(\frac{f t^{3}}{\min }\right) \text { and } \Delta P \text { in Pascals. }
$$

### 1.9.3.9 ACM Manual relationship between CFM50 and ELA

Using Equation 142 (in volume flow form) the infiltration volume flow, CFS50, with 50 Pa pressurization is:

$$
\text { CFS50 }=\mathrm{ELA} \sqrt{\frac{2 g_{c}}{\rho}} P_{r}^{\frac{1}{2}-n} \Delta P^{n}=E L A \sqrt{\frac{64.4}{0.075}} 0.08354^{-0.15} 1.04428^{0.65}=43.738 * E L A
$$

Equation 149
where

$$
\Delta P=50 \mathrm{~Pa}=1.04428 \mathrm{psf}
$$

CFS50 $=$ flow in units of $\frac{f t^{3}}{s e c}$ or cfm units, and ELA in square inches,
CFM50 = (60/144)CFS50 = 18.224*ELA

Equation 150
or alternately,

$$
\mathrm{ELA}\left[\mathrm{in}^{2}\right]=0.055^{*} \mathrm{CFM} 50
$$

This is the equation used to get ELA from blower door data at 50 Pa pressure difference.

### 1.9.3.10 Heat Flow

When the flow $m_{i, j}$ is positive, the heat delivered to zone j by this flow is given by

$$
Q_{j}=m_{i, j} C_{p}\left(T_{i}-T_{j}\right)
$$

Equation 152
while the heat delivered to zone $i$ by the flow $m_{i, j}$ is zero:

$$
Q_{i}=0
$$

Equation 153
When the flow $m_{i, j}$ is negative, the heat delivered to zone j is zero,

$$
Q_{j}=0
$$

Equation 154
while the heat delivered to zone $i$ by the flow $m_{i, j}$ is:

$$
Q_{i}=m_{i, j} C_{p}\left(T_{i}-T_{j}\right)
$$

Equation 155

### 1.9.4 Large Horizontal Openings

An additional set of equations is needed to model large horizontal openings such as stairwells. The algorithm used is similar to that implemented in Energy Plus, which is based on that given by Cooper (1989). In addition to pressure driven flow using the power law equations of Section 1.9.3.3 this algorithm involves buoyancy induced flows that can occur when the density of the air above the opening is larger than the density of the air below the opening, causing Rayleigh-Taylor instability.

For a given rectangular opening this algorithm can produce three separate flows components between the zones:
a) a forced orifice flow in the direction dictated by the zone to zone pressure difference, $\Delta \mathrm{p}$. This flow is independent of the following instability induced flows.
b) a buoyancy flow downward when the air density in the upper zone is greater than that is the lower zone, i.e., Tupper-zone < Tlower-zone. This flow is maximum when $\Delta p$ is zero, and linearly decreases with increasing $\Delta p$ until the buoyancy flow is zero, which occurs when the pressure difference is large enough that the forced flow "overpowers" the instability flow. The latter occurs if $\Delta p$ is greater than the "flooding" pressure $\Delta \mathrm{pF}$.
c) an upward buoyancy flow equal to the downward buoyancy flow.

These three flows are modeled by two flow-elements. The first element handles the forced flow (a) and in addition whichever of the buoyancy flow component, (a) or (c), that is in the same direction as the forced flow. The second element handles the alternate buoyancy flow component.

### 1.9.4.1 Pressure driven flow

The pressure forced flow is modeled as orifice flow using Equation 143, except the area $A$ in the Section 1.9.3.5 is replaced by:

$$
\text { Aeff }=L 1 * L 2 * \sin (\text { StairAngle }) *(1+\cos (\text { StairAngle }))
$$

Equation 156
$\angle 1$ and $\angle 2$ are the dimensions of the horizontal rectangular hole. To include the effect of stairs, a StairAngle can be set, where StairAngle $=90$ deg corresponds to vertical stairs. The angle can be set to 90 degrees to exclude the effect of the stairs.

Equation 144 is used for the partial derivative of the flow, with the area $A_{e}$ using Aeff in place of $A$.

### 1.9.4.2 Buoyancy flow

When the zone on top has a higher density than the zone on the bottom, the maximum possible buoyancy flow, mbm, occurs when the pressure difference across the hole is zero:

$$
m b m=0.055 \sqrt{g \bar{\rho}|\Delta \rho| D h y d^{5}}
$$

Equation 157
The 0.055 factor is dimensionless ; $\mathrm{g}=32.2 \mathrm{ft} / \mathrm{s}^{\wedge} 2$.
The hydraulic diameter of the hole is defined as:

$$
\text { Dhyd }=2 * \frac{\text { Aeff }}{L 1+L 2}
$$

Equation 158
When the zone on top has a higher density than the zone on the bottom, and the pressure difference is lower than the flooding pressure, then the buoyancy flow is given by:

$$
m b=m b m *\left(1-\frac{|\Delta p|}{\Delta p F}\right)
$$

Equation 159
The flooding pressure difference $\Delta p F$ is defined as:

$$
\Delta p F=\frac{C_{s}^{2} g|\Delta \rho| D h y d^{5}}{2 \mathrm{~A}_{\mathrm{eff}}^{2}}
$$

The shape factor $C_{s}$ is

$$
C_{s}=0.942\left(\operatorname{minimum}\left(\frac{L 1}{L 2}, \frac{L 2}{L 1}\right)\right)
$$

Equation 161
If the top zone density is lower than the bottom zones, or if $|\Delta p|>\Delta p F$ then the buoyancy flow mb is zero.

The partial derivatives of the buoyancy flows with respect to adjacent zone pressures are all zero since the buoyancy flows are equal and opposite. That is, although the buoyancy flow magnitudes are sensitive to zone pressures, they have no influence on the zone mass balance.

Although the buoyancy flows don't directly influence zone pressures, they do affect the heat transfer rates.

When buoyancy flows exists, the heat transfer due to the buoyancy flow to the upper zone, isay, is

$$
\mathrm{Q}_{\mathrm{i}}=\mathrm{mb} * \mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{\mathrm{j}}-\mathrm{T}_{\mathrm{i}}\right)
$$

Equation 162
and to the lower zone is

$$
Q_{j}=m b * C_{p}\left(T_{i}-T_{j}\right)
$$

Equation 163

### 1.9.5 Large Vertical Openings

The flow through large vertical rectangular openings are handled using the method suggested by Woloszyn (1999).

Woloszyn uses a simplified version of the common integrate-over-pressure-distribution scheme as used by Walker for example. Rectangular holes are divided in two, with the flow through the top half driven by a constant $\Delta \mathrm{p}$ equal to the pressure difference $3 / 4$ the way up the opening (the midpoint of the top half of the opening area). Similarly, the flow through the bottom half uses the $\Delta \mathrm{p}$ at $1 / 4$ the way up the hole, and assumes it is constant over the bottom half. Although approximate compared to the integration methods, it is expected to be able to reasonably accurately, if not precisely, portray one and two way flows through such elements. This procedure has the virtue of eliminating the calculation of the neutral level, thereby greatly reducing the number of code logic branches and equations. It also eliminates a divide by zero problem when $\Delta \rho$ à $O$ in the exact integration methods.

Besides being used for large vertical holes, like open windows and doorways, the method is also used for distributed infiltration. That is, a rectangular wall with an effective leakage area ELA is represented by two holes, each of area ELA/2, located at
the $1 / 4$ and $3 / 4$ heights. These holes are then modeled using Equation 138, Equation 141, Equation 144, and Equation 145.

### 1.9.5.1 Triangular surfaces

The method is generalized further to treat the tilted triangular surfaces assumed for hip roofs. In this case the lower Woloszyn hole, of area ELA/2, is placed at the height that is above $1 / 4$ of the area of the triangle. This can be shown to be a height of:

$$
H_{\text {lower hole }}=Z_{\text {soffit }}+\left(1-\frac{\sqrt{3}}{2}\right)\left(Z_{\text {ridge }}-Z_{\text {soffit }}\right)
$$

Equation 164
Similarly, the top hole is placed at the height above $3 / 4$ of the area of the triangle:

$$
H_{\text {upper hole }}=Z_{\text {soffit }}+\left(\frac{1}{2}\right)\left(Z_{\text {ridge }}-Z_{\text {soffit }}\right)
$$

Equation 165

### 1.9.6 Newton-Raphson Solution

Assume there are a total of nuc conditioned and unconditioned zones with unknown pressures. The outside conditions, of known pressure, are assigned a zone number nout $=n u c+1$.

The mass flow rate from zone $i$ to zone $j$ (including $j=n o u t$ ) is designated as $m_{i, j}$, and can be positive (flow out of zone i) or negative (flow into zone i).

$$
m_{i, j}=\sum_{k=1}^{K_{i, j}} m_{i, j, k}
$$

Equation 166
where $m_{i, j, k}$ is the flow rate through the $k$ th element of the $K_{i, j}$ elements in surface $i, j$. By symmetry,

$$
m_{i, j, k}=-m_{j, i, k}
$$

Equation 167
and

$$
m_{\mathrm{I}, \mathrm{j}}=-\mathrm{m}_{\mathrm{j}, \mathrm{i}}
$$

Equation 168
From Equation $167, \mathrm{~m}_{\mathrm{I}, \mathrm{j}, k}$ values are functions of the zone pressure difference $\left(P_{i}-P_{j}\right)$.

$$
\begin{array}{ll}
m_{i, j, k}=+A_{e} \sqrt{2 \rho_{i n}}\left(P_{i}-P_{j}\right)^{n} & \text { for positive } \Delta P \\
m_{i, j, k}=-A_{e} \sqrt{2 \rho_{i n}}\left(P_{j}-P_{i}\right)^{n} & \text { for negative } \Delta P
\end{array}
$$

This shows that in general,

$$
\frac{\partial m_{i, j, k}}{\partial P_{i}}=-\frac{\partial m_{i, j, k}}{\partial P_{j}}
$$

Equation 170
The net flow leaving zone $i\left(i=1\right.$ to nuc) is the defined as the residual $r_{i}$ :

$$
r_{i}=\sum_{j=1, j \neq i}^{\text {nout }} m_{i, j}=\sum_{j=1, j \neq i}^{j=\text { nout }} \sum_{k=1}^{K_{i, j}} m_{i, j, k}
$$

Equation 171
The $j \neq i$ criterion on the sums eliminates summing $m_{i, i}$ terms which are zero by definition. The zone pressures $P_{I}$ are to be determined such that the residuals $r_{I}$ all become zero.

Equation 169 and Equation 171 constitute a set of $n=$ nuc nonlinear equations with $n$ $=$ nuc unknown pressures. To linearize the equations, a Taylor's series is used to determine the residual $r_{i}^{\prime}$ at the pressure $P_{j}^{\prime}$ near the guessed value of pressures $P_{j,}$, where the residual is $r_{i}$. Keeping only first order terms:

$$
r_{i}^{\prime}=r_{i}+\sum_{j=1}^{n u c} \frac{\partial r_{i}}{\partial P_{j}}\left(P_{j}^{\prime}-P_{j}\right)
$$

Equation 172
In matrix form this is written:

$$
\boldsymbol{r}^{\prime}=\boldsymbol{r}+\boldsymbol{J}\left(\boldsymbol{P}^{\prime}-\boldsymbol{P}\right)
$$

Equation 173
where $\boldsymbol{r}^{\prime}$ is the vector with elements $r_{i}^{\prime}$, and $\boldsymbol{r}$ is the vector with elements $r_{i}$.
$J$ is the nuc-by-nuc Jocobian matrix with elements:

$$
J_{i, l}=\frac{\partial r_{i}}{\partial P_{l}}=\sum_{j=1}^{\text {nout }} \frac{\partial m_{i, j}}{\partial P_{l}}=\sum_{j=1, j \neq i}^{j=\text { nout }} \sum_{k=1}^{k=K_{i, j}} \frac{\partial m_{i, j, k}}{\partial P_{l}}
$$

Equation 174
where $i=1$ to nuc, and $l=1$ to nuc.
Setting $r_{i}^{\prime}=0$ and solving for $P_{j}{ }^{\prime}$, Equation 173 becomes

$$
P^{\prime}=P-J^{-1} r
$$

Equation 175

$$
P^{\prime}=P-C
$$

Equation 176
where $\boldsymbol{C}$ is the correction vector:

$$
C=-J^{-1} r
$$

$$
P_{i}^{\prime}=P_{i}-C_{i}
$$

Equation 178
Equation 178 gives the pressures $P_{i}^{\prime}$ that are predicted to make $r_{i}^{\prime}$ zero.

### 1.9.6.1 Convergence

Convergence is attained when the residuals $r_{i}$ are sufficiently small. As employed by Energy Plus and Clarke, both absolute and relative magnitude tests are made.

Convergence is assumed when the absolute magnitude of the residual in each zone $i$ is less than a predetermined limit ResMax:

$$
\mid \text { res }_{i} \mid<\text { ResMax }
$$

Equation 179
OR, the magnitude of the residual divided by the sum of the magnitudes of the flow through each element connected to zone $i$, is less than a predetermined limit ResErr:

$$
\frac{\left|r e s_{i}\right|}{\text { resmag }_{i}}<\text { ResErr }
$$

Equation 180
where

$$
\operatorname{resmag}_{i}=\sum_{j=1}^{\text {nout }}\left|m_{i, j}\right|
$$

Equation 181
(code uses: sum of magnitude of flows to \& from zone iz, resmag(iz) += ABS(mdot(iz,jz,ke)).

### 1.9.6.2 Relaxation

Equation 178 is more generally written as

$$
P_{i}^{\prime}=P_{i}-\operatorname{relax} * C_{i}
$$

Equation 182
where relax is the relaxation coefficient, a factor less than one that reduces the correction applied to $P_{i}$. Relaxation factors on the order of 0.75 have been shown to reduce the number of iterations in cases normally having slowly decreasing and oscillating corrections. But a fixed value of 0.75 can slow what were formerly rapidly converging cases. The following approach is used to reduce the relaxation factor only when necessary.

Following Clarke, when the corrections $C_{i}$ from one iteration to the next changes sign, and the latest $C_{i}$ has a magnitude over half as big as the former $C_{i}$, then it is assumed
that the convergence is probably slow and oscillating. This symptom is typically consistent over a few iterations, and if this were precisely the case, the correction history would follow a geometric progression with a negative common ratio $\frac{c_{i}}{c_{i}^{\text {last }}}$. Thus by extrapolation a better estimate of correct solution will be obtained if the relaxation factor is taken as the sum of the infinite termed geometric progression:

$$
\text { relax }=\frac{1}{1-\frac{C_{i}}{C_{i}^{\text {last }}}}
$$

Equation 183
Thus, whenever, during an iteration for zone $i$,

$$
\frac{C_{i}}{C_{i}^{\text {last }}}<-0.5
$$

Equation 184
then Equation 178 is replaced by

$$
P_{i}^{\prime}=P_{i}-\frac{1}{1-\frac{C_{i}}{C_{i}^{\text {last }}}} * C_{i}
$$

Equation 185
Insofar as the extrapolation is warranted, this should give a better prediction of the pressure $P_{i}^{\prime}$ than would using relax $=1$ for this iteration. For the iteration following that using Equation 185, relax $=1$ is reverted to (i.e., Equation 178) so that only unrelaxed correction values are used to evaluate $\frac{c_{i}}{c_{i}^{\text {last }}}$. The following iteration, if any, is then again tested by Equation 184. The first iteration is always done with relax $=0.75$ since at this point there is no value available for $C_{i}^{\text {last }}$.

It would be reasonable to add a max Ci limit; i.e., max pressure change allowed, a la Clarke, but code testing has not shown the need.

### 1.9.6.3 Off diagonal terms

Consider Equation 174 for off-diagonal terms. Since $i \neq l$, zone $i$ 's flow $m_{i, j}$ varies with $P_{l}$ only if $j=/$. Thus setting $j=l$, and $i \neq l$, Equation 174 reduces to:

$$
J_{i, l, i \neq l}=\sum_{k=1}^{K_{i, l}} \frac{\partial m_{i, l, k}}{\partial P_{l}}
$$

Equation 186
where $i=1$ to nuc, and $l=1$ to nuc.
Equation 186, along with Equation 170, shows that all off diagonal terms have a negative magnitude. Since $m_{i, l, k}=-m_{l, i, k}$,

$$
J_{i, l, i \neq l}=\sum_{k=1}^{K_{i, l}} \frac{\partial m_{i, l, k}}{\partial P_{l}}=-\sum_{k=1}^{K_{i, l}} \frac{\partial m_{l, i, k}}{\partial P_{l}}
$$

Equation 187
Using Equation 170, Equation 187 becomes:

$$
J_{i, l, i \neq l}=\sum_{k=1}^{K_{i, l}} \frac{\partial m_{l, i, k}}{\partial P_{i}}=J_{l, i, i \neq l}
$$

Equation 188
Thus the Jacobian matrix is symmetric:

$$
J_{i, l}=J_{l, i}
$$

Equation 189
Thus only the upper (or lower) diagonal terms need be determined, with the other half determined by transposition. The off diagonal terms only involve partials of flows between zones with unknown pressures.

### 1.9.6.4 Diagonal terms

For $i=j$ Equation 174 gives:

$$
J_{i, i}=\frac{\partial r_{i}}{\partial P_{i}}=\sum_{j=1}^{\text {nout }} \frac{\partial m_{i, j}}{\partial P_{i}}=\sum_{j=1}^{j=\text { nout }} \sum_{k=1}^{k=K_{i, j}} \frac{\partial m_{i, j, k}}{\partial P_{i}}
$$

Equation 190
where $i=1$ to nuc.
Equation 190 terms can be regrouped to show a simpler numerical way to determine $J_{i, i}$, by using the off diagonal terms already calculated:

$$
J_{i, i}=\frac{\partial m_{i, n o u t}}{\partial P_{i}}-\sum_{k=1, k \neq i}^{k=n u c} J_{k, i}
$$

Equation 191
This shows that the diagonal elements use the derivatives of mass flows to the outdoors minus the off diagonal terms in the same column of the Jacobian.

Equation 191 shows that matrix will be singular if $\frac{\partial m_{\text {inout }}}{\partial P_{i}}=0$, so that at least one connection to outdoors is necessary.

### 1.10 Duct System Model

### 1.10.1 Description of Model

The duct model builds on the procedure given by Palmiter (see Francisco and Palmiter, 2003), that uses a steady state heat exchanger effectiveness approach to get analytical expressions for instantaneous duct loss and system efficiencies. The duct model, developed for this program by Palmiter, makes use of many of the same fundamental steady state equations and approach, but given the considerable complexity of the multiple duct systems, does not do a simultaneous solution of all the equations which a generalized Francisco and Palmiter scheme may imply. Instead the approach takes advantage of the small time steps used in the code, and in effect decouples the systems from each other and the zone by basing all losses and other heat transfers occurring during the time step on the driving conditions of Tair and Tmrt known at the beginning of the time step, similar to how heat transfers are determined during mass temperature updates.

Other assumptions made in the duct program: mass and thermal siphon effects in the duct system are ignored.

The duct system performance is analyzed at every time step. The duct air temperatures are calculated assuming they are operating at steady state, in equilibrium with the thermal conditions at the beginning of the time-step in the attic. Heat capacity effects of the ducts are ignored.

During each time step, the following steps are taken to find the duct system operating conditions such as the air temperatures in each duct, the losses, the heating or cooling delivered, etc.

