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Plume Vertical Velocity Assessment for the Air Cooled Condensers

Palmdale Energy Project

Palmdale, California

Submitted to

California Energy Commission

Submitted by

Palmdale Energy, LLC

Prepared by

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Introduction

Palmdale Energy, LLC is proposing to develop the Palmdale Energy Project (PEP), located near the Palmdale Airport. The combined-cycle project will utilize two (2) Siemens SCC6-5000F natural gas-fired combustion turbine generators (CTG) and two (2) heat recovery steam generators (HRSG) with supplemental duct firing and a 32 cell air cooled condenser (ACC). The PEP site will be located on an approximately 50-acre undeveloped parcel west of the northwest corner of U.S. Air Force Plant 42. An analysis of the ACC plume characteristics was prepared based on the vertical plume-averaged velocities as described below.

Atmospheric Dynamics, Inc. (ADI) has prepared screening level plume vertical velocity assessments based on the calm wind Spillane methodology outlined in the *"Aviation Safety and Buoyant Plumes"* paper (Peter Best, et. al., presented at the Clean Air Conference, Newcastle, New South Wales, Australia, 2003). This methodology is also recognized as a screening tool for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in *"AC 139-5(1) Plume Rise Assessments (CASA, 2012)".*

The aim of this screening assessment is to conservatively determine the potential for turbulence generated by the ACC waste heat exhaust plumes. Part 139.370 of the Australian Civil Aviation Safety Regulations (1998, 2004) provides that CASA may determine that plume velocities in excess of 4.3 m/s is or will be a potential hazard to aircraft operations. The *Manual of Aviation Meteorology* (Australian Bureau of Meteorology 2003) defines severe turbulence as a vertical wind gust velocity in excess of 10.6 m/s. The assumed critical velocity used as a CEC significance threshold is 4.3 meters per second (m/s) but it should be noted that the basis of this CASA derived threshold has been lost in antiquity and that CASA no longer relies on the 1998 and 2004 regulations that established this critical threshold other than to note that a more rigorous analysis, which includes site specific meteorology, should be used if the 4.3 m/s screening threshold is exceeded. The screening method uses absolute worst-case assumptions of calm winds and neutral atmospheric conditions for the entire vertical extent of the plume to determine these worst-case impacts. It should be noted that these results are extremely conservative in that these worst-case conditions typically only occur during a few hours each year.

The Spillane methodology is generally applied to a limited number of plume source geometry's (turbines, power plant boilers, etc.) with the stacks arranged linearly (in a single straight-line) and separated by distances that typically exceed the individual stack diameters. For this assessment, a conservative assumption was made in order to use the Spillane methodology on an atypical ACC plume configuration which is made up of 32 plumes or cells arranged on a two dimensional surface. Here, the methodology, as described below, assumed all operating ACC cells were merged into a single equivalent ACC cell with an effective diameter based on the combined diameters of all operating cells. In other words, a single large cell was assumed to initially describe the release parameters of the ACC.

Screening Methodology and Vertical Plume Velocity Calculations for ACC

The ACC is comprised of 32 individual cells, arranged along four rows of eight cells each in 4 x 8 matrix. Thus, the 32 cells or radiators are arranged along two axis of direction producing a two dimensional plane in both the x and y directions. ACC stack parameter data (plume velocity, plume temperature) was provided by Siemens and the ACC manufacturer. The ACC will utilize variable speed fans. Additionally, the number of fans that are operational are dependent upon ambient temperature and plant load. For all ambient conditions, plant operation was assumed to be at full load. Thus, during cold winter conditions, only 10 fans would be operational at low speeds. During annual average conditions, up to 16 fans would be operational while during hot summer days, all 32 fans would be on at maximum speed. This data is summarized in Table 1.

Table 1 ACC Stack Characteristics for Vertical Plume Velocity Analysis

Case #	1	2	3
Ambient Temp (°F)*	23	64	98
Number of ACC Cells in Use*	10	16	32
Heat Rejection (MW/hr)*	447.24	447.38	445.28
Exhaust Flow Rate (lb/hr)*	5.12E+07	7.79E+07	1.51E+08
Cell Exit Temperature (°F)*	146.1	145.2	140.1
Cell Height (ft)*	130	130	130
Individual Cell Diameter (ft)*	36.09	36.09	36.09
Effective Stack Diameter (ft)**	114.12	144.36	204.15
Stack Exit Velocity (ft/s)*	3.18	5.24	10.83

*ACC stack data provided by Siemens
** Calculated value based on the cell diameter of 36 feet to the square of the number of operating cells or for example, Case #1: $D_{eff} = 36.09 * \sqrt{10