Initially, for each time step, the duct systems performance is determined when operating at full capacity, independent of the load. The procedure starts at the return registers in each conditioned zone, where the duct air temperatures are the current timesteps conditioned zone air temperatures. The conditioned zone air entering the return register heats or cools, or both, as it traversed through each component of the duct system: the return duct, the return plenum, the heating/cooling device, and the supply ducts. That is, the duct air temperature rises or drops immediately downstream of the return register (where returns leaks are assigned to occur) due to mixing of leakage air at the air temperature in the unconditioned zone in which the return duct is located with the return air from the conditioned zone. It may also increase or decrease in temperature in the return plenum as it mixed with the air from the return duct in another unconditioned zone. After being heated or cooled by the air handler at its applicable heating/cooling capacity, it is then additionally heated or cooled by supply duct conductive gains/losses to the interior of the unconditioned zone.

Summing all the gains and losses in temperature of the duct air as it travels through the system gives the supply temperature for the supply duct, allowing the heat delivered at full capacity, Qdel, to be determined.

If the above useful heat delivered at full capacity is more than required by the load, then the equipment capacity is reduced to meet the load by assuming the system is only running the fraction $\frac{\text { Qload }}{Q d e l}$ of the time step. The needed capacity, Qneed, is this fraction of the nominal capacity. The duct losses for the time step are also reduced by this fraction.

The above calculations are done each time step and the average Qneed summarized in the hourly output.

The above steps are presented in detail in the following sections, in the same sequence as described above.

### 1.10.2 Duct System Inputs

### 1.10.2.1 Subscripts

In most cases in this section, the subscripted variables stand for arrays.
The subscript u stands for the unconditioned zone in which the duct is located.
The subscript c stands for conditioned zone number and its associated air handler system.
The subscript m stands for the mode of air handler operation: 0 off, 1 heating, 2 cooling.

### 1.10.2.2 Annual run inputs

The following data is input to model the duct/air handler system(s):

## Duct inside areas

Asd $d_{c, u}=$ supply duct inside area for air handler c in unconditioned zone u .
$\operatorname{Ard}_{c, u}=$ return duct inside area for airhandler c in unconditioned zone u.

## Duct insulation rated $R$ values

$R s d_{c, u}=$ supply duct rated R for air handler c in unconditioned zone u ; hr - $\mathrm{ft}{ }^{2}-$ F/Btu.
$\operatorname{Rr} d_{c, u}=$ return duct rated R for air handler c in unconditioned zone u ; $\mathrm{hr}^{-\mathrm{ft}^{2}-}$ F/Btu.

## Inside duct area and inside area based resistance, and the outside duct area and outside area based resistance when there is a single duct segment in the return and supply branches

Consider one duct of constant inside diameter, $d_{i}$, and length $L$. The duct is insulated with insulation having a thermal conductivity k , and rated R value, $R_{\text {rate }}$. All R values herein are in the units of (hr-ft2-F/Btu).

Layed flat, the thickness the insulation layer is:

$$
t=\text { Rrate } * k
$$

Equation 192
If the insulation is wrapped at this thickness around a duct of diameter $d_{i}$, the outside diameter, $d_{o}$, of the insulation will be:

$$
d_{o}=d_{i}+2 * \text { Rrate } * k
$$

so,

$$
\frac{d_{o}}{d_{i}}=1+\frac{2 k R_{\text {rate }}}{d_{i}}
$$

Equation 193
Conduction heat transfer texts gives the overall conductance $C$ of length $L$ of an annular insulation layer as:

$$
C=\frac{2 \pi k L}{\ln \left(\frac{d_{o}}{d_{i}}\right)}
$$

Equation 194
Dividing this by inside area, $A_{i}=\pi d_{i} L$, gives the conductance per unit inside area:

$$
C_{i}=\frac{2 k}{d_{i} \ln \left(\frac{d_{o}}{d_{i}}\right)}
$$

The duct resistance value per unit inside area is the reciprocal,

$$
R_{i}=\frac{d_{i} \ln \left(\frac{d_{o}}{d_{i}}\right)}{2 k}
$$

Equation 195
This can be written in terms of areas, and length $L$, as:

$$
R_{i}=\frac{A_{i} \ln \left(\frac{A_{o}}{A_{i}}\right)}{2 \pi k L}
$$

Equation 196
The duct resistance value based on outside area can be determined from $R_{i}$ and $A_{i}$ as:

$$
R_{o}=\frac{d_{o}}{d_{i}} R_{i}
$$

Equation 197

$$
A_{o}=\frac{d_{o}}{d_{i}} A_{i}
$$

The $R$ values of Equation 195 and Equation 197, divided by $R_{\text {rated }}$, are plotted in Figure 25 as a function of the inside diameter of the duct branch.

Figure 25: Ratio of Actual $\mathbf{R}$ to Rated $\mathbf{R}$


## Duct system composed of mu/tiple segments in the supply and return branches

Suppose the supply ducts from an air handler system are branched, with each branch having different sizes, lengths, rated insulation Rrate, and conductivity k values, and all the branches are in one unconditioned zone. These could be combined into one equivalent duct as follows.

The duct branches, $\mathrm{j}=1 \mathrm{a} \mathrm{a} \mathrm{n}$, are combined, each of inside areas $A_{i}(j)$, outside areas $A_{o}(j)$, conductivities $k(j)$, and inside area based resistances $R_{i}(j)$. Using the method of

Palmiter and Kruse (2003), the overall conductance of the branched duct system, based on inside area, is the sum of the conductances of each branch:

$$
{ }^{\prime} U A^{\prime}=\sum_{j=1 \rightarrow n}\left(\frac{A_{i}(j)}{R_{i}(j)}\right)
$$

Equation 198
where $R_{i}$ for each branch segment $j$ is given by Equation 196 as

$$
R_{i}(j)=\frac{A_{i}(j) \ln \left(\frac{A_{o}(j)}{A_{i}(j)}\right)}{2 \pi k(j) L(j)}
$$

Equation 199
With $A_{i}(j)=\pi d_{i}(j) L(j)$, and using Equation 193, this can be written as

$$
R_{i}(j)=\frac{d_{i}(j) \ln \left(1+\frac{2 k(j) R_{\text {rate }}(j)}{d_{i}(j)}\right)}{2 k(j)}
$$

Equation 200
The total branch inside area is:

$$
A_{i}=\sum_{j=1 \rightarrow n} A_{i}(j)
$$

Equation 201
The effective overall resistance of the branched duct, based on inside area $A_{i}$, is thus:

$$
R_{i}=\frac{A_{i}}{{ }^{\prime} U A^{\prime}}=\frac{A_{i}}{\sum_{j=1 \rightarrow n}\left(\frac{A_{i}(j)}{R_{i}(j)}\right)}=\frac{A_{i}}{\sum_{j=1 \rightarrow n}\left[\frac{A_{i}(j)}{\frac{d_{i}(j) \ln \left(1+\frac{2 k(j) R_{\text {rate }}(j)}{d_{i}(j)}\right)}{2 k(j)}}\right]}
$$

Equation 202
The values of the terms needed for each branch segment, shown on the right hand side of Equation 202 are not available since the former ACM manual only requires that the following $R$ is known:

$$
\text { "R" }=\frac{A_{i}}{\sum_{j=1 \rightarrow n} \frac{A_{i}(j)}{R_{\text {rate }}(j)}}
$$

Equation 202 and Equation 203 would be equivalent if Equation 203 had the term $R_{i}(j)$ in place of $R_{\text {rate }}(j)$. As it is, Equation 203 gives the area weighted average $R_{\text {rate }}$, not $R_{i}(j)$.
The total outside area is:

$$
A_{o}=\sum_{j=1 \rightarrow n} A_{o}(j)
$$

Equation 204
Based on outside area, the effective duct system resistance would be:

$$
R_{o}=\frac{A_{o}}{A_{i}} R_{i}
$$

Equation 205

### 1.10.2.3 Emissivities

epss $c_{c, u}=$ supply duct emissivity for air handler c in unconditioned zone u .
$e p s r_{c, u}=$ return duct emissivity for air handler c in unconditioned zone u .

### 1.10.2.4 Duct leakage

$L s_{c, u}=$ the fraction of the flow through the system c air handler fan that is leaked from the supply duct in unconditioned zone $u$. The leak is assigned to occur near the supply register so that the leakage air is at the supply register temperature.
$L r_{c, u}=$ the fraction of the flow through the system c air handler fan that is leaked into the return duct in unconditioned zone $u$. The leak is assigned to occur at the return register. The air leaking into the duct is at the unconditioned zone temperature.

### 1.10.2.5 System flow

Flow $_{m, c}=$ the flow rate in cfm (at standard conditions) through the air handler for the cooling and heating modes, for of each system.

### 1.10.2.6 Flow distribution

How much of the air handler flow of system c goes through each of its return and supply ducts is given by the per run input flow fractions:
$F m r_{c, u}=$ fraction of flow of system c in the return duct located in unconditioned zone u.
$F m s_{c, u}=$ fraction of flow of system c in the supply duct located in unconditioned zone u.
$F m r c_{c}=$ fraction of flow of system c in the return duct located in conditioned zone c .
$\mathrm{Fmsc}_{c}=$ fraction of flow of system c in the supply duct located in conditioned zone c.

For a given system c , the sum of the return duct fractions must add to one: $F m r_{c, 1}+$ $F m r_{c, 2}+F m r c_{c}=1$. Similarly for the supply duct fractions.

### 1.10.3 Return Duct Air Temperatures

Following the procedure indicated in Section 1.10.1, the return duct air temperatures are determined first. Utilizing the heat exchanger effectiveness approach (see Mills (1992), andA), the temperature of the system c return duct air entering the return plenum from a return duct located in unconditioned zone number $u$ is given by:

$$
\operatorname{Tout}_{c, u}=E r_{m, c, u} \text { Teqr }_{c, u}+\left(1-E r_{m, c, u}\right) \cdot \operatorname{Tmix}_{c, u}
$$

where $E r_{m, c, u}$ is the effectiveness of the return duct of system c in unconditioned zone u when operating in mode m :

$$
E r_{m . c . u}=1-e^{\frac{- \text { Urtot }_{c, u}}{\text { Mcpr } r_{m, c}, u}}
$$

where $\operatorname{Urtot}_{c, u}$ is the total conductance between the return duct air and the equivalent surroundings temperature $T e q r_{c, u}$ :

$$
\operatorname{Teqr}_{c, u}=\left(F r d a_{c, u} \cdot \text { Tair }_{u}+F r d r_{c, u} \cdot \operatorname{Tmr}_{u}\right)
$$

Frd $a_{c, u}$ is the fraction of return duct (dissolved surface node) conductance that goes to the Tair ${ }_{u}$ node.

$$
F r d a_{c, u}=\frac{U r c_{c, u}}{\left(U r c_{c, u}+U r r_{c, u}\right.}
$$

$F r d r_{c, u}$ is the fraction of the conductance from the c,u return duct air that goes to the $T m r t_{u}$ radiant node.

$$
F r d r_{c, u}=\frac{U r r_{c, u}}{U r c_{c, u}+U r r_{c, u}}
$$

The $U$ terms are the conductances from the duct air to the mrt and air nodes, determined as described inA. These conductance values, and the similar supply duct values of Section 1.10 .6 are used in the energy balance of the unconditioned zone(s) containing ducts.

$$
\begin{aligned}
& U r r_{c u}=\text { conductance from return duct air to Tmrt. } \\
& U r c_{c, u}=\text { conductance from return duct air to Tair. } \\
& U r t o t_{c, u}=U r c_{c, u}+U r r_{c, u}
\end{aligned}
$$

The term $M c p r_{m, c, u}$ is the flow conductance (see below) for the return duct flow:

$$
M c p r_{m, c, u}=M c p_{m, c} F m r_{c, u}
$$

The total system flow, $\mathrm{Mcp}_{\mathrm{m}, \mathrm{c}}$ is in the "flow conductance" form with the units Btu/hr-F:

$$
M_{c} c p_{m, c}=\text { Flow }_{m, c} \cdot c_{p}
$$

where $c_{p}$ is the volumetric heat capacity, which is taken as $1.08 \mathrm{Btu} /(\mathrm{hr}-\mathrm{F}$-cfm) for dry air at the ASHRAE standard conditions of density $=0.075 \mathrm{Ib}_{\mathrm{m}} / \mathrm{ft}^{3}$ and $\mathrm{c}_{\mathrm{p}}=0.24 \mathrm{Btu} / \mathrm{b}_{\mathrm{m}}-$ F.

The term $T m i x_{c, u}$ is the mixed air just downstream of the return duct leakage, given by:

$$
\operatorname{Tmix}_{c, u}=L r_{c, u} \text { Tair }_{u}+\left(1-L r_{c, u}\right) \text { Temp }_{c}
$$

where $T_{e m p}^{c}$ is the temperature of conditioned zone c's air, assumed to be well-mixed.

### 1.10.4 Return Plenum Temperature and Return Duct Conductive Heat Losses

The heat loss rate from the return duct via convection and radiation, needed in the unconditioned zone energy balance, is:

$$
q l r_{c, u}=M c p r_{m, c, u} \cdot\left(\text { Tmix }_{c, u}-\text { Tout }_{c, u}\right)
$$

The final return plenum temperature of system c is found by summing the contributions to its plenum temperature from the return ducts in each unconditioned zone and the return ducts located in the conditioned zone. That is,

$$
\operatorname{Trp}_{c}=F m r c_{c} \cdot \operatorname{Temp}_{c}+\sum_{\text {allu }} F m r_{c, u} \cdot \text { Tout }_{c, u}
$$

### 1.10.5 Temperature Rise through Air Handler Heating or Cooling Equipment

If the mode is heating or cooling, the temperature rise through the air handler heating or cooling equipment of system c at sensible capacity $\mathrm{Cap}_{c}$ is given by:

$$
d t e_{c}=\frac{C a p_{c}}{M c p_{m, c}}
$$

Equation 206
The program considers no heat losses or gains from the air handler components other than from the ducts.

### 1.10.6 Supply Plenum and Supply Register Temperatures

The supply plenum temperature is given by:

$$
T s p_{c}=T r p_{c}+d t e_{c}
$$

The supply register temperature for the supply duct of system c in unconditioned space u is:

$$
T s r_{c, U}=\text { Teqs }_{c, u}+\left(1-E s_{m, c, u}\right) \cdot\left(T s p_{c}-\text { Teqs }_{c, u}\right)
$$

Equation 208
where $E s_{m, c, u}$ is the effectiveness of the supply duct of system c in unconditioned zone u when operating in mode m :

$$
E s_{m, c, u}=1-e^{\frac{-U_{s t o t}^{c, u}}{M c p s_{m, c}, u}}
$$

Substituting the $T s p_{c}$ equation above into Equation 208and rearranging gives:

$$
T s r_{c, u}=\left(1-E s_{m, c, u}\right) d t e_{c}+T s r h x_{m, c, u}
$$

Equation 209
Where

$$
\operatorname{Tsrhx}_{m, c, u}=\left(1-E s_{m, c, u}\right) T r p_{c}+E s_{m, c, u} T e q s_{c, u}
$$

Tsrhx is the temperature that would be delivered to the supply register with the current mode's flow rate but with zero capacity such that $d t e_{c}=0$. The duct system is then acting as a heat exchanger (thus the ' hx ') between the connected conditioned and unconditioned zones.

The term $s_{c, u}$, similar to $T e q r_{c, u}$ of Section 1.10.3, is an equivalent environmental temperature defined by

$$
\operatorname{Teqs}_{c, u}=\left(\text { Fsda }_{c, u} \cdot \text { Tair }_{u}+F s d r_{c, u} \cdot \text { Tmrt }_{u}\right)
$$

where

$$
\begin{aligned}
& F s d a_{c, u}=\frac{U s c_{c, u}}{U s c_{c, u}+U s r_{c, u}} \\
& F s d r_{c, u}=\frac{U s r_{c, u}}{U s c_{c, u}+U s r_{c, u}} \\
& U s r_{c, u}=\text { conductance from supply duct air to Tmrt. } \\
& U s c_{c, u}=\text { conductance from supply duct air to Tair. } \\
& U s t o t_{c, u}=U s c_{c, u}+U s r_{c, u}
\end{aligned}
$$

The supply duct flow rate is:

$$
M c p s_{m, c, u}=M c p_{m, c} \cdot F m s_{c, u}
$$

### 1.10.7 Heating/Cooling Delivered and Supply Duct Conductive Heat Loss

Given $T s r_{c, u}$ from above, the heat delivered to the conditioned zones by way of the supply ducts located in one or more of the unconditioned zones is given by summing the sensible heat delivered via each unconditioned zones:

$$
\mathrm{Q} \text { delivered from ducts }=\sum_{u} M c p s r_{m, c, u} \cdot\left(T s r_{c, u}-\text { Temp }_{c}\right)
$$

Equation 210
where $M c p s r_{m, c, u}$, the flow out the supply register after the supply leakage is removed, is given by:

$$
M c p s r_{m, c, u}=\left(1-L s_{c, u}\right) \cdot M c p s_{m, c, u}
$$

The heat delivered to the conditioned zones by way of ducts in the conditioned zone, which are assumed to have no losses or unbalanced leakage, is given by:

Q delivered directly to conditioned zone $=\mathrm{Fmsc}_{c} \cdot$ Cap $_{c}$
Equation 211
Adding the Q's of Equation 210 and Equation 211 gives the net heating (+), or cooling $(-)$, delivered by the system c as:

$$
\text { Qdel }_{c}=\text { Fmsc }_{c} \cdot \text { Cap }_{c}+\sum_{\text {over } u}\left[\text { Mcpsr }_{m, c, u} \cdot\left(\text { Tsrc }_{c, u}-\text { Temp }_{c}\right)\right]
$$

Substituting the expression for $\mathrm{Tsr}_{c, u}$ from Equation 209 into this, $Q \operatorname{del}_{c}$ can be put in the form:

$$
\text { Qdel }_{c}=\text { Qdel }_{c}+\text { Qdel2 }_{c},
$$

where $Q d e l 1_{c}$ is the part of Qdel that is independent of air handler capacity. That is, it is the Q delivered if dte is zero, and is the heat exchanged between the unconditioned and conditioned zones via the duct system:

$$
\text { Qdel } 1_{c}=\sum_{\text {all u }}\left[\operatorname{Mcpsr}_{m, c, u}\left(T \operatorname{srh} x_{m, c, u}-\operatorname{Temp}_{c}\right)\right]
$$

Qdel2 ${ }_{c}$ is the part of $Q d e l$ that is linearly dependent on the air handler capacity:

$$
\text { Qdel2 }_{c}=F m s c_{c} \cdot \operatorname{Cap}_{c}+\sum_{\text {over u }}\left[M c p s r_{m, c, u} \cdot\left(1-E s_{m, c, u}\right) \cdot d t e_{c}\right]
$$

The rate of supply duct conduction losses this time step is given by:

$$
q l s_{c, u}=M c p s_{m, c, u} \cdot\left(T s p_{c}-T s r_{c, u}\right)
$$

### 1.10.8 Duct System Performance when the Load is Less than the Heat Delivered at Full Capacity

If $Q l d_{c}$ is smaller than the capacity $Q d e l_{c}$, then the system runs only part of the time step. In this case the run time fraction is:

$$
\text { Frun }_{c}=\frac{Q l d_{c}}{Q d e l_{c}}
$$

The capacity required to meet the load is Qneedc:

$$
\text { Qneed }_{c}=\text { Frun }_{c} \cdot \text { Cap }_{c}
$$

The duct conductive and leakage losses are also reduced by the same Frun ${ }_{c}$ fraction.

### 1.10.9 Duct System Performance when the Load is Greater than the Heat Delivered at Full Capacity

In principle this won't occur because the conditioned zone load is limited to the system capacity when it is calculated by the conditioned zone thermostat logic However, the capacity thus calculated is based on the duct efficiency [defined as $\eta=Q l o a d / Q n e e d]$ determined for the unconditioned zone during the last time-step, and as a result the load might exceed the capacity determined by the duct model efficiency this time-step.
That is, when the conditioned zone energy balance is performed, and for example heating is called for, then the output capacity of the heating system needs to be known, which requires knowing the duct system efficiency. But the duct efficiency is only known after the attic simulation is run.

To avoid iteration between the conditioned zone and attic zone modules, the most recent duct efficiency is used to determine the capacity in the conditioned zones thermostat calculations. When the attic simulation is next performed, if the conditioned zone was last running at capacity, and if the efficiency now calculated turns out to be higher than was assumed by the thermostat calculations, then the load will have exceeded the limiting capacity by a small amount depending on the assumed vs. actual efficiency. In cases like this, to avoid iteration, the limiting capacity is allowed to exceed the actual limit by a small amount, so that the correct air handler input energy demand is determined for the conditioned zone load allowed.

In this case, the system is set to run for the full sub-hour time step and the air handler meets the load by increasing its capacity with the following procedure. This procedure, a carryover from the 2008 Residential Building Standards ACM procedures, wherein no capacity limits were imposed on the air handler systems, is as follows.

From the Qdel1 and Qdel2 equations it can be seen that the capacity needed in this case is:

$$
\text { Qneed }_{c}=\frac{\text { Qld }_{c}-\text { Qdel1 }_{c}}{\text { del2 }_{c}} \text { Cap }_{c}
$$

Thus, the temperature rise through the air handler needs to be:

$$
\text { dte }_{c}=\frac{\text { Qneed }_{c}}{M c p_{m, c}}
$$

The supply plenum temperature becomes:

$$
\operatorname{Tsp}_{c}=\operatorname{Tr} p_{c}+d t e_{c}
$$

The supply register temperatures is determined reusing Equation 208:

$$
T s r_{c, u}=\text { Teqs }_{u}+\left(1-E s_{m, c, u}\right) \cdot\left(T s p_{c}-\text { Teqs }_{c, u}\right)
$$

The supply duct losses now become:

$$
q l s_{c, u}=M c p s_{m, c, u} \cdot\left(T s p_{c}-T s r_{c, u}\right)
$$

The Qneed ${ }_{c}$ 's from each of the time steps during the hour are summed over the hour and reported in the output as Qneed $_{c}$. The supply and return duct conduction loss terms $q l s_{c, u}$ and $q l r_{c, u}$ are used in the energy balance of the unconditioned zone each time step.

### 1.11Variable Insulation Conductivity

The following correlation is used. It is based on the correlation used in EnergyGauge USA (Parker, et al, 1999) which is based on Wilkes (1981) data:

$$
k=(k n) \cdot\left(1+0.00418\left(T_{\text {insul }}-70\right)\right) ; \text { temperatures in }{ }^{\circ} F .
$$

where,

```
\(k=\) insulation conductivity (Btu/hr-ft-R) at the average insulation temperature,
    \(T_{\text {insul }}(\mathrm{F})\).
\(k n=\) nominal insulation conductivity \((\mathrm{Btu} / \mathrm{hr}-\mathrm{ft}-\mathrm{R})\) for insulation at 70 F
```


### 1.12 Ceiling Bypass Model

A simple model was implemented to simulate ceiling bypass heat transfer, the heat that is transported from the conditioned zone to the attic via miscellaneous inter-wall cavities in the conditioned zone that may be partially open to the attic, as for example around a fireplace unit. Natural convection in the cavity when the conditioned zone is hotter than the attic is assumed to be the main mechanism for the bypass heat transfer. The conductance, when the conditioned zone air temperature Tair $_{c}>$ Tair $_{u}$, the attic air temperature:

$$
q b p=U\left(\text { Tair }_{c}-\text { Tair }_{u}\right)
$$

where, the conductance follows a simple power law dependence on the temperature difference:

$$
U=U_{b p}\left(T e m p_{1}-T a i r_{u}\right)^{n b p}
$$

$U b p$ is a coefficient depending on the cavity geometry. Although an exponent of $n b p$ on the order of $1 / 4$ can be assumed for laminar convection, there is no current empirical basis for determining the exponent. If the ACM rule of $U=0.02$ Aceil were implemented, then $n b p$ would be chosen as zero.

### 1.13 Zone Humidity Balance

### 1.13.1 Zone Humidity Balance

Given a zone with various flows, $m_{j}$, with humidities $w_{j}$, entering the zone, and with a scheduled source of water vapor, $m_{\text {sched }}$, a water mass balance on the zone gives:

$$
\frac{d M w}{d t}=\sum_{j} m_{j}\left(w_{j}-w\right)+m_{\text {sched }}
$$

Equation 212
which can be written as:

$$
M \frac{d w}{d t}=\sum_{j} m_{j}\left(w_{j}-w\right)+m_{\text {sched }}-w \frac{d M}{d t}
$$

Equation 213
where,
$M=$ mass of dry air in the zone; lbm of dry air.
$\frac{d w}{d t}=$ the rate of change of humidity ratio in zone.
$m_{j}=$ air flow rate from source $j$ into zone; Ibm-dry-air/unit-time. Source $j$ can be outdoors, a supply register, adjacent zone, etc.
$\mathrm{w}_{j}=$ humidity ratio of air coming from source $j_{;}$lbm $\mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air.
$w$ = humidity ratio of air in zone; lbm $\mathrm{H}_{2} \mathrm{O} / \mathrm{lbm}$ dry air.
$m_{\text {sched }}=$ scheduled rate of moisture addition to zone; lbm H $\mathrm{H}_{2} \mathrm{O} /$ unit time .
Using the air perfect gas equation the last term in Equation 213 can be written

$$
w \frac{d M}{d t}=-w \frac{M}{T} \frac{d T}{d t}
$$

so that Equation 213 becomes

$$
M \frac{d w}{d t}=\sum_{j} m_{j}\left(w_{j}-w\right)+m_{\text {sched }}+w \frac{M}{T} \frac{d T}{d t}
$$

Equation 214
where $T$ is the air temperature in absolute degrees.
This equation is solved using a forward difference rather than a backward or central difference since a forward difference uncouples the moisture balance equations of each of the zones. Integrating from time $t$ to time $t+\delta t$, where $\delta t$ is the time step, using a forward difference, gives:

$$
\begin{aligned}
w(t+\delta t)= & \left(m_{\text {sched }}(t)+\sum_{j} m_{j}(t) w_{j}(t)\right) \frac{\delta t}{M(t)} \\
& +w(t)\left(1-\frac{\delta t}{M(t)} \sum_{j} m_{j}(t)-\frac{T(t+\delta t)-T(t)}{T(t)[\operatorname{deg} R]}\right)
\end{aligned}
$$

Notice that all of the values on the right hand side of Equation 215 are determined at $t$ (the beginning of the integration period) except for the $T(t+\delta t)$ term which represents the zone air temperature at the end of the integration period. $T(t+\delta t)$ is known from the zone energy sensible energy balance at time $t$ (see Section 1.3). The term $\frac{T(t+\delta t)-T(t)}{T(t)[\operatorname{deg} R]}$ is assumed to be negligible and not included in the CSE code.

### 1.13.2 Stability of Solution

The time series solution of Equation 215 will become unstable unless the second term is positive. That is, stability requires

$$
\left(\frac{\delta t}{M(t)} \sum_{j} m_{j}(t)+\frac{T(t+\delta t)-T(t)}{T(t)[\operatorname{deg} R]}\right)<1
$$

Equation 216
Solving for $\delta t$, stability requires

$$
\delta t<\frac{M(t)}{\sum_{j} m_{j}(t)}\left(1-\frac{T(t+\delta t)-T(t)}{T(t)[\operatorname{deg} R]}\right)
$$

Equation 217
Since the zone air changes per unit time is $A C=\frac{\sum_{j} m_{j}(t)}{M(t)}$ then the stability requirement can be written in terms of air changes as:

$$
A C<\frac{1}{\delta t}\left(1-\frac{T(t+\delta t)-T(t)}{T(t)[\operatorname{deg} R]}\right)
$$

Equation 218
If the solution is unstable at the given $\delta t$, the zone air mass $M(t)$ can be temporarily boosted up such that:

$$
M(t)>\frac{\delta t \sum_{j} m_{j}(t)}{\left(1-\frac{T(t+\delta t)-T(t)}{T(t)[\operatorname{deg} R]}\right)}
$$

This will lead to a higher latent capacity for the zone air, introducing some error in the zone humidity prediction. This will also lead to a zone latent heat imbalance unless this artificial increase in zone air is accounted for.

### 1.13.3 Hygric Inertia of Zone

The absorption/desorption of moisture in the zone is accounted for using the hygric inertial model of Vereecken et al. whereby a multiplier $X$ is added to the $M(t)$ term of Equation 10 and Equation 11. An appropriate value of $X$ can be measured for the
complete zone and all of its furnishings by using the protocol given by [Vereecken E , Roels S, Janssen H, 2011. In situ determination of the moisture buffer potential of room enclosures, Journal of Building Physics, 34(3): 223-246.]

### 1.14 Zone Comfort Algorithm

CSE includes an implementation of the ISO 7730 comfort model. The model is documented in ASHRAE Standard 55-2010 (ASHRAE 2010) among other places. The model calculates Predicted Mean Vote (PMV) and Predicted Percent Dissatisfied (PPD) for each zone at each time step. These statistics are averaged over days, months, and the full year.
The inputs to the ISO 7730 model are:

- Air dry-bulb temperature
- Air humidity ratio
- Mean radiant temperature
- Air velocity
- Occupant metabolic rate
- Occupant clothing level

Zone conditions calculated by CSE are used for the first three of these inputs. The remaining inputs are set by user input. They can be varied during the simulation using the CSE expression capability.

### 1.15HVAC Equipment Models

Air conditioning systems shall be sized, installed, tested and modeled according to the provisions of this section.

### 1.15.1 Compression Air-Conditioner Mode/

The Compliance Software calculates the hourly cooling electricity consumption in kWh using Equation 219. In this equation, the energy for the air handler fan and the electric compressor or parasitic power for the outdoor unit of a gas absorption air conditioner are combined. The Compliance Software calculates the hourly cooling gas consumption in therms using Equation 219.

$$
\mathrm{AC}_{\mathrm{kWh}}=\frac{\mathrm{Fan}_{\mathrm{Wh}}+\mathrm{Comp}_{\mathrm{Wh}}}{1,000}
$$

Where:
$\mathrm{AC}_{\mathrm{kWh}}=$ Air conditioner kWh of electricity consumption for a particular hour of the simulation. This value is calculated for each hour, combined with the TDV multipliers, and summed for the year.

Fan $_{w h} \quad=$ Indoor fan electrical energy for a particular hour of the simulation, Wh.
Compwh = Electrical energy for all components except the indoor fan for a particular hour of the simulation, Wh. This value includes consumption for the compressor plus outdoor condenser fan and is calculated using Equation 221.

CSE calculates the energy for electrically driven cooling using the algorithms described in this section.

Primary model parameters. The following values characterize the AC unit and are constant for a given unit:

Cap95 = AHRI rated total cooling capacity at $95^{\circ} \mathrm{F}$, Btuh
CFM $_{\text {per ton }}=$ Air flow rate per ton of cooling capacity, cfm/ton.

$$
=\frac{\text { Operating air flow rate, cfm }}{\text { Cap } 95-12000}
$$

EFan $\quad=$ Fan operating electrical power, W/cfm. Default $=0.365$.
SEER = AHRI rated Seasonal Energy Efficiency Ratio, Btuh/W. EER shall be used in lieu of the SEER for equipment not required to be tested for a SEER rating.

EER $\quad=$ AHRI rated energy efficiency ratio at $95^{\circ} \mathrm{F}$, Btuh/W. If EER is not available, it is derived from SEER as follows:

SEER >=16 EER $=13$
SEER $>=13$ and $<16 \quad$ EER $=11.3+0.57 \times($ SEER -13$)$
SEER $<13 E E R=10+0.84 \times($ SEER -11.5$)$
$F_{\text {chg }} \quad=$ Refrigerant charge factor, default $=0.9$. For systems with a verified charge indicator light (Reference Residential Appendix RA3.4) or verified refrigerant charge (Reference Residential Appendix RA3), the factor shall be 0.96 .
$F_{\text {size }} \quad=$ Compressor sizing factor, default $=0.95$. For systems sized according to the Maximum Cooling Capacity for compliance software Credit (see Section <TODO>), the factor shall be 1.0.

Derived model parameters. The following values are used in the formulas below and depend only on model parameters.

Tons $\quad=$ Nominal cooling capacity defined as Cap95 / 12000
QFan $_{\text {rat }}=$ Assumed fan heat included at AHRI test conditions, Btuh

| Fan motor type | QFan $_{\text {rat }}$ |
| :---: | :---: |
| PSC | $500 \times$ Cap95 / 12000 |
| BPM | $283 \times$ Cap95 / 12000 |

Source: NORESCO for California Energy Commission
$\mathrm{QFan}_{\mathrm{op}}=$ Fan heat assumed during operation (i.e., during simulation), Btuh

$$
\text { QFan }_{o p}=\frac{\text { CFM }_{\text {perton }} \times \text { Cap } 95 \times \text { EFan } \times 3.413}{12000}
$$

Equation 220
Model inputs. The following values vary at each time step in the simulation and are used in the formulas below to determine AC unit performance under for that time step.
$\mathrm{DB}_{\mathrm{t}} \quad=$ Dry bulb temperature of air at the condensing unit, ${ }^{\circ} \mathrm{F}$ (typically outdoor air temperature).
$\mathrm{WB}_{\text {ec }} \quad=$ Coil entering air wet bulb temperature, ${ }^{\circ} \mathrm{F}$ (return air temperature adjusted for blow-through fan heat if any)
$\mathrm{DB}_{\mathrm{ec}} \quad=$ Coil entering air dry bulb temperature, ${ }^{\circ} \mathrm{F}$ (return air temperature adjusted for blow through fan heat if any)

Qneed = Cooling system sensible cooling output, Btuh. Qneed is calculated across the unit and thus includes both the building load and distribution losses.

Compressor energy for a particular time step of the simulation shall be calculated using Equation 221.


Equation 221
Where:
Fan $_{\text {wh }} \quad=$ Fan power for this time step, Wh.
$\mathrm{CE}_{\mathrm{t}} \quad=$ Sensible energy efficiency at current conditions, Btuh/W. This is calculated using Equation 222 below.

$$
C E_{t}=E E R_{t} \times S H R
$$

Where:
EER $\mathrm{t}_{\mathrm{t}}$ = Energy efficiency ratio at current conditions, Btuh/W. EER $\mathrm{R}_{\mathrm{t}}$ is calculated using Equation 226 below.

SHR = Sensible Heat Ratio (sensible capacity / total capacity), derived as follows:
SHR $=\operatorname{minimum}\left(1, \quad A_{S H R} \times \mathrm{DB}_{\text {ec }}+\right.$ $\mathrm{B}_{\mathrm{SHR}} \times \mathrm{WB}_{\mathrm{ec}}+$ $\mathrm{C}_{\text {SHR }} \times \mathrm{DB}_{\mathrm{t}}+$ $\mathrm{D}_{\text {SHR }} \times \mathrm{CFM}_{\text {per ton }}+$ $\mathrm{Eshr} \times \mathrm{DB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+$ $\mathrm{F}_{\text {SHR }} \times \mathrm{DB}_{\text {ec }} \times \mathrm{CFM}_{\text {per ton }}+$ $\mathrm{G}_{\mathrm{SHR}} \times \mathrm{WB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+$ $\mathrm{H}_{\text {SHR }} \times \mathrm{WB}_{\text {ec }} \times \mathrm{CFM}_{\text {per ton }}+$ $\mathrm{I}_{\text {SHR }} \times \mathrm{DB}_{\mathrm{t}} \times \mathrm{CFM}_{\text {per ton }}+$ $\mathrm{J}_{\mathrm{SHR}} \times \mathrm{WB}_{\mathrm{ec}}{ }^{2}+$ $\mathrm{K}_{\text {SHR }} / \mathrm{CFM}_{\text {per ton }}+$ LSHR)
SHR coefficients:

|  |  |
| :--- | :--- |
| $A_{\text {SHR }}$ | 0.0242020 |
| $\mathrm{~B}_{\text {SHR }}$ | -0.0592153 |
| $\mathrm{C}_{\text {SHR }}$ | 0.0012651 |
| $\mathrm{D}_{\text {SHR }}$ | 0.0016375 |
| $\mathrm{E}_{\text {SHR }}$ | 0 |
| $\mathrm{~F}_{\text {SHR }}$ | 0 |
| $\mathrm{G}_{\text {SHR }}$ | 0 |
| $\mathrm{H}_{\text {SHR }}$ | -0.0000165 |
| $\mathrm{I}_{\text {SHR }}$ | 0 |
| $\mathrm{~J}_{\text {SHR }}$ | 0.0002021 |
| $\mathrm{~K}_{\text {SHR }}$ | 0 |
| $\mathrm{~L}_{\text {SHR }}$ | 1.5085285 |

CAP ${ }_{\text {nf }}=$ Total cooling capacity across coil (that is, without fan heat) at current conditions, Btuh

$$
\text { CAP }_{n f}=\left(\text { Cap } 95+\text { QFan }_{\text {rat }}\right) \times F_{\text {chg }} \times F_{\text {size }} \times F_{\text {cond }}^{\text {cap }} \text { }
$$

Equation 223
$\mathrm{F}_{\text {cond_cap }}=\mathrm{A}_{\text {CAP }} \times \mathrm{DB}_{\text {ec }}+$
$\mathrm{B}_{\text {cap }} \times \mathrm{WB}_{\text {ec }}+$
$\mathrm{C}_{\mathrm{CAP}} \times \mathrm{DB}_{\mathrm{t}}+$

$$
\begin{aligned}
& \mathrm{D}_{\mathrm{CAP}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{E}_{\mathrm{CAP}} \times \mathrm{DB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{F}_{\mathrm{CAP}} \times \mathrm{DB}_{\mathrm{ec}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{G}_{\mathrm{CAP}} \times \mathrm{WB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{H}_{\mathrm{CAP}} \times \mathrm{WB}_{\mathrm{ec}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{I}_{\mathrm{CAP}} \times \mathrm{DB}_{\mathrm{t}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{J}_{\mathrm{CAP}} \times \mathrm{WB}_{\mathrm{ec}}{ }^{2}+ \\
& \mathrm{K}_{\mathrm{CAP}} / \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{L}_{\mathrm{CAP}}
\end{aligned}
$$

Coefficients as follows:

| SHR <br> Condition | SHR < 1 | SHR $=1$ |
| :---: | :---: | :---: |
| Acap | 0 | 0.009483100 |
| $\mathrm{B}_{\text {cap }}$ | 0.009645900 | 0 |
| Ccap | 0.002536900 | -0.000600600 |
| DCAP | 0.000171500 | -0.000148900 |
| ECAP | 0 | -0.000032600 |
| Fcap | 0 | 0.000011900 |
| Gcap | -0.000095900 | 0 |
| $\mathrm{H}_{\text {cap }}$ | 0.000008180 | 0 |
| ICAP | -0.000007550 | $-0.000005050$ |
| JCAP | 0.000105700 | 0 |
| K CAP | $53.542300000^{-}$ | $52.561740000^{-}$ |
| Lcap | 0.381567150 | 0.430751600 |

Source: NORESCO for California Energy Commission
$\mathrm{CAP}_{\text {sen }}=$ Sensible capacity including fan heat, Btuh

$$
C A P_{\text {sen }}=C A P_{n f} \times S H R-Q F a n_{o p}
$$

Equation 224
CAPlat = Latent capacity, Btuh

$$
C A P_{\text {lat }}=C A P_{n f}-C A P_{\text {sen }}
$$

Note: The air leaving the AC unit is limited to $95 \%$ relative humidity. If that limit is invoked, CAP lat is reduced and CAP ${ }_{\text {sen }}$ is increase.

EER ${ }_{t}$ is calculated as follows:
When
$\mathrm{DB}_{\mathrm{t}} £ 82^{\circ} \mathrm{F} \quad \mathrm{EER}_{\mathrm{t}}=$ SEER $_{\text {nf }}$
$82<\mathrm{DB}_{\mathrm{t}}<95^{\circ} \mathrm{F}$ EER $_{\mathrm{t}}=$ SEER $_{\mathrm{nf}}+\left(\left(\mathrm{DB}_{\mathrm{t}}-82\right) *\left(\right.\right.$ EER $_{\mathrm{nf}}-$ SEER $\left.\left._{\mathrm{nf}}\right) / 13\right)$
$D B_{t}{ }^{3} 95{ }^{\circ} \mathrm{F} \quad E E R_{t}=E E R_{n f}$
Equation 226
Where:
SEER $_{n f} \quad=$ Seasonal energy efficiency ratio at current conditions without distribution fan consumption ("nf" = no fans). This is calculated using Equation 227.
$E E_{\text {nf }} \quad=$ Energy efficiency ratio at current conditions without distribution fan consumption ("nf" = no fans) and adjusted for refrigerant charge and airflow. This is calculated using Equation 228.

$$
S E E R_{n f}=\frac{F_{\text {chg }} \times F_{\text {size }} \times F_{\text {cond }}^{\text {SEER }}}{} \times\left(1.09 \times{\text { Cap } \left.95+Q F a n_{r a t}\right)}_{1.09 \times \text { Cap } 95 / \text { SEER }-Q F a n_{r a t} / 3.413}\right.
$$

Equation 227

$$
\begin{aligned}
& \mathrm{F}_{\text {cond_SEER }}=\mathrm{F}_{\text {cond_cap }} / \quad\left(\mathrm{A}_{\text {seer }} \times \mathrm{DB}_{\text {ec }}+\right. \\
& \mathrm{B}_{\text {SEER }} \times \mathrm{WB}_{\text {ec }}+ \\
& \mathrm{C}_{\text {SEER }} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{D}_{\text {SEER }} \times \text { CFM }_{\text {per ton }}+ \\
& \mathrm{E}_{\text {SEER }} \times \mathrm{DB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{F}_{\text {SEER }} \times \mathrm{DB}_{\text {ec }} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{GsEER} \times \mathrm{WB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{H}_{\text {SEER }} \times \mathrm{WB}_{\text {ec }} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{I}_{\text {SEER }} \times \mathrm{DB}_{\mathrm{t}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{J}_{\text {SEER }} \times \mathrm{WB}_{\mathrm{ec}}{ }^{2}+ \\
& K_{\text {seer }} / \text { CFM }_{\text {per ton }}+ \\
& \text { Lseer) }
\end{aligned}
$$

Coefficients as follows:

| SHR <br> Condition | SHR < 1 | SHR $=1$ |
| :---: | :---: | :---: |
| $\mathrm{A}_{\text {SEER }}$ | 0 | 0.0046103 |
| $\mathrm{B}_{\text {SEER }}$ | -0.0202256 | 0 |
| Cseer | 0.0236703 | 0.0125598 |
| DSEER | -0.0006638 | -0.000512 |
| EsEER | 0 | -0.0000357 |
| $\mathrm{F}_{\text {SEER }}$ | 0 | 0.0000105 |
| $\mathrm{G}_{\text {Seer }}$ | -0.0001841 | 0 |
| $H_{\text {SEER }}$ | 0.0000214 | 0 |
| IsEER | -0.00000812 | 0 |
| $J_{\text {SEER }}$ | 0.0002971 | 0 |
| $\mathrm{K}_{\text {SEER }}$ | -27.95672 | 0 |
| Lseer | 0.209951063 | -0.316172311 |

Source: NORESCO for California Energy Commission

$$
\left.E E R_{n f}=\frac{\operatorname{Cap}_{n f}}{F_{\text {cond }_{E E R} \times\left(\text { Cap9 }^{25 / E E R-Q F a n ~}\right.}^{\text {rat }}} 13.413\right)
$$

Where:

$$
\begin{array}{ll}
\mathrm{F}_{\text {cond_EER }}=\quad\left(\mathrm{A}_{\text {EER }} \times \mathrm{DB}_{\mathrm{ec}}+\right. \\
& \mathrm{B}_{\text {EER }} \times \mathrm{WB}_{\mathrm{ec}}+ \\
& \mathrm{C}_{\text {EER }} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{D}_{\text {EER }} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{E}_{\text {EER }} \times \mathrm{DB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{F}_{\text {EER }} \times \mathrm{DB}_{\mathrm{ec}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{G}_{\text {EER }} \times \mathrm{WB}_{\mathrm{ec}} \times \mathrm{DB}_{\mathrm{t}}+ \\
& \mathrm{H}_{\text {EER }} \times \mathrm{WB}_{\text {ec }} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{I}_{\text {EER }} \times \mathrm{DB}_{\mathrm{t}} \times \mathrm{CFM}_{\text {per ton }}+ \\
& \mathrm{J}_{\text {EER }} \times \mathrm{WB}_{\text {ec }}{ }^{2}+ \\
& \mathrm{K}_{\text {EER }} / \mathrm{CFM}_{\text {per ton }}+
\end{array}
$$

Leer)
Coefficients as follows:

| SHR <br> Condition | SHR < 1 | SHR = 1 |
| :---: | :---: | :---: |
| Aeer | 0 | 0.004610300 |
| Beer | -0.020225600 | 0 |
| Ceer | 0.023670300 | 0.012559800 |
| DEER | -0.000663800 | -0.000512000 |
| Eeer | 0 | -0.000035700 |
| Feer | 0 | 0.000010500 |
| GeER | -0.000184100 | 0 |
| $\mathrm{H}_{\text {EER }}$ | 0.000021400 | 0 |
| $\mathrm{I}_{\text {Eer }}$ | -0.000008120 | 0 |
| J EER | 0.000297100 | 0 |
| Keer | $27.956720000^{-}$ | 0 |
| Leer | 0.015003100 | -0.475306500 |

Source: NORESCO for California Energy Commission

### 1.15.2 Air-Source Heat Pump Model (Heating mode)

The air source heat pump model is based on methods presented in AHRI Standard 210/240-2008.

Primary model parameters. The following values characterize the ASHP and are constant for a given unit:

Cap47 $=$ Rated heating capacity at outdoor dry-bulb temperature $=47^{\circ} \mathrm{F}$
COP47 = Coefficient of performance at outdoor dry bulb $=47^{\circ} \mathrm{F}$ (if available, see below)

Cap35 = Heating capacity under frosting conditions at outdoor dry-bulb temperature $=35^{\circ} \mathrm{F}$ (if available, see below)

COP35 = Coefficient of performance at outdoor dry bulb $=35^{\circ} \mathrm{F}$ (if available, see below)

Cap17 $=$ Rated heating capacity at outdoor dry-bulb temperature $=17^{\circ} \mathrm{F}$

COP17 = Coefficient of performance at outdoor dry bulb $=17^{\circ} \mathrm{F}$ (if available, see below)

HSPF = Rated Heating Seasonal Performance Factor, Btuh/Wh
COPbu $=$ COP of backup heating, default $=1$ (electric resistance heat)
Capbu = Available backup heating capacity, Btuh

## Derived model parameters.

Inp47 = Electrical input power at $47{ }^{\circ} \mathrm{F}=$ Cap47 / COP47, Btuh (not W)
Inp17 = Electrical input power at $17^{\circ} \mathrm{F}=$ Cap17 / COP17, Btuh (not W)

## Estimation of unavailable model parameters.

$$
\begin{aligned}
& \text { COP47 }=0.3038073 \times \mathrm{HSPF}-1.984475 \times \frac{\mathrm{Cap} 17}{\mathrm{Cap} 47}+2.360116 \\
& \text { COP17 }=0.2359355 \times \mathrm{HSPF}+1.205568 \times \frac{\mathrm{Cap} 17}{\mathrm{Cap} 47}-0.1660746
\end{aligned}
$$

$$
\begin{aligned}
& \operatorname{Cap} 35=0.9 \times[\operatorname{Cap} 17+0.6 \times(\operatorname{Cap} 47-\operatorname{Cap} 17)] \\
& \operatorname{Inp} 35=0.985 \times[\operatorname{Inp} 17+0.6 \times(\operatorname{Inp} 47-\operatorname{Inp} 17)] \\
& \operatorname{COP} 35=\frac{\operatorname{Cap} 35}{\operatorname{Inp} 35}
\end{aligned}
$$

## Simulation

Full-load capacity and input power of the ASHP is determined each time step as a function of outdoor dry-bulb temperature T , as follows --

If ( $17^{\circ} \mathrm{F}<\mathrm{T}<45^{\circ} \mathrm{F}$ )

$$
\begin{aligned}
& \operatorname{Cap}(T)=\operatorname{Cap} 17+\frac{(\operatorname{Cap} 35-\operatorname{Cap} 17) \times(T-17)}{35-17} \\
& \operatorname{Inp}(T)=\operatorname{Inp} 17+\frac{(\operatorname{Inp} 35-\operatorname{Inp} 17) \times(T-17)}{35-17}
\end{aligned}
$$

Else

$$
\begin{aligned}
& \operatorname{Cap}(T)=\operatorname{Cap} 17+\frac{(\operatorname{Cap} 47-\operatorname{Cap} 17) \times(T-17)}{47-17} \\
& \operatorname{Inp}(T)=\operatorname{Inp} 17+\frac{(\operatorname{Inp} 47-\operatorname{Inp} 17) \times(T-17)}{47-17}
\end{aligned}
$$

## Resistance heat.

Load in excess of $\operatorname{Cap}(T)$ is met with backup heating at COPbu.

### 1.15.3 Equipment Sizing

CSE determines the capacity of HVAC equipment via an auto-sizing capability. Autosizing is conducted prior to the main annual simulation. It is done by using the hourly simulator for a set of design days and increasing capacity as needed to maintain thermostat set points. Each design day is repeated several times until the required capacity converges. The set of design days includes one cold day with no solar gain and several hot days at with clear-sky solar at different times of the year. This ensures that maximums of both heating and cooling loads will be found. Equipment characteristics other than capacity are specified on a per-unit basis (e.g. "cfm per ton"), so a full description of the system can be derived from the primary capacity.

The sizing procedure uses the equipment models in an inverse mode. For example, the sensible cooling load for a given set up conditions is back-converted to the required rated total capacity (Cap95) by using inverted forms of the model equations. The general simulation calculation sequence is used, but the logic of the HVAC models is altered during the autosizing phase.

Note that for air-source heat pumps, only the backup heating capacity is autosized. In addition, modeled duct sizes are not sized and must be specified.

The equipment sizes calculated by CSE are used for compliance analysis only and are not substitutes for load calculations used for selecting equipment or meeting other code requirements.

## 2 Compliance Manager

### 2.1 Overview

### 2.2 One-dimensional Roof Edge Heat Transfer Model

### 2.2.1 Construction Practice

This document describes the one-dimensional model used to represent the heat flow between the conditioned zone and the outdoors through the portion of the ceiling insulation, along the outside edge of the attic, through which the heat flows to the outdoors without passing through the attic air. This portion will be modeled in CSE as cathedral ceilings, and is referred herein as the roof edge. The rest of the heat flow path through the ceiling insulation will be modeled as part of the attic zone, and is not discussed here.

Two types of roof construction are considered, standard-heel and raised-heel trusses, shown in Figure 26 and Figure 27, with the geometries assumed to be representative of current practice. The roof trusses are assumed to be framed with $2 \times 4$ 's. Although the figures are for a roof with a $4-\mathrm{in}-12$ pitch, the 1-D model will handle any standard pitch. The distance between the wall plate and roof deck (shown, for example, as 12 inches in Figure 27) is also not restricted to the distances implied by Figure 26 and Figure 27.

The 1-D model is developed in order to simplify the heat transfer calculation for roof edges, while preserving the steady state and transient characteristics (layer mass) of the typical roof constructions addressed. The 1-D model produces the dimensions of the construction layers needed to represent the roof edges.

Figure 26: Standard-Heel Geometry

$L$ and $D$ determine where the top of the ceiling insulation meets the roof deck plane.

Figure 27: Raised-Heel Geometry


L and D determine where the top of the ceiling insulation meets the roof deck plane.

### 2.2.2 One-Dimensional Mode/

Using the parallel path method, the heat transfer is determined separately for the insulation and framing paths of the constructions.
First consider modeling the standard-heel truss of Figure 26.

### 2.2.2.1 Standard heel insulation path

For the path through the insulation, Figure 26 is approximated as the simpler 2-D configuration of Figure 28 and Figure 30, with the left vertical edge assumed to be adiabatic and of height $Y$. The right vertical edge is also assumed to be adiabatic. To partly compensate for not allowing heat flow out the left side tilted edge board in Figure 26 , the ceiling is assumed to extend to the outer edge of the vertical wall.

The width, $W_{A}$ of roof edge path A , is taken as the width of the vertical path of solid wood in the framing section view of Figure 30.

Figure 28: Standard-Heel Simplified Geometry for Insulation Path


Figure 28 is then reduced to the 1-D form shown in Figure 29, where the layer thicknesses are taken as the average height of the layer in Figure 28.

Figure 29: Standard-Heel 1-D Geometry for Insulation Path


The left hand portion represents the insulation path of the 1-D model of roof edge $A$. The right side represents the insulation path of the 1-D model of roof edge $B$.

The 1-D model just considers the ceiling insulation and framing. When implemented as part of a cathedral ceiling in CSE, a sheetrock layer would be added to the bottom of Figure 29 paths. Layers added to the tops of the layers in Figure 29 would be decking, asphalt shingles, and tile, for example.

### 2.2.2.2 Standard heel framing path

Similar to the insulation path, the framing heat transfer path starts with Figure 30, which is reduced to Figure 31 . The widths of $A$ and $B$ are the same as for the insulation
path figures. $H_{B 1}$, and $H_{B 3}$ are the vertical thickness of the $2 x 4^{\prime} s$ and $H_{B 2}$ is the average thickness of insulation.

Figure 30: Standard-Heel Simplified Geometry for Framing Path


Figure 31: Standard-Heel 1-D Geometry for Framing Path

2.2.2.3 Raised heel

The 1-D model for the raised-heel case of Figure 27 is different than for the standardheel case of Figure 26. The geometry is illustrated in Figure 32 and Figure 33 for a ceiling insulation of R38.

As the edge height, $Y$, in Figure 30 is increased, the deck and ceiling $2 \times 4$ 's separate vertically near the roof edge, and a vertical $2 \times 4$ is assumed to fill the gap. That is, as $Y$ is increased, the standard truss geometry of Figure 28 and Figure 30 morphs into the raised truss geometry of Figure 32 and Figure 33. The 1-d roof edge algorithm below, gives the layering outputs for both the standard-heel truss and the raised-heel truss and everything in between.
2.2.2.4 Raised heel insulation path

Figure 32: Raised-Heel 1-D Insulation Path Geometry

2.2.2.5 Raised heel frame path

Figure 33: Raised-Heel 1-D Framing Path Geometry

$H_{B 1}$, and $\mathrm{H}_{\mathrm{B} 3}$ are the vertical thickness of the $2 \times 4{ }^{\prime} \mathrm{s}$ and $\mathrm{H}_{\mathrm{B} 2}$ is the average thickness of insulation.

### 2.2.2.6 Roof edge algorithm

The following algorithm, written in succinct pseudo-code form, determines the layer widths and thicknesses for roof edge paths $A$ and $B$. [This algorithm is implemented in RoodEdgeAlgorithm-1.xlsx].

All dimensions are assumed to be feet.

## Input:

$Y=$ edge height, ft.
$w=$ framing width, ft . [nominally (3.5/12)-ft]
Rtot $=$ total R value of ceiling insulation, $\mathrm{hr}-\mathrm{ft}{ }^{2}-\mathrm{F} / \mathrm{Btu}$.
$R_{\text {perin }}=\mathrm{R}$ value of the ceiling insulation for one inch thickness, $\mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}$.
$P=$ pitch $=$ rise $/ 12$

## Calculation of $W_{A}, W_{B}, H_{A}, H_{B}, H_{B 1}, H_{B 2}$, and $H_{B 3}$ :

$D=\frac{\text { Rtot }}{12 * R_{\text {perin }}} \quad$ insulation depth, ft
$Y_{I}=w\left(1+\sqrt{1+P^{2}}\right) \quad$ vertical thickness at position $x=X_{I}$.
$X_{I}=\frac{Y_{I}-Y}{P}$ horiz distance from left edge to intersection of deck and ceiling $2 \times 4$ 's (see * in Figure 30). In Figure 33, the corresponding point would be outside the roof section, and near the left edge of the page, and as opposed to the * point, this $X_{I}$ will is a negative number since it is to left of origin at outside surface of top wall plate.

IF $X_{I} \geq w$ :

$$
\begin{gathered}
Y_{M}=Y_{I} \quad Y_{M} \text { is the height of roof section at the vertical line between } \mathrm{A} \text { and } \\
\mathrm{B} .
\end{gathered}
$$

ELSE $X_{I}<w$ :

$$
Y_{M}=\left(w-X_{I}\right) P+Y_{I}
$$

END IF
IF $D \geq Y_{M}$ :

$$
\begin{aligned}
& W_{B}=\frac{D-Y_{M}}{P} \\
& H_{A}=\frac{Y+Y_{M}}{2} \\
& H_{B}=\frac{Y_{M}+D}{2} \\
& H_{B 1}=w \sqrt{1+P^{2}} \\
& H_{B 2}=H_{B}-Y_{I} \\
& H_{B 3}=w \\
& \text { IF } X_{I} \geq w:
\end{aligned}
$$

$$
W_{A}=X_{I}
$$

ELSE $X_{I}<w$ :

$$
W_{A}=w
$$

END IF
ELSE IF $D<Y_{M}$ AND $D>Y$ :

$$
\begin{aligned}
& W_{A}=\frac{D-Y}{P} \\
& H_{A}=\frac{D+Y}{2} \\
& W_{B}=H_{B}=H_{B 1}=H_{B 2}=H_{B 3}=0
\end{aligned}
$$

ELSE ( $\mathrm{D} \leq \mathrm{Y}$ )

$$
\begin{aligned}
& W_{A}=H_{A}=0 \\
& W_{B}=H_{B}=H_{B 1}=H_{B 2}=H_{B 3}=0
\end{aligned}
$$

END IF
PRINT OUTPUT: $W_{A}, W_{B}, H_{A}, H_{B}, H_{B 1}, H_{B 2}, H_{B 3}$

### 2.2.3 Roof Edge Model Validation

The roof edge heat transfer is basically a 3-D problem. The 1-D model makes a number of simplifications. For example, the parallel insulation and framing path assumption ignores lateral heat transfer between the insulation and framing path, and leads to an underestimation of the overall heat transfer. The assumption of an adiabatic right hand border, where in reality the heat flow lies are not quite vertical, also underestimates the heat transfer through the cathedral ceilings with an accompanying overestimation of the remaining heat transfer through the attic portion of the ceiling insulation. The assumption of the layer thicknesses taken as the average layer thickness ignores 2-D effects. The complicated 2-D heat transfer at the junction of the vertical wall and roof is simplified by assuming the left border is adiabatic, and the ceiling continuation to the outside of the wall. Corner effects for the roof edge, where vertical walls meet at right angles, results in a 3-D heat flow situation that can only be estimated.

Because of these complexities, it is difficult to assess the accuracy of the 1-D model.
However, in order to obtain some perspective on the accuracy of the 1-D model, the heat transfer was calculated for two cases, of different insulation depths, using both the 1-D roof edge algorithm, and a 2-D (using FEHT finite-element program) solution with the roof edge 2-D geometry of Figure 26. The 2-D model still requires many of the assumptions made in the 1-D model, including the parallel path assumption.
2.2.3.1 1-D model, Rtot $=30, Y=4$-inches

Using the 1-D Roof Edge Algorithm, the heat transfer rates through roof edges A and B was calculated for the following inputs.

## Input to Roof Edge Algorithm

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{tot}}=30 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu} . \\
& \mathrm{P}=\frac{4}{12} \\
& Y=0.3333 \mathrm{ft} \\
& w=0.2917 \mathrm{ft} \\
& R_{\text {perin }}=2.6 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu} .
\end{aligned}
$$

## Output of Roof Edge Algorithm

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{A}}=0.797 \mathrm{ft} \\
& \mathrm{~W}_{\mathrm{B}}=1.087 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{A}}=0.466 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{B}}=0.780 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{B} 1}=0.292 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{B} 2}=0.182 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{B} 3}=0.307 \mathrm{ft}
\end{aligned}
$$

## Insulation Path Results

The insulation conductivity is $k=\frac{1}{12 R_{\text {perin }}}=0.03205 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}-\mathrm{F}$.
The thermal resistance of A and B are:

$$
\begin{aligned}
& R_{A}=\frac{H_{A}}{k_{\text {insul }}}=14.546 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu} \\
& R_{B}=\frac{H_{B}}{k_{\text {insul }}}=24.346 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}
\end{aligned}
$$

If no additional layers are added (sheetrock, etc.), and the top and bottom surface temperature difference is 100 F , the heat transfer rate in this case, per foot of roof perpendicular to the section, becomes:

$$
\begin{aligned}
& Q_{A}=\frac{W_{A} * \Delta T}{R_{A}}=\frac{0.797 * 100}{14.546}=5.48 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft} \\
& Q_{B}=\frac{W_{B} * \Delta T}{R_{B}}=\frac{1.087 * 100}{24.346}=4.465 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
\end{aligned}
$$

The total is:

$$
Q_{\text {insulpath }}=Q_{A}+Q_{B}=9.944 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
$$

## Framing Path Results

Assume wood framing conductivity $\mathrm{k}=0.084 \mathrm{hr}-\mathrm{ft}-\mathrm{F} / \mathrm{Btu}$.
The thermal resistance of $A$, per foot of roof edge perpendicular to the section:

$$
R_{A}=\frac{H_{A}}{k_{\text {wood }}}=5.55 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}
$$

The thermal resistance of path B ; sum of layer resistances:

$$
R_{B}=\frac{H_{B 1}}{k_{\text {wood }}}+\frac{H_{B 2}}{k_{\text {insul }}}+\frac{H_{B 3}}{k_{\text {wood }}}=3.66+5.654+3.473=12.786 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}
$$

The sum of the heat transfers in this case is (from CathedralWorksheet.xlsx).

$$
\begin{aligned}
& Q_{A}=\frac{W_{A} * \Delta T}{R_{A}}=\frac{0.797 * 100}{5.550}=14.36 \\
& Q_{B}=\frac{W_{B} * \Delta T}{R_{B}}=\frac{1.087 * 100}{12.786}=8.50 \\
& Q_{\text {framingpath }}=Q_{A}+Q_{B}=22.86 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
\end{aligned}
$$

2.2.3.2 1-D model, Rtot $=60, Y=4$-inches

## Input to Roof Edge Algorithm

Rtot $=60$; other inputs the same as in Rtot $=30$ case above.

## Output of Roof Edge Algorithm

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{A}}=0.797 \mathrm{ft} \\
& \mathrm{~W}_{\mathrm{B}}=3.972 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{A}}=0.466 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{B}}=1.261 \mathrm{ft} \\
& \mathrm{H}_{\mathrm{B} 1}=0.292 \mathrm{ft} \\
& H_{B 2}=0.662 \mathrm{ft} \\
& H_{B 3}=0.307 \mathrm{ft}
\end{aligned}
$$

## Insulation Path Results

Similar to the Rtot $=30$ case, the thermal resistance of $A$ and $B$ are:

$$
\begin{aligned}
& R_{A}=\frac{H_{A}}{k_{\text {insul }}}=14.546{\mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}}^{R_{B}=\frac{H_{B}}{k_{\text {insul }}}=39.346 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}}
\end{aligned}
$$

The heat transfer rates are:

$$
\begin{aligned}
& Q_{A}=\frac{W_{A} * \Delta T}{R_{A}}=\frac{0.797 * 100}{14.546}=5.48 \\
& Q_{B}=\frac{W_{B} * \Delta T}{R_{B}}=\frac{3.972 * 100}{39.346}=10.095 \\
& Q_{\text {framingpath }}=Q_{A}+Q_{B}=15.57 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
\end{aligned}
$$

## Framing path results

The thermal resistance of $A$, per foot of roof edge perpendicular to the section:

$$
R_{A}=\frac{H_{A}}{k_{\text {wood }}}=5.55 \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu}
$$

The thermal resistance of path $B$ is the sum of layer resistances. $k=0.084 \mathrm{hr}$-ft-F/Btu is assumed.

$$
\begin{array}{r}
R_{B}=\frac{H_{B 1}}{k_{\text {wood }}}+\frac{H_{B 2}}{k_{\text {insul }}}+\frac{H_{B 3}}{k_{\text {wood }}}=\mathrm{hr}-\mathrm{ft}^{2}-\mathrm{F} / \mathrm{Btu} \\
\quad=3.472+20.66+3.66=27.79
\end{array}
$$

The sum of the heat transfers in this case is:

$$
\begin{aligned}
& Q_{A}=\frac{W_{A} * \Delta T}{R_{A}}=\frac{0.797 * 100}{5.55}=14.36 \\
& Q_{B}=\frac{W_{B} * \Delta T}{R_{B}}=\frac{3.972 * 100}{27.79}=14.29 \\
& Q_{\text {framingpath }}=Q_{A}+Q_{B}=28.65 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
\end{aligned}
$$

### 2.2.3.3 2-D Model, Rtot $=30, Y=4$-inches

## Insulation Path

The Figure 26 case is modeled with the simplified geometry of Figure 34, shown for a ceiling insulation of R30. The top of the plate capping the vertical wall is assumed to be adiabatic. The tilted block assumed to be wood, exposed to ambient conditions on its outside sides. The outside of the wood and insulation assumed to be at a uniform 100F. The ceiling side of the insulation is set to at 0 F . The same material properties were used as in the 1-D model.

Figure 34: Standard Truss, Insulation Path, 2-Dimensional Heat Transfer Model Geometry


The resulting isotherms and heat transfer vectors are shown in Figure 35.
Figure 35: Standard-Heel, Insulation Path, 2-Dimensional Heat Transfer Isotherms and Heat Transfer Vectors


The overall heat transfer, per foot of perimeter, for this case was determined (RUN std30.FET) to be:

$$
\mathrm{Q}=9.67 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
$$

Equation 229

## Framing Path

The frame path was modeled similarly, with the Figure 36 graphic results.
Figure 36. Standard-Heel, Frame Path, 2-Dimensional Heat Transfer Isotherms and Heat Transfer Vectors


The overall heat transfer, per foot of perimeter, for this case was determined (RUN std30F.FET) to be:

$$
\mathrm{Q}=21.94 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
$$

Equation 230
2.2.3.4 2-D Model, Rtot $=60, Y=4$-inches

Similar to the R30 case above, Figure 37 and Figure 38 show the insulation and framing path 2-D results.

Figure 37: 2-D Results for Insulation Path of R-60 Standard-Heel


The overall heat transfer, per foot of perimeter, for this case was determined (RUN: std60.fet) to be:

$$
\mathrm{Q}=16.342 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}
$$

Equation 231
Figure 38: 2-D Results for Framing Path of R-60 Standard-Heel


The overall heat transfer, per foot of perimeter, for this case was determined (RUN: std60F.FET) to be:

$$
\mathrm{Q}=31.543 \mathrm{Btu} / \mathrm{hr} \text {-ft }
$$

### 2.2.3.5 Comparison of 1-D and 2-D results

The above 2-D results are compared with the simplified 1-D model in Table 8.

Table 8: Comparison of 1-D and 2-D Results

|  | R-30 Ceiling <br> Insulation <br> Path | R-30 Ceiling <br> Framing <br> Path | R-60 Ceiling <br> Insulation <br> Path | R-30 Ceiling <br> Framing Path |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1 D}$ | 9.94 | 22.86 | 15.57 | 28.65 |
| $Q_{2 D}$ | 9.67 | 21.94 | 16.34 | 31.543 |
| $Q_{1 D}$ is | $3 \%>Q_{2 d}$ | $4 \%>Q_{2 d}$ | $5 \%<Q_{2 d}$ | $9 \%<Q_{2 d}$ |

Source: NORESCO for California Energy Commission
Because the 2-D model used is itself of limited accuracy considering the numerous approximations made, the above results are not considered to be definitive. However they do indicate that a number of the assumptions made in the 1-D model are reasonably accurate. While this comparison is limited to steady state heat transfer performance, mass effects are expected to have comparable accuracy.

### 2.3 How to Build an Airnet

### 2.3.1 Background

### 2.3.2 Approach

IZXFER is the building block input for an Airnet. There will be many IZXFERs in an input file, each representing a single air transfer object (leak, vent, window, fan, duct leak). Each IZXFER needs a unique name if detailed reports on its activity are needed. IZXFER is a command at the same level as HOLIDAY.

MATERIAL, CONSTRUCTION, METER, ZONE and REPORT which means that it can be located anywhere except inside another object (like a zone).

The main objects in the Airnet are:

- Infiltration
- Window ventilation
- IAQ ventilation
- Mechanical cooling ventilation
- Duct leakage

Units are $\mathrm{ft}^{2}$.

CBECC inputs to add for Airnet:

| Input | Description |
| :--- | :--- |
| WinHHTop | Head height of the highest windows in the zone. Used to get the <br> vertical location of the window ventilation holes. In the <br> development program this was done on a building wide basis: <br> \#define WnHeadHeight 7.67 // Average head height above the <br> floor of operable windows |
| Ventilation Height <br> Difference | This rule needs to be changed to refer to Zone instead of <br> Building: "The default assumption for the proposed design is 2 <br> feet for one story buildings and 8 feet for two or more stories. <br> Greater height differences may be used with special ventilation <br> features such as high, operable clerestory windows. In this case, <br> the height difference entered by the user is the height between <br> the average center height of the lower operable windows and the <br> average center height of the upper operable windows. Such <br> features shall be fully documented on the building plans and <br> noted in the Special Features Inspection Checklist of the CF-1R." <br> (2008 RACM pp 3-9) |
| Floor Height | The height of each floor over outdoors, crawl or garage is needed <br> to set the Z dimension of the hole in the floor. |
| Soffit height | The height of the attic floor. Probably can be determined by the <br> height of the ceiling below attic. Trouble for Split level? |
| Interzone Door | May need input for whether an interzone door exists between <br> each 2 conditioned zones. Assuming it for now. |
| ReturnRegister | The conditioned zone(s) where the return/exhaust register is <br> located. Make this an input on the HVAC System Data screen |

### 2.3.2.1 Problems

- The window scheme doesn't work for 3 story zones!!!
- The Econ and NightBreeze cooling ventilation systems are multizone and use ducts. I suggest we set them up as part of the duct system.


### 2.3.3 Inputs

| Input | Description |
| :--- | :--- |
| ACH50 | 7.6 (Air Changes per Hour at 50 Pascals pressure difference that <br> leak through the envelope of the conditioned zones) |
| Avent | $1 / 300$ (ratio of "free area of attic vents to AceilGross) |
| Fraction High | 0.3 (fraction of the attic vent area located in the upper part of the <br> attic, check precise definition) |

1. Infiltration Setup. Infiltration is uncontrolled air leakage through the cracks and intentional vents in the building. The first step is to determine the total size of the openings and then distribute them over the conditioned zones in proportion to surface areas.

It is modeled in a single conditioned zones with 8 holes (IZXFERs) to represent the leakage in vertical walls and 1 hole each in the floor and ceiling.

## Calculate:

a. For conditioned zones the total Effective Leakage Area ELAtot = CFA*ACH50/(2*10000) (CFA is conditioned floor area)
b. Determine Envelope Areas

1. ExCeiltotSF =sum (AceilGross + area of exterior ceilings) (exterior ceilings are surfaces in conditioned zones of type ceiling whose outside condition is Ambient, Ignore Knee walls for infiltration (walls between the conditioned zone and the attic)
2. ExWalltotSF =sum (Gross Area of Exterior Walls) (walls in conditioned zones whose outside condition is Ambient)
3. ExFloortotSF =sum (Gross Area of Exterior floors) (floors in conditioned zones whose outside condition is Ambient, Crawl or GROUND)
4. ExFloorSlabSF =sum (Gross Area of Exterior slab on grade floors) (slab on grade floors in conditioned zones)
5. SlabRatio =ExFloorSlabSF/ExFloortotSF
6. GaragetotSF =sum (Gross Area of Surfaces to Garage) (walls and floors in conditioned zones whose outside condition is Garage)
c. Determine leakage distribution:
7. ELAceilsf = ELAtot* (.4+.1*SlabRatio)/( AceilGross + area of exterior ceilings)
8. ELAraisedFloorsf = ELAtot* (.2* 1-SlabRatio)/( ExFloortotSF-ExFloorSlabSF)
\#If there is a garage zone
9. ELAGaragesf = ELAtot* 0.1/GaragetotSF
10. ELAwallsf = ELAtot* ( $.3+.1^{*}$ SlabRatio)/ExWalltotSF
\#Else
11. ELAGaragesf $=0$
12. ELAwallsf = ELAtot* (. $4+.1^{*}$ SlabRatio)/ExWalltotSF \#endif
13. Cooling Ventilation Setup: Four types: Windows only (all types have windows for some part of the year), Whole house fan, Smart Vent, NightBreeze

Set up seasonal window control
\#if Smart Vent or NightBreeze //Windows are on in Winter, but off in summer when mechanical ventilation is on
\#redefine Windowmode select( @weather.taDbAvg07 >60., 0.00001,default 1.)
\#define VentDiffMult select( @top.tDbOSh < (@znRes[Single].prior.S.tAirVentDiff), 1,default 0.000001) //Vent off if Tin-Vendiff > Tout
\#Else //everything but Econ and NightBreeeze Windows are on year round
\#reDefine Windowmode 1.//Always available
\#Define VentDiff 0 // Differential. No differential for windows or WWF // multiplier for window and whole house fan vent availability, . 00001 is proxy for Off Revised to start at dawn end at 11 PM.
\#redefine Win_hr select( \$hour < 24, select(\$radDiff <1., select(\$hour>12,1.0, default . 00001 ), default 1.0 ), default . 00001 )
3. Airnet for Each Conditioned Zone:
a. Calculate

$$
\begin{aligned}
& \text { ELA_Aceil(zone) = ELAceilsf * AceilGross(zone) } \\
& \text { ELA_Xceil(zone) = ELAceilsf * (AEdge(zone) + area of exterior }
\end{aligned}
$$

b. Exteror wall of conditioned zones infiltration objects Calculate height of bottom and top holes.
// All infiltration leaks in walls are assumed to be spread uniformly over the exposed wall surfaces areas. There are no LEAKS associated with windows, doors etc.
//8 Wall Holes in each zone to Outdoors 1 upwind, 2 side walls, 1 downwind. Sidewalls are identical so combine them into 1 hole with 2*area
// Low is at $1 / 4$ of wall height, high is at $3 / 4$ of wall height
$/ /$ izCpr $($ default $=0)=$ Wind Coef Upwind wall +0.6 Side walls -0.65 Downwind Wall -0.3

ELAXwall(zone) = ELAwallsf * Gross Area of Exterior Walls(zone)
WH = ELAXwall(zone)*1.45/8 //Wall Hole size. Conversion from ELA to airnet infiltration opening is 1.45*
Write Airnet Objects to CSE Input WILU stands for Wall Low Upwind etc.
IZXFER (ZoneName)WILU izNVTYPE = AirNetExt izZN1 = (ZoneName)
izALo $=$ WH izHD $=$ ZoneBotZ $+(0.25 *$ ZoneHeightZ $)$ izNVEff $=1$
izExp=0.65 izCpr=0.6
IZXFER (ZoneName)WILS izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo $=2 *$ WH izHD $=$ ZoneBotZ $+(0.25 *$ ZoneHeightZ) izNVEff $=1$ izExp=0.65 izCpr=-. 65

IZXFER (ZoneName)WILD izNVTYPE = AirNetExt izZN1 = (ZoneName)
izALo $=$ WH izHD $=$ ZoneBotZ $+(0.25 *$ ZoneHeightZ $)$ izNVEff $=1$ izExp=0.65 izCpr=-0.3

IZXFER (ZoneName)WIHU izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo $=$ WH izHD $=$ ZoneBotZ $+(0.75$ * ZoneHeightZ) izNVEff $=1$ izExp=0.65 izCpr=0.6

IZXFER (ZoneName)WIHS izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo $=2 *$ WH izHD $=$ ZoneBotZ $+(0.75 *$ ZoneHeightZ)izNVEff $=1$ izExp=0.65 izCpr=-. 65

IZXFER (ZoneName)WIHD izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo $=$ WH izHD $=$ ZoneBotZ $+(0.75 *$ ZoneHeightZ $)$ izNVEff $=1$ izExp=0.65 izCpr=-0.3
c. Windows
// Operable window openings for ventilation. Assumes effect of screens is included in open area Revised 120409 BAW
// IZXFER izALo and izAHi are the min and max vent areas. Both are hourly.
//8 Window Holes in zone Single to Outdoors Assumes no orientation so $1 / 4$ each orientation, $1 / 8$ low and $1 / 8$ high. Sidewalls are identical so combine them into 1 hole with 2*area
// high is at $1 / 2$ default Hdiff below Window WinHHTop, Low is at WinHdiff below.high
//Note that this scheme doesn't work for 3 story zones!!!
Inputs
WnVentArea // ft2, Nonzero - operable window open area. Default is 10 percent of the window area. Assume a single window is 4 feet high with openings centered at -1 and -3 from the top

WnVentHDiff 2.0 // Window vent height difference between center of high opening and low opening

WinHHTop // Head height of highest windows in the zone Calculate

WnHole $=0.5 *$ (WnVentArea/8.)*Win_hr*Windowmode // 1/8th in each hole, ft2. $1 / 2$ of nominal area to account for screens etc. Hourly and seasonal availability

Write Airnet Objects to CSE Input WnLU stands for Window Low Upwind etc
IZXFER (ZoneName)WnLU izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo=. 00001 izAHi $=$ WnHole izHD $=$ WinHHTop -(3+ WnVentHDiff) izNVEff $=.5$ izCpr=0.6

IZXFER (ZoneName)WnLS izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo=. 00001 izAHi $=2 *$ WnHole izHD $=$ WinHHTop - ( $3+$ WnVentHDiff) izNVEff $=.5 \mathrm{izCpr}=-.65$

IZXFER (ZoneName)WnLD izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo=. 00001 izAHi $=$ WnHole izHD $=$ WinHHTop - (3+WnVentHDiff) izNVEff $=.5$ izCpr=-0.

IZXFER (ZoneName)WnHU izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo=. $00001 \mathrm{izAHi}=$ WnHole izHD $=$ (WinHHTop-1) izNVEff $=.5 \mathrm{izCpr}=0.6$

IZXFER (ZoneName)WnHS izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo=. 00001 izAHi $=2 *$ WnHole izHD $=$ (WinHHTop-1) izNVEff $=.5 \mathrm{izCpr=}=.65$

IZXFER (ZoneName)WnHD izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo=. 00001 izAHi $=$ WnHole izHD = (WinHHTop-1) izNVEff $=.5$ izCpr=-0.3
d. Ceiling

Calculate
If ceiling below attic
ELA_Aceil(zone) = ELAceilsf * AceilGross(zone) //knee walls to attic not included in determining the conditioned to attic leakage distribution of

If Whole House fan, Ceiling leak through WWF when off
If Improved WHFela $=.05$ // Average of Motor Damper Models else WHFela = . 11 // Average of Gravity Damper Models

If ceiling to outside

$$
\begin{aligned}
& \text { ELA_Xceil(zone) = ELAceilsf * AEdge(zone) + area of exterior } \\
& \text { ceilings(zone) }
\end{aligned}
$$

CeilHole $=(\text { ELA_Aceil(zone) }+ \text { WHFela })^{*} 1.45 / /$ Ceil Hole size.
Conversion from ELA to airnet infiltration opening is 1.45*

Conversion from ELA to airnet infiltration opening is 1.45*
Write Airnet Objects to CSE Input
IZXFER (ZoneName)xAttic izNVTYPE = AirNetIZ
izZN1=(ZoneName) izALo=CeilHole izHD = ZoneTopZ izNVEff=1. izExp=0.65 izZN2 $=$ Attic

IZXFER (ZoneName)CC izNVTYPE = AirNetExt izZN1=(ZoneName) izALo=CathCeilHole izHD $=$ ZoneTopZ izNVEff=1. izExp=0.65
e. Floor over outside

For each floor over outside calculate:
ELAfloor(Name) $=$ ELAraisedFloorsf * AreaExtfloor * 1.45 (floors whose outside condition is Ambiant)

Write Airnet Object to CSE Input
IZXFER (Name) izNVTYPE = AirNetExt izZN1 = (ZoneName) izALo =
ELAfloor(Name) izHD = Floor Height izNVEff = 1 izExp=0.65
izCpr=0. // located at the Extfloor elevation, no wind effect
f. Floor over Crawl

For each floor over outside calculate:
ELAfloor(Name) = ELAraisedFloorsf * AreaCrawlfloor * 1.45 (floors whose outside condition is Crawl)

Write Airnet Object to CSE Input
IZXFER (Name)xCrawl izNVTYPE = AirNetIZ izZN1=(ZoneName)
izALo=ELAfloor(Name) izHD = Floor Height izNVEff=1. izExp=0.65
izZN2 = Crawl
g. Floor over Garage

For each floor over garage calculate:
ELAfloor(Name) = ELAGaragesf * AreaGarfloor * 1.45 (floors whose outside condition is Garage)

Write Airnet Object to CSE Input
IZXFER (Name)xGarage izNVTYPE = AirNetIZ izZN1=(ZoneName)
izALo=ELAfloor(Name) izHD = Floor Height izNVEff=1. izExp=0.65
izZN2 = Garage
h. Garage wall

Calculate:

| GWH | = ELAGaragesf * Gross Area of walls next to the Garage(zone)/2 // size of the 2 holes (high and low) betwen zone and garage |
| :---: | :---: |
| GWalltopz | $=$ Min(ZoneTopZ(zone),(ZoneTopZ(Garage)) //The top of the shared wall |
| GWallBotz | $\begin{aligned} & =\text { Max(ZoneBotZ(zone),(ZoneBotZ(Garage)) } \quad / / \text { The bottom } \\ & \text { of the shared wall } \end{aligned}$ |
| Gwalllh | = GWalltopZ - GWallBotZ // Height of shared wall |
| GWHhZ | = GwallBotZ + . 75 Gwalll // Height of top hole |
| GWHIZ | = GwallBotZ + . 25 GwallH // Height of bottom hole |

Write Airnet Objects to CSE Input
IZXFER (ZoneName)xGarageH izNVTYPE = AirNetIZ izZN1=(ZoneName)
izALo=GWH izHD = GWHhZ izNVEff=1. izExp=0.65 izZN2 = Garage
IZXFER (ZoneName)xGarageL izNVTYPE = AirNetIZ izZN1=(ZoneName)
izALo=GWH izHD = GWHIZ izNVEff=1. izExp=0.65 izZN2 = Garage

## 4. Airnet for Each Unconditioned Zone:

a. Attic

If Ventilated attic Calculate ( 4 soffit vents at attic floor elevation plus sloped deck vents at $2 / 3$ of Attic high if frac high > 0
// Pitch types for roof wind pressure coeffs: 0 deg, <10deg, <15 deg,<25, $<35$,all the rest. Flat same as low slope.
\#define PitchType select( Pitch <= 0, 1,Pitch <= 0.18, 1,Pitch <= 0.27, 2,Pitch $<=0.47,3$,Pitch $<=0.7,4$,default 5)

AventTot $=$ AceilGross $*$ AVent
SoffitVent 0.5*0.25 *(1.-FracHigh)*Max(AventTot, AtticRelief) //Attic relief is minimum vent needed to vent mechanical cooling air dumped to attic

Deckvent 0.5*0.25*FracHigh*Max(AventTot, AtticRelief)
If sealed attic [to be developed]

## Write Airnet Objects to CSE Input

IZXFER AtticSU izNVTYPE = AirNetExt izZN1 = Attic izALo = SoffitVent izHD $=$ SoffitHeight izNVEff $=.6 \mathrm{izExp}=0.65 \mathrm{izCpr=}=0.6$

IZXFER AtticSS izNVTYPE $=$ AirNetExt izZN1 $=$ Attic izALo $=2 *$ SoffitVent izHD $=$ SoffitHeight izNVEff $=.6$ izExp= $0.65 \mathrm{izCpr=}=.65$

IZXFER AtticSD izNVTYPE = AirNetExt izZN1 = Attic izALo = SoffitVent izHD $=$ SoffitHeight izNVEff $=.6 \mathrm{izExp}=0.65 \mathrm{izCpr}=-0.3$

IZXFER AtticDU izNVTYPE $=$ AirNetExt izZN1 $=$ Attic izALo $=$ DeckVent izHD $=0.67$ * AtticHeight + SoffitHeight izNVEff=. 6 izExp=0.5 izCpr=testx*choose(Pitchtype,-.5,-.8,-.5,-.3,.1,.3)
IZXFER AtticDS izNVTYPE $=$ AirNetExt izZN1 $=$ Attic izALo $=2 *$ DeckVent izHD $=0.67$ * AtticHeight + SoffitHeight izNVEff=. 6 izExp=0.5
izCpr=testx*choose(Pitchtype,-.5,-.5,-.5,-.5,-.5,-.5)
IZXFER AtticDD izNVTYPE = AirNetExt izZN1 = Attic izALo = DeckVent izHD $=0.67$ * AtticHeight + SoffitHeight izNVEff=. 6 izExp=0.5 izCpr=testx*choose(Pitchtype,-.5,-.3,-.5,-.5,-.5,-.5)
b. Garage - Assume California garage with a water heater and combustion air vents so it is pretty leaky Guess 1 ft 2 of free area. Ignore other infiltration Calculate

$$
\text { Gvent }=1 / 4
$$

```
Write Airnet Objects to CSE Input
IZXFER GarageU izNVTYPE = AirNetExt izZN1 = Garage izALo = Gvent izHD \(=\) GarageBotZ +1 izNVEff \(=.6\) izExp=0.65 izCpr=0.6
IZXFER GarageS izNVTYPE \(=\) AirNetExt izZN1 \(=\) Garage izALo \(=2 *\) Gvent izHD = GarageBotZ +1 izNVEff = . 6 izExp=0.65 izCpr=-. 65
IZXFER GarageD izNVTYPE = AirNetExt izZN1 = Garage izALo = Gvent izHD \(=\) GarageBotZ +1 izNVEff \(=.6\) izExp=0.65 izCpr=-0.3
```

c. Vented crawl space [To Be Developed]
d. Sealed crawl space [To Be Developed]
e. Basement [To Be Developed]
5. Interzone Holes - Assume an open door or stair between any twp conditioned zones with common surfaces, except between units in multi-family

If 2 or more conditioned zones
Error if not at least one common surface for every conditioned zone (a surface in zone A whose outside condition is another conditoned zone)

Door calculation for each pair of zones with a common wall surface (zoneA<>zoneB, zoneB<>zoneC, zoneA<>zoneC, etc)
DoortopZ = Min(ZoneTopZ(zone A),(ZoneTopZ(zone B)) //The top of the shared wall

DoorBotZ = Max(ZoneBotZ(zone A),(ZoneBotZ(zone B)) //The bottom of the shared wall

DoorH = DoortopZ - DoorBotZ // Height of shared opening
DH $\quad=20 / / /$ Area of half of assumed door
DHhZ $=$ GwallBotZ +.75 GwallH // Height of top hole
DHHIZ = GwallBotZ + . 25 GwallH // Height of bottom hole
For each zone pair write Airnet Objects to CSE Input
IZXFER (ZoneNameA)DHx(ZoneNameB)DH izNVTYPE = AirNetIZ izZN1=(ZoneNameA) izALo=DH izHD = DHhZ izNVEff=1. izExp=0.5 izZN2 = (ZoneNameB)

IZXFER (ZoneNameA)DLx(ZoneNameB)DL izNVTYPE = AirNetIZ izZN1=(ZoneNameA) izALo=DH izHD = DHIZ izNVEff=1. izExp=0.5 izZN2 = (ZoneNameB)

Stair calculation for each pair of zones with only a floor/ceiling surface (zoneA<>zoneB, zoneB<>zoneC, zoneA<>zoneC, etc)

StairZ = Max(ZoneBotZ(zone A),(ZoneBotZ(zone B)) //The height of the stair hole is at the upper floor

For each zone pair write Airnet Objects to CSE Input Note that izZN1 MUST be the lower of the 2 zones or the model doesn't work

IZXFER (ZoneNameA)Sx(ZoneNameB)S izNVType = AIRNETHORIZ izZN1=(ZoneName of lowerzone) izZN2 = (ZoneName of upper zone) izL1=3 izL1=10 izHD =StairZ

## 6. IAQ ventilation

Inputs for each zone
IAQVentCFM // CFM of IAQ vent
IAQfanWperCFM // W/CFM of IAQ vent
Type IAQExhaust // "IAQExhaust", "IAQSupply", "IAQBalanced" "NoIAQVent"
IAQVentHtRcv 0.0 // Heat recovery efficiency of Balanced type, frac
Write Airnet Objects to CSE Input
If Exhaust
IZXFER (Zone)IAQfan izNVTYPE = AirNetExtFan izZN1 = (Zone) izVFmin=IAQVentCFM izVFmax=-IAQVentCFM izFanVfDs=IAQVentCFM izFanElecPwr=IAQfanWperCFM izFanMtr=IAQventMtr

If IAQSupply
IZXFER (Zone)IAQfan izNVTYPE = AirNetExtFan izZN1 = (Zone)
izVFmin=IAQVentCFM izVFmax=IAQVentCFM izFanVfDs=IAQVentCFM
izFanElecPwr=IAQfanWperCFM izFanMtr=IAQventMtr
If IAQBalanced // Needs heat recovery
IZXFER (Zone)IAQfanS izNVTYPE = AirNetExtFan izZN1 = (Zone)
izVFmin=IAQVentCFM izVFmax=IAQVentCFM izFanVfDs=IAQVentCFM
izFanElecPwr=IAQfanWperCFM izFanMtr=IAQventMtr
IZXFER (Zone)IAQfanE izNVTYPE = AirNetExtFan izZN1 = (Zone) izVFmin=IAQVentCFM izVFmax=-IAQVentCFM izFanVfDs=IAQVentCFM izFanElecPwr=IAQfanWperCFM izFanMtr=IAQventMtr
7. Mechanical Cooling Ventilation // The following does not work for multi-zone systems with Econ, NightBreeze. Revise along with ducts model

For each Cooling Ventilation System
Inputs

CoolVentType //type of MECHANICAL cooling ventilation, Choice of WHF, Econ, NightBreeze

CoolVentCFM //Rated air flow of the mechanical cooling system CoolVent W/CFM

ReturnRegister // If WHF the conditoned zone where it is located Calculate

Relief = CoolVentCFM/750 // The minimum size of the attic vents required to let the WHF flow out of the attic

If WHF // Whole House Fan
Calculate
Relief = CoolVentCFM/750 // The minimum size of the attic vent required for this fan to let the WHF flow out of the attic

Write Airnet Objects to CSE Input
IZXFER (Zone)WHF izNVTYPE=AirNetIZFan izZN1=(Zone) izVFmin=0. izVFMax=-CoolVentCFM*Win_hr izFanVfDs=CoolVentCFM izZn2=Attic izFanElecPwr=CoolVentWperCFM izFanMtr=CoolVentMtr
If Econ // Economizer ventilation option on the Central Forced Air System such as Smartvent

Calculate
Relief = CoolVentCFM/750 // The minimum size of the attic vent required for this fan to let the WHF flow out of the attic

Write Airnet Objects to CSE Input
ZXFER Econ\# izNVTYPE=AirNetExtFan izZN1=(zone) izVFmin=0. izVFMax=CoolVentCFM*Coolmode*VentDiffMult izFanVfDs=CoolVentCFM izFanElecPwr=CoolVentWperCFM izFanMtr=CoolVentMtr //!! 110413

IZXFER Relief\# izNVTYPE=AirNetIZFlow izZN1=ReturnRegister izZn2=Attic izVFmin=0 izVFmax=CoolVentCFM*Coolmode*.9*VentDiffMult

If NightBreeze //Model for NightBreeze variable flow night ventilation system !!needs lower limit @ CFA<1000/unit and multiple systems @ CFA> $3333 \mathrm{ft2}$

Calculate
Relief $=$ CoolVentCFM/750 // The minimum size of the attic vent required for this fan to let the WHF flow out of the attic

Write Airnet Objects to CSE Input
IZXFER NightBreeze izNVTYPE=AirNetExtFan izZN1=(zone) izVFmin=0. izFanMtr=CoolVentMtr izFanVfDs=CoolVentCFM * CFA //CoolVentCFM = CFM/CFA for NightBreeze. Default is 0.6
izFanElecPwr $=(616.47-0.6159 *$ CFA +.000246 *CFA*CFA)/(CoolVentCFM * CFA) //W/CFM DEG 9/29/2010 Equation 1
izVFMax=CoolVentCFM*Coolmode*VentDiffMult*CFA / max((17.91554 3.67538 * logE(@weather.taDbPvPk)),1)//DEG 9/29/2010 Equation 3 110411
izFanCurvePy $=0,-0.026937155,0.187108922,0.839620406,0$ //Fit to DEG flow^2.85

IZXFER NBRelief izNVTYPE=AirNetIZFlow izZN1=ReturnRegister izZn2=Attic izVFmin=0 izVFmax=-CoolVentCFM * CFA * Coolmode*. 9 *VentDiffMult

Next Zone
Calculate
AtticRelief $=$ Sum(CoolVentCFM)/750 // The sum of all zonal cool vent CFM determines the minimum size of the attic vents required to let the vent air out of the attic
//Used in Attic Zone AirNet above
//Min Attic Vent area for relief Tamarac
http://www.tamtech.com/userfiles/Fan\ size\ and\ venting\ requir ements(3).pdf

## 7. Duct system leaks and pressurization.

These are generated automatically by CSE based on the duct system inputs.

### 2.4 How to Create CSE Conditioned Zone Internal Mass Inputs

### 2.4.1 Background

### 2.4.2 Approach

Internal mass objects are completely inside a zone so that they do not participate directly in heat flows to other zones or outside. They are conected to the zone radiantly
and convectively and participate in the zone energy balance by passively storing and releasing heat as conditions change. For now only in Conditioned Zones.

The main internal mass objects in the are:

- Interior walls
- Interior floors
- Furniture
- Cair
- Specific masses (for addition later)

CBECC inputs to add:

- Specific masses (for addition later)


### 2.4.3 Inputs

Floor Area of zone
For each Conditioned Zone

1. Interior Floor Setup. Input for inside the conditioned zone interior floors as mass elements.

Calculate:
a. Xflr = sum of the area of floors to ground, crawl space, exterior or other zones
b. IntFlr = Floor Area-Xflr
2. Interior Wall Setup. Input for inside the conditioned zone interior walls as mass elements.

Calculate:
a. IZwall $=$ sum of the area of interior walls to other conditioned zones
b. Intwall(zone) floor area - .5 * IZwall
3. Write objects to the CSE input

Light stuff

1. znCAir=floor area * 2
2. Interior wall if Intwall(zone) >0

SURFACE IntWallC(zone) sfType=Wall sfArea=0.75*Intwall(zone) sfCon=IntwallCav; sfAZM=0 sfExCnd=ADJZN sfAdjZn=(zone)

SURFACE IntWallF(zone) sfType=Wall sfArea=0.25*Intwall(zone) sfCon=IntwallFrm; sfAZM=0 sfExCnd=ADJZN sfAdjZn=(zone)
3. Furniture

SURFACE Furniture(zone) sfType=wall sfArea= Floor Area * 2.; sfCon=FurnCon; sfAZM=0 sfExCnd=ADJZN sfAdjZn=Zone

Interior Floor if IntFIr(zone) >0
// floor construction for interior mass. Assumes $2 \times 10$ @ 16" OC.
Both floor and ceiling are in the conditioned zone
SURFACE IntFlrFrm sfType=Floor sfCon=IntFFrm2x10 sfArea=0.1 * RaisedFIr; sfExCnd=ADJZN sfAdjZn=(Zone)

SURFACE IntFIrCav sfType=Floor sfCon=IntFCav2x10 sfArea=0.9 * RaisedFI; sfExCnd=ADJZN sfAdjZn=(Zone)
4. Constructions

CONSTRUCTION FurnCon // 2.5" wood Revised Layers
Layer IrMat="SoftWood" IrThk=2.5/12
CONSTRUCTION IntwallCav // 2x4 Revised Layers
Layer IrMat="Gypsum Board"
Layer IrMat="Gypsum Board"
CONSTRUCTION IntwallFrm // 2x4 Revised Layers
Layer IrMat="Gypsum Board"
Layer IrMat="SoftWood" IrThk=3.5/12.
Layer IrMat="Gypsum Board"
CONSTRUCTION IntFFrm2x10 // 9.25" (2x10)
Layer IrMat="Carpet"
Layer IrMat="Wood layer"
Layer IrMat="SoftWood" IrThk=9.25/12.
Layer IrMat="Gypsum Board"
CONSTRUCTION IntFCav2x10 // 9.25" (2x10)
Layer IrMat="Carpet"
Layer IrMat="Wood layer"
Layer IrMat="Carpet" // Air space with 1 psf of stuff (cross bracing wiring, plumbing etc) approximated as 1 " of carpet
Layer IrMat="Carpet" // Air space with 1 psf of stuff (cross bracing wiring, plumbing etc) approximated as 1 " of carpet

## Layer IrMat="Gypsum Board"

### 2.5 Appliances, Miscellaneous Energy Use, and Internal Gains

### 2.5.1 Background

This model is derived from the 2008 HTM (California Energy Commission, HERS Technical Manual, California Energy Commission, High Performance Buildings and Standards Development Office. CEC-400-2008-012). This is a major change from the 2008 RACM in that internal gains are built up from models for refrigerator, people, equipment and lights instead of the simple constant plus fixed $\mathrm{BTU} / \mathrm{ft}^{2}$ used there. The HTM derived model has been used in the 2013 Development Software throughout the 2013 revision process.
This model has another significant change beyond the HTM model with the addition of latent gains required as input for the new CSE air conditioning model. There was no information on latent gains in either the 2008 RACM or the HTM. The latent model here was created by applying the best available information on the latent fraction of internal gains to the HTM gains model.

### 2.5.2 Approach

The approach here is to calculate the Appliances and Miscellaneous Energy Use (AMEU) for the home and use that as the basis for the internal gains. This will facilitate future expansion of the procedure to calculate a HERS Rating.

### 2.5.2.1 Problems

The procedure here (also used in the 2013 development program) does not work correctly for multifamily buildings unless all of the units are the same (CFA and number of bedrooms). I don't believe this problem was considered in developing the HTM. I believe that the only exactly correct solution involves simulating each unit as a separate zone with a different internal gain. For now we will ignore this problem and assume that average values are OK.

The HTM equations do not work if there is a gas range and electric oven.
The allocation of internal gain to zones is not specified in either the RACM or the HTM. A proposed approach is presented here.

### 2.5.3 Inputs

Units Number of dwelling units in the building.
BRperUnit Bedrooms/DwellingUnits rounded to an integer

CFA Conditioned Floor Area in the building
CFAperUnit CFA/DwellingUnits
New CBECC input at the building level: an Appliances Input Screen (for a single conditioned zone, most of these default, we are assuming that MF buildings will be done as one zone):

| Input | Description |
| :---: | :---: |
| Refrigerator/Freezer | Efficiency (Choice of Default $=669 \mathrm{kWh} /$ year, no other choices at this time), Location (Choice of zones if multiple conditioned zones). // HTM assumes all Dwelling units have refrigerators. Different for additons and alterations when we get to them. |
| Dishwasher | Efficiency (Choice of Default, no other choices at this time), Location (Choice of zones if multiple conditioned zones). // HTM assumes all Dwelling units have refrigerators. Different for additons and alterations when we get to them. |
| Clothes Dryer | Location (Choice of zones if multiple zones, No Dryer space or hookup provided) Dryer power (Choice Electric, Gas or other) //Assuming gas for now |
| Clothes Washer | Location (Choice of zones if multiple zones), No Washer space or hookup provided) |
| Range/Oven | Location (Choice of zones if multiple conditioned zones, No Range/Oven space and hookup provided) Range/Oven power (Choice Electric, Gas or other) Assumes gas for now. |
| Assumes CSE Meters are set up elsewhere: |  |
| Mtr_Elec |  |
| Mtr_NatGas |  |
| Mtr_Othewr //PropaNE |  |
| Write Constants to the CSE input: |  |
| 1.19,1.11,1.02,0.93,0.84,0.8,0.82,0.88,0.98,1.07,1.16,1.21) //The monthly internal gain multiplier (same as 2008 RACM). |  |
| \#redefine Lights_hr hourval ( $0.023,0.019,0.015,0.017,0.021,0.031,0.042,0.041,0.034,0.029,0.027,0.025, \backslash$ |  |
| 0.021,0.021,0.021,0.026,0.031,0.044,0.084,0.118,0.113,0.096,0.063,0.038) // Changed 0.117 to 0.118 to add to 1 |  |
| \#redefine OutdoorLights_hr |  |

\#redefine People_hr
hourval( $0.035,0.035,0.035,0.035,0.035,0.059,0.082,0.055,0.027,0.014,0.014,0.014, \backslash$
0.014,0.014,0.019,0.027,0.041,0.055,0.068,0.082,0.082,0.070,0.053,0.035)
\#redefine Equipment_hr
hourval( $0.037,0.035,0.034,0.034,0.032,0.036,0.042,0.044,0.037,0.032,0.033,0.033, \backslash$
0.032,0.033,0.035,0.037,0.044,0.053,0.058,0.060,0.062,0.060,0.052,0.045)

1. Setup the gains that are distributed across the zones per CFA of the zone and write to CSE input: Calculations are generally more complicated in future for HERS
a. //Lights Returns Btu/day-CFA - based on ElectricityInteriorLights $=(214+$ $0.601 \times$ CFA $) \times($ FractPortable $+(1-$ FractPortable $) \times$ PAMInterior ) //HTM Eqn 11, p. 30
\#define FractPortable . 22 //fixed for now, variable later for HERS
\#define Paminterior 0.625 //fixed for now, variable later for HERS
\#Redefine LightsGainperCFA (((214. + 0.601 * CFAperUnit) * (FractPortable $+(1$-FractPortable) $*$ Paminterior ) * 3413. / 365) * DwellingUnits /CFA)
b. People Returns BTU/day-CFA - 100\% is internal gain $57.3 \%$ sensible, $42.7 \%$ latent Based on HTM and BA existing bldgs Sensible 220, Latent 164 BTU

> \#Redefine PeopleperUnit (1.75 + 0.4 * BRperUnit)
> \#Redefine PeopleGainperCFA (( $3900 / 0.573) ~$ PeopleperUnit * DwellingUnits / CFA)
c. Misc Electricity Returns BTU/day-CFA - $100 \%$ is internal gain
\#Redefine MiscGainperCFA ((723. + (0.706 * CFAperUnit))* DwellingUnits * 3413. / 365.)/CFA
2. Setup the gains that are point sources located in a particular zone and write to CSE input. Calculations are generally more complicated in future for HERS
a. Refrigerator. In the HTM all Standard Design refrigerators use the same amount of electricity ( $669 \mathrm{kWh} /$ year) regardless of the size of dwelling unit or number of bedrooms. The proposed use is based on the energy label of the actual refrigerator installed or if that is not available the default. For existing home HERS calculations the default is ( $775 \mathrm{kWh} /$ year). Refrigerators run at a constant power 24 hours per day, regardless of the interior air temperature or number of times the door is opened.

Returns BTU/day - 100\% is internal gain. Installed refrigerator rating is input for proposed in HERS later
\#Redefine RefrigeratorGain (DwellingUnits * 669. * (3413. / 365.))
b. Dishwasher. 0 based choose returns BTU/day // uses Table based in INTEGER bedrooms per dwelling.
\#Redefine DishwasherGain (choose
(BRperUnit,90,90,126,126,126,145,145,174,174,174,default 203) *
DwellingUnits * 3413. / 365.)
c. Stove and Oven - Assumes both are gas with electonic igniter Returns BTU/day - Full Energy Use, $90 \%$ is internal Gain
define CookGain (((31. + (.008 * CFAperUnit))* 0.43* 0.9)* DwellingUnits * 100000. / 365.) //Added the 0.43 for the electronic ignition 12/4 BAW
d. Clothes Washer - // Returns BTU/day
\#Redefine WasherGain ((-64 + 0.108 * CFAperUnit) * DwellingUnits * 3413. / 365.)
e. Clothes Dryer - Assumes gas with electonic igniter Returns BTU/day - Full energy Use, $30 \%$ is internal gain
define DryerGAin (13. + (.01 * CFAperUnit))* DwellingUnits * 100000. / 365. //Added the 0.43 for the electronic ignition //120831
f. Exterior Lights Returns Btu/day - based on HTM Eqn 14
\#define PamExterior 0.49 //fixed for now, variable later for HERS
\#Redefine ExtLightGain ( $-81+0.152 \times$ CFA) $\times$ PAMExterior * 3413 . / 365)
3. For each conditioned zone: //Write GAIN objects inside each conditioned zone

GAIN Lights(zone) gnPower= LightsGainperCFA*CFA(Zone)*Lights_hr*Intgain_mo gnFrRad=0.4 gnEndUse=Lit gnMeter= Mtr_Elec

GAIN People(zone) gnPower=
PeopleGainperCFA*CFA(Zone)*People_hr*Intgain_mo gnFrRad=0.3 gnFrLat=0.427 // Free Energy so not metered

GAIN Misc(zone) gnPower= MiscGainperCFA*CFA(Zone)*Equipment_hr*Intgain_mo gnFrRad=0.3 gnFrLat=0.03 gnEndUse=Rcp gnMeter= Mtr_Elec

Write any of the following if the source is located in this zone:
GAIN Refrigerator gnPower= RefrigeratorGain/24 gnFrRad=0 gnEndUse=Refr gnMeter= Mtr_Elec // No *Intgain_mo, change fro 2013 DevProg

GAIN Dishwasher gnPower= DishwasherGain*Equipment_hr*Intgain_mo gnFrRad=0 gnFrLat=0.25 gnEndUse=Dish gnMeter= Mtr_Elec

GAIN Cooking gnPower=CookGain*Equipment_hr*Intgain_mo gnFrRad=0 gnFrLat=0.67 gnEndUse=Cook gnMeter= Mtr_NatGas gnFrZn=. 9 //
GAIN Washer gnPower= WasherGain*Equipment_hr*Intgain_mo gnFrRad=0 gnEndUse=Wash gnMeter= Mtr_Elec //

GAIN Dryer gnPower= DryerGAin*Equipment_hr*Intgain_mo gnFrRad=0 gnFrLat=0.5 gnEndUse=Dry gnMeter= Mtr_NatGas gnFrZn=. 3 //

Write the following to the 1st zone only (one gain per building):
GAIN ExtLights gnPower= ExtLightGain*OutdoorLights_hr gnFrZn=. 0 gnEndUse=Ext gnMeter= Mtr_Elec // outside lights, no internal gain
4. For each unconditioned zone write the following if the source is located in this zone: //Garage or Basement Maybe 2nd refrigerator in garage later?

GAIN Washer gnPower= WasherGain*Equipment_hr*Intgain_mo gnFrRad=0 gnEndUse=Wash gnMeter= Mtr_Elec //

GAIN Dryer gnPower= DryerGAin*Equipment_hr*Intgain_mo gnFrRad=0 gnFrLat=0.5 gnEndUse=Dry gnMeter= Mtr_NatGas gnFrZn=. 3 //

### 2.6 Seasonal Algorithm

These are constant control rules. You could substitute values for defined terms in some cases like Winter_Vent Winter_Cool Summer_heat and Sumr_Vent_Temp
//Thermostats and associated controls
//Heat Mode
\#redefine SZ_Heat_hr
hourval(65,65,65,65,65,65,65,68,68,68,68,68,68,68,68,68,68,68,68,68,68,68,68,65)
\#redefine Liv_Heat_hr
hourval( $65,65,65,65,65,65,65,68,68,68,68,68,68,68,68,68,68,68,68,68,68,68,68,65$ )
\#redefine SIp_Heat_hr
hourval(65,65,65,65,65,65,65,68,65,65,65,65,65,65,65,65,65,65,65,65,65,68,68,65)
\#redefine Winter_Vent 77
\#redefine Winter_Cool 78
//Cool Mode
\#redefine SZ_Cool_hr
hourval(78,78,78,78,78,78,78,83,83,83,83,83,83,82,81,80,79,78,78,78,78,78,78,78)
\#redefine Liv_Cool_hr
hourval( $83,83,83,83,83,83,83,83,83,83,83,83,83,82,81,80,79,78,78,78,78,78,78,83)$
\#redefine Slp_Cool_hr hourval(78,78,78,78,78,78,78,83,83,83,83,83,83,83,83,83,83,83,83,83,83,78,78,78)
\#redefine Summer_Heat 60
\#redefine Sumr_Vent_Temp 68 //
// Summer Winter mode switch based on 7 day average temp. Winter<=60>Summer \#redefine Coolmode select( @weather.taDbAvg07 >60., 1,default 0)
\#redefine HeatSet select( @weather.taDbAvg07 >60., Summer_Heat, default SZ_Heat_hr )
\#redefine CoolSet select( @weather.taDbAvg07 >60., SZ_Cool_hr, default Winter_Cool )
\#redefine Tdesired select( @weather.taDbAvg07 >60., Sumr_Vent_Temp, default Winter_Vent)
// Window interior shade closure
\#define SCnight 0.8 // when the sun is down. 80\%
\#define SCday 0.5 // when the sun is up $50 \%$
\#define SCcool 0.5 // when cooling was on previous hour. $50 \%$ ?

## TECHNICAL APPENDICES

## Appendix A. Derivation of Duct Loss Equations Using Heat Exchanger Effectiveness and Y-Delta Transformations

This derivation is for one zone only, and the nomenclature is specific to this appendix alone.

Heat transfer through the duct walls can be illustrated in the electrical analogy in Figure $\mathrm{A}-1$. The first node on the left represents the temperature of the air in the duct ( $\mathrm{T}_{\mathrm{d}}$ ) and is connected to the temperate on the surface of the duct $\left(T_{s}\right)$ by the conductance through the duct wall $\left(U_{d}\right)$. The convective heat transfer coefficient $\left(\mathrm{h}_{\mathrm{c}}\right)$ connects the surface temperature to the duct zone air temperature $\left(\mathrm{T}_{\mathrm{a}}\right)$. The radiation heat transfer coefficient ( $\mathrm{h}_{\mathrm{r}}$ ) connects the surface temperature to the duct zone radiant temperature ( $T_{r}$ ).

Figure A-1: Electrical Analogy of Heat Transfer through a Duct Wall


The temperatures of the duct zone are assumed to be constant; the duct surface temperature is not. The duct surface temperature can be removed from the analysis by using a $\mathrm{Y}-\Delta$ transform. Figure $\mathrm{A}-2$ shows the result of this transformation with direct connections between the duct air temperature, the duct zone radiant and air temperatures through combined coefficients defined in Equation A-1.

$$
U_{r}=\frac{U_{d} h_{r}}{D} \quad U_{c}=\frac{U_{d} h_{c}}{D} \quad U_{x}=\frac{h_{c} h_{r}}{D}
$$

where

$$
D=U_{d}+h_{c}+h_{r}
$$

Figure A-2: Heat Transfer through a Duct Wall with Surface Temperature Removed


Using an energy balance, the rate of change of heat flow along the length $(x)$ of duct must equal the heat flow through the duct wall, or

$$
-m c_{p} \frac{d T_{d}(x)}{d x}=U_{c} P\left(T_{d}(x)-T_{a}\right)+U_{r} P\left(T_{d}(x)-T_{r}\right)
$$

Equation A- 2
where
$m c_{p}=$ capacitance flow rate of the air in the duct
$T_{d}=$ temperature of air in the duct
$U_{c}=$ equivalent heat transfer coefficient (see Equation A-1)
$P=$ perimeter of duct
$T_{a}=$ temperature of air in duct zone
$U_{r}=$ equivalent heat transfer coefficient (see Equation A-1)
$T_{r}=$ radiant temperature in duct zone
Regrouping by temperature terms

$$
m c_{p} \frac{d T_{d}(x)}{d x}=-\left(U_{c} P+U_{r} P\right) T_{d}(x)+U_{c} P T_{a}+U_{r} P T_{r}
$$

Equation A- 3
and dividing through by the quantity $\left(U_{c} P+U_{r} P\right)$ gives

$$
\frac{m c_{p}}{\left(U_{c} P+U_{r} P\right)} \frac{d T_{d}(x)}{d x}=-T_{d}(x)+T_{a m b}
$$

where

$$
T_{a n b}=\frac{U_{c} P}{\left(U_{c} P+U_{r} P\right)} T_{a}+\frac{U_{r} P}{\left(U_{c} P+U_{r} P\right)} T_{r}
$$

Equation A-5
Let $y(x)$ be

$$
y(x)=T_{a m b}-T_{d}(x)
$$

Equation A-6
The derivative of which is

$$
d y=-d T_{d}
$$

Equation A-7
Substituting Equation A- 6 and Equation A- 7 into Equation A- 4 gives

$$
-\frac{m c_{p}}{\left(U_{c}+U_{r}\right) P} \frac{d y}{d x}=y(x)
$$

Equation A-8
Rearranging

$$
\frac{1}{y(x)} d y=-\frac{\left(U_{c}+U_{r}\right) P}{m c_{p}} d x
$$

Equation A-9
and integrating from entrance ( $x=0$ ) to exit ( $x=L$ )

$$
\int_{0}^{L} \frac{1}{y(x)} d y=\int_{0}^{L}-\frac{\left(U_{c}+U_{r}\right) P}{m c_{p}} d x
$$

Equation A-10
Gives

$$
\ln y(L)-\ln y(0)=-\frac{\left(U_{c}+U_{r}\right) P L}{m c_{p}}
$$

Recalling the definition in Equation A-6 and replacing the product of the perimeter and length with the surface area $(A)$ of the duct, and a bit of manipulation yields the following relationships

$$
\frac{y(L)}{y(0)}=\frac{T_{d}(L)-T_{a m b}}{T_{d}(0)-T_{a m b}}=\exp \left(-\frac{\left(U_{c}+U_{r}\right) A}{m c_{p}}\right)
$$

Let

$$
\beta=\exp \left(-\frac{\left(U_{c}+U_{r}\right) A}{m c_{p}}\right)
$$

Equation A-13
Then

$$
\frac{T_{d}(L)-T_{a m b}}{T_{d}(0)-T_{a m b}}=\beta
$$

Equation A-14
Solving for the exit temperature gives

$$
T_{d}(L)=\beta\left(T_{d}(0)-T_{a m b}\right)+T_{a m b}
$$

Equation A-15
The temperature change in length $L$ of duct is

$$
T_{d}(0)-T_{d}(L)=-\beta\left(T_{d}(0)-T_{a m b}\right)-T_{a m b}+T_{d}(0)
$$

Equation A-16
This can be rewritten as

$$
T_{d}(0)-T_{d}(L)=(1-\beta)\left(T_{d}(0)-T_{a m b}\right)
$$

Equation A-17
Let $\varepsilon$ be the sensible heat exchanger effectiveness

$$
\varepsilon=(1-\beta)
$$

Equation A-18
Then the conduction loss from the duct to the duct zone can then be written as

$$
Q_{\text {loss }}=m c_{p}\left(T_{d}(0)-T_{d}(L)\right)=\varepsilon m c_{p}\left(T_{d}(0)-T_{a m b}\right)
$$

Equation A-19

## Appendix B. Screen Pressure Drop

NOTE; The following algorithms are not currently implemented in the code, but are here for future code use, and in the interim are useful to manually determine the effects of screens on window ventilation flow pressure drop.

The references cited are a few of the papers reviewed to ascertain state of the art regarding screen pressure drop. In one of the more recent papers, Bailey et al. (2003) give the pressure drop through a screen as:

$$
\Delta p=K \frac{\rho w^{2}}{2 g_{c}}=K \frac{m^{2}}{2 g_{c} \rho A^{2}}
$$

where,

$$
K=\left(\frac{1}{\beta^{2}}-1\right)\left[\frac{18}{R e}+\frac{0.75}{\log (R e+1.25)}+0.055 \log (R e)\right]
$$

Equation B- 2
and,

$$
\beta=\text { screen porosity }=\text { open area/total area perpendicular to flow direction. }
$$

$$
\beta=\left(1-\frac{d}{t 1}\right)\left(1-\frac{d}{t 2}\right)
$$



$$
\begin{aligned}
& R e=\frac{w d}{v}=\frac{m d}{\rho A v}=\text { Renolds number } . \\
& w=\text { face velocity }=\frac{m}{\rho A^{\prime}}, \mathrm{ft} / \mathrm{sec} . \\
& m=\text { mass flow rate, } \mathrm{lb}_{\mathrm{m}} / \mathrm{sec} . \\
& d=\text { wire diameter, } \mathrm{ft} . \\
& v=\text { viscosity } \approx 1.25 \mathrm{E}-4+5.54 \mathrm{E}-07 \mathrm{~T}(\mathrm{degF}) ; \mathrm{ft}^{2} / \mathrm{sec} .=1 / 6100 \mathrm{ft} 2 / \mathrm{sec} \text { at } 70-\mathrm{F} . \\
& \rho=\text { air density, } \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3} . \\
& g_{c}=32.2 \mathrm{lb}_{\mathrm{m}} \mathrm{ft} / \mathrm{lb}_{\mathrm{f}}-\mathrm{sec}^{2} .
\end{aligned}
$$

The first term, intended for portraying $\mathrm{Re}<1$ pressure drops, is the dominate term.
The third term, intended for Re > 200, is relatively negligible, and the second term is a bridge between the first and third terms.

The Reynolds number for the screen flow $R e=\frac{w d}{v}$ is roughly 5 times the face velocity in $\mathrm{ft} / \mathrm{sec}$. For a velocity of $1 \mathrm{ft} / \mathrm{sec}, \operatorname{Re} \sim 5$. For the expected range of wind speeds of concern for ventilation (see note \#1), and with the motive of making the partial derivatives simple (see below), Equation B- 2 is approximated as

$$
K=\left(\frac{1}{\beta^{2}}-1\right) \frac{25}{R e}
$$

Equation B-3
[note \#1: California CZ12 ave yearly Vmet $\approx 11 \mathrm{ft} / \mathrm{sec}$. Correcting for height and shielding gives Vlocal $\approx 11 * 0.5 * 0.32 \approx 1.8 \mathrm{ft} / \mathrm{sec}$. For max flow case of windows on windward and leeward walls, with typical wall wind pressure coefficients, $\mathrm{w} \approx$ $0.5^{*}$ Vlocal. Thus, the maximum window velocity expected for the annual average wind velocity of $11 \mathrm{ft} / \mathrm{sec}$ is $\mathrm{w} \approx 0.5^{*} 1.8=0.9 \mathrm{ft} / \mathrm{sec}$. In a building with windows in multiple directions, the average $w$ is expected to be much lower, perhaps $0.5 \mathrm{ft} / \mathrm{sec}$. Stack

Effect: Using old ASHRAE equation,

$$
w(f p m)=9 \sqrt{H[f t](\Delta T[F])}=9 * \frac{\sqrt{10 * 10}}{60}=1.5 \frac{\mathrm{ft}}{\mathrm{sec}}
$$ together, the wind and stack may be on the order of $1-\mathrm{ft} / \mathrm{sec}$.

The constant 25 in this approximate formula was determined by forcing Equation B- 2 and Equation B- 3 to match when the window air velocity is at the characteristic value $w_{c}=21 \psi v C_{d}^{2}$, defined below. As discussed there, at this velocity the pressure drop through the screen is equal to that through the window orifice.

Figure B-1: Screen Pressure Drop


Equation $B$ - 3 can be written as

$$
K=\frac{25 \psi v}{w}, \text { or alternately } \frac{25 \psi \rho A v}{m}
$$

Equation B- 4
where, as a result of the approximation Equation $B$ - 3 , the screen inputs can be combined into one characteristic screen parameter $\psi$ (of dimension $\mathrm{ft}^{-1}$ ):

$$
\psi=\frac{1}{d}\left(\frac{1}{\beta^{2}}-1\right)
$$

$\psi$ encapsulates all that needs to be known about the screen for pressure drop purposes. This is only true when $K$ varies in the form assumed by Equation $B-3$.
The flow rate through a screenless window is modeled by Airnet as a sharp edged orifice of opening area A .

$$
m=C_{d} A\left(2 \rho g_{c} \Delta p\right)^{\frac{1}{2}}
$$

Equation B- 6
[Idelchik says this is valid for $\mathrm{Re}>10^{\wedge 4] .}$
At the equal pressure point the window orifice Reynolds number is $\mathrm{Re}_{\mathrm{wdw}}=$ ( $\mathrm{D}_{\mathrm{h}} / \mathrm{d}$ ) $25 \psi \mathrm{dC}_{\mathrm{d}}^{2}$.

$$
\begin{array}{ll}
\mathrm{w}_{\mathrm{c}}=1.89 \mathrm{ft} / \mathrm{sec} & \text { for std } 14 \times 18 \times 0.011 \text { screen } \& C d=0.6 ; 14 \& 18 \text { are wires/inch. } \\
\operatorname{Re}_{\mathrm{c}}=10.6 & \text { for std } 14 \times 18 \times 0.011 \text { screen } \& C d=0.6 \\
\operatorname{Re}_{\mathrm{wdw}}=\left(\sim 1.5^{*} 12 / 0.011\right)^{*}(10.6)=1,7345>10^{4} .
\end{array}
$$

Thus Re is not $>10^{4}$ when $\mathrm{w}<\sim 1 \mathrm{ft} / \mathrm{sec}$. But this is when the pressure drop starts to be dominated by the screen, so the orifice drop accuracy is not so important.

Solving for $\Delta p$,

$$
\Delta \mathrm{p}=\frac{1}{\mathrm{C}_{\mathrm{d}}^{2}} \frac{\rho \mathrm{w}^{2}}{2 \mathrm{~g}_{\mathrm{c}}}=\frac{1}{\mathrm{C}_{\mathrm{d}}^{2}} \frac{\mathrm{~m}^{2}}{2 \mathrm{~g}_{\mathrm{c}} \rho \mathrm{~A}^{2}}
$$

Equation B- 7
Adding Equation B- 1 and Equation B- 7 gives the total pressure drop for a window and screen in series:

$$
\Delta p=\left(K+\frac{1}{C_{d}^{2}}\right) \frac{m^{2}}{2 g_{c} \rho A^{2}}
$$

Equation $B-8$
Solving for mass flow rate,

$$
\mathrm{m}=\mathrm{AC}_{\mathrm{d}}\left(\frac{1}{1+\mathrm{C}_{\mathrm{d}}^{2} \mathrm{~K}}\right)^{\frac{1}{2}}\left(2 \rho_{\mathrm{c}} \Delta \mathrm{p}\right)^{\frac{1}{2}}
$$

Equation B- 9
or,

$$
m=A C_{d} S\left(2 \rho g_{c} \Delta p\right)^{\frac{1}{2}}
$$

Equation B-10
where S is the ratio of the flow with a screen to the flow rate without a screen, as a function of velocity $w$, viscosity, and screen and window orifice parameters.

$$
S=\left(\frac{1}{1+C_{d}^{2} K}\right)^{\frac{1}{2}}=\left(\frac{1}{1+C_{d}^{2} \frac{25 \psi v}{w}}\right)^{\frac{1}{2}}
$$

Equation B-11
Equating Equation B-1 and Equation B- 7 shows that the velocity when the screen and window pressure drops are equal is given by:

$$
\mathrm{w}_{\mathrm{c}}=25 \psi v \mathrm{C}_{\mathrm{d}}^{2}
$$

Equation B-12
The corresponding Reynolds number is $R e_{c}=25 \psi d C_{d}^{2}$.
Substituting Equation $\mathrm{B}-12$ into Equation B - 11shows that for this condition,

$$
S_{c}=\frac{1}{\sqrt{2}}=0.707
$$

Equation B-13
Equation B- 12 and Equation B- 13 show that $w_{c}$, which, besides viscosity, only depends on the screen constant $\psi$ and window-orifice coefficient $C_{d}$, can be considered a "characteristic" velocity, the velocity at which the flow is reduced by ( $1-0.707$ ) ~ $29.3 \%$ by the addition of a screen to the window.

Figure $\mathrm{B}-2$ shows that the typical $\Delta \mathrm{p}$ vs. flow curves for a screen and an orifice separately, not in series. The curves cross at velocity $w_{c}$. To the left of $w_{c}$, the laminar flow pressure drop dominates the window orifice pressure drop; to the right the orifice pressure drop progressively dominates. Screens give a greater flow reduction at low wind speeds than at high wind speeds.

Figure B-2: Pressure vs. Flow Characteristics


In Figure $B-2$, to the left of $w_{c}$ the laminar-flow screen pressure drop is higher, and to the right the orifice pressure drop dominates.

Figure B-3 shows $S$ as a function of air velocity $w$ for two common screen sizes. For the Standard screen, $w_{c}$ is $1.9-\mathrm{ft} / \mathrm{sec}$. [At $\mathrm{w}=\sim 1-\mathrm{ft} / \mathrm{sec}$ taken as typical according to note $\# 1, S=0.6$, corresponding to a $40 \%$ reduction in flow].

Figure B-3: Standard Screen Flow Reduction
Flow Reduction for Std screen ( $14 \times 18$ Mesh, 0.011" dia)
and fine screen ( $20 \times 20$ mesh, 0.011 dia). Mesh $=$ wires per inch.


## Partial Derivatives for use in Airnet:

From Equation B- 4 and Equation B- 8,

$$
\Delta p=\left(\frac{25 \psi \rho A v}{m}+\frac{1}{C_{d}^{2}}\right) \frac{m^{2}}{2 g_{c} \rho A^{2}}
$$

Equation B-14
This can be written in the quadratic form for m :

$$
m^{2}+b m-a \Delta p=0
$$

Equation B-15
Where,

$$
\begin{aligned}
& a=2 g_{c} \rho C_{d}^{2} A^{2} \\
& b=25 \psi \rho v C_{d}^{2} A
\end{aligned}
$$

The single real root of the quadratic Equation B-15 gives the mass flow rate through a screen in series with a window-orifice as a function of screen and window properties and overall pressure drop:

$$
m=\frac{b}{2}\left\{\left(1+\frac{4 a \Delta p}{b^{2}}\right)^{\frac{1}{2}}-1\right\}
$$

Equation B-16
If $\Delta p$ is taken as $\Delta p=P_{1}-P_{2}$, then the partial derivative of $m$ with respect to $P_{1}$ is

$$
\begin{aligned}
& \frac{\partial m}{\partial P_{1}}=\frac{a}{b \sqrt{1+\frac{4 a \Delta p}{b^{2}}}} \\
& \frac{\partial m}{\partial P_{2}}=-\frac{\partial m}{\partial P_{1}}
\end{aligned}
$$

Equation B-17

Equation B-18

These derivatives are needed in the Newton-Raphson procedure. The derivatives Equation $\mathrm{B}-17$ and Equation $\mathrm{B}-18$ do not become unbounded when $\Delta p=0$, as does the orifice Equation $\mathrm{B}-6$, so that no special treatment is needed near $\Delta p=0$.

But the derivatives do become a little peculiar near zero $\Delta p$ as shown in Figure B-4 and Figure $B-5$. The value of $\frac{a}{b} \approx 25$, and $\frac{4 a}{b^{2}} \approx 88$ for thse plots for the standard screen of Figure B-3. It is possible this could cause problems in the N-R method, but testing AirNet with this type of element is the easiest way to find out.

Figure B-4: For Small $\Delta p$


Figure B-5: For Large $\Delta p$


## Appendix C. Exact Longwave Radiation Model

Figure C-1 shows the standard Heat Transfer Engineering method of determining the long wavelength radiation exchange between black-body surfaces at uniform temperatures (Oppenheim(1956), Mills(1992)). The areas need not be equal, or symmetrically disposed, but are drawn that way for simplicity. The surfaces are assumed to be isothermal, and each surfaces temperature node is connected to all other surface temperature nodes via conductances $h_{b} A_{i} F_{i j}$.

The following methodology is referred to as the "exact" solution in the discussions of Section 1.6. However, it is recognized that it still is an idealization. For instance, surfaces are generally not isothermal. Although the heat transfer $q_{i j}$ [Btu/hr], of Equation $\mathrm{C}-2$ is accurate if surfaces $i$ and $j$ are isothermal, the local surface heat transfer $q_{i j}^{\prime}\left[\mathrm{Btu} / \mathrm{hr}-\mathrm{ft}^{2}\right]$ on the surfaces is nonuniform because the local view factors are different than the integrated value $F_{i j}$. For example, if the two surfaces are connected along a common edge, then near the edge $q_{i j}^{\prime}$ will be higher than the average $\frac{q_{i j}}{A_{i}}$, which will tend to change the temperatures of each wall near the edge faster than away from the edge. For the same reason, the radiation intensities are also non-uniform over a surface, which affects the accuracy of the treatment of the emissivity effects by the Oppenheim surface conductance term, which assumes uniform irradiation.

From the Stefan-Boltzmann equation, the net heat transfer rate between surfaces $i$ and $j$ is:

$$
q_{i j}=h_{b} A_{i} F_{i j}\left(T_{i}-T_{j}\right)
$$

Equation C- 1
where,

$$
\begin{aligned}
& h_{b}=4 \sigma \bar{T}^{3} \\
& \sigma=0.1714 \times 10^{-8} \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}^{2}-\mathrm{R}^{4} \text {, the Stefan-Boltzmann constant. } \\
& \bar{T}^{3} \approx \frac{T_{i}+T_{j}}{2} ; \text { degrees } \mathrm{R} .
\end{aligned}
$$

The $F_{i j}$ term is the standard view factor, equal to the fraction of radiation leaving surface $i$ that is intercepted by surface $j . F_{i j}$ depends on the size, shape, separation, and orientation of the surfaces, and at worst requires a double integration. Reciprocity requires that $A_{i} F_{i j}=A_{j} F_{j i}$.

Equation ( $\mathrm{C}-1$ ) is in the linearized form of the Stefan-Boltzmann equation, where for small temperature differences, $\left(T_{i}^{4}-T_{j}^{4}\right)$ is approximated by $4 \bar{T}^{3}\left(T_{i}-T_{j}\right)$.

Figure C-1: View-Factor Method's Radiant Network for Black-Body Surfaces


Figure C-2 shows the Figure C-1 black surface case extended to handle diffuse gray surfaces ( $\varepsilon=\mathrm{a}=$ constant over temperature range of interest) with emissivities $\varepsilon_{i}$, by adding the Oppenheim radiant surface conductances $\frac{A_{i} \varepsilon_{i}}{1-\varepsilon_{i}}$ between the surface temperature nodes and the black body network. (Figure C-2 also represents the unlinearized Stefan-Boltzmann circuit if the surface temperatures are replaced by the emissive power of the surfaces. In this case the surface radiosities are the potentials at the floating nodes.)

Figure C-2: View-Factor Method's Network for Grey Surfaces


By dissolving the radiosity nodes using Y-delta transformations, Figure $\mathrm{C}-2$ converts into Figure C-3 showing the same circuit form as the black surface circuit of Figure C-1. The transformation provides the conductances $A_{i} F_{i j}^{\prime}$ implicit in the conductances of Figure C2.

Figure C-3: View-Factor Method's Network for Grey Surfaces Reduced to Star Network


T3
$F_{i j}^{\prime}$ are the 'radiant interchange factors'. As with the black surfaces view factors, reciprocity holds: $A_{i} F_{i j}^{\prime}=A_{j} F_{j i}^{\prime}$. The net heat transfer between surface $i$ and $j$ (both directly and via reflections from other surfaces) is given by:

$$
q_{i j}=h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right)
$$

Equation C- 2
The total net heat transfer from surface $i$ (i.e., the radiosity minus the irradiation for the un-linearized circuit) is given by summing Equation C-2 for all the surfaces seen by surface $i, j \neq i$ :

$$
q_{i}=\sum_{j=1}^{n} h_{b} A_{i} F_{i j}^{\prime}\left(T_{i}-T_{j}\right)
$$

Equation C- 3
The above methodology is referred to as the "exact" solution in the discussion of Section 1.6.1. However, as discussed by Carroll, it is recognized that it is still an idealization. For instance, surfaces are generally not isothermal. Although the heat transfer, $q_{i j}[\mathrm{Btu} / \mathrm{hr}]$, of Equation $\mathrm{C}-2$ is accurate if surfaces $i$ and $j$ are isothermal, the local surface heat transfer $q_{i j}^{\prime} q^{\prime}\left[\mathrm{Btu} / \mathrm{hr}-\mathrm{ft}^{2}\right]$ on the surfaces is nonuniform because the local view factors are different than the integrated value $F_{i j}$. For example, if the two surfaces are connected along a common edge, then near the edge $q_{i j}^{\prime} q^{\prime}$ will be higher than the average $\frac{q_{i j}}{A_{i}}$, which will tend to change the temperatures of each wall near the edge faster than away from the edge. For the same reason, the radiation intensities are also non-uniform over a surface, which affects the accuracy of the treatment of the emissivity effects by the Oppenheim surface conductance term, which assumes uniform irradiation.

## Appendix D. Determining the Form of the Self-weighting Term $\mathrm{F}_{\mathrm{i}}$

Consider a flat black surface of area $A_{1}$ and temperature $T_{1}$ viewing the rest of the room of area $A_{s}$ and surface temperature $T_{s}$, with the view factor $F_{1 s}=1$. By Equation C- 1 of C , the net q from surface $A_{1}$ is given by:

$$
q_{1}=h_{b} A_{1} F_{1 s}\left(T_{1}-T_{s}\right)=h_{b} A_{1}\left(T_{1}-T_{s}\right)
$$

Equation D- 1
With Carroll's model applied to this geometry,

$$
q_{1}=h_{b} A_{1} F_{1}\left(T_{1}-T_{r}\right)
$$

Equation D- 2
where

$$
T_{r}=\frac{A_{1} F_{1} T_{1}+A_{s} F_{s} T_{s}}{A_{1} F_{1}+A_{s} F_{s}}
$$

Equating Equation D-1 and Equation D- 2 and solving for $F_{1}$ gives:

$$
F_{1}=\frac{1}{1-\frac{A_{1} F_{1}}{A_{1} F_{1}+A_{s} F_{s}}}
$$

Equation D- 4
The net heat transfer rate from surface 1 is $q_{1}=h_{b} A_{1} F_{1}\left(T_{1}-T_{r}\right)$, with similar expressions for $F_{s}$ and $q_{s}$. This is then generalized to the form of Equation 78 of Section1.6.1.

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[^0]:    ${ }^{1}$ TDV during hours when there is a net generation (i.e., the load is negative) is adjusted to account for net energy metering rules.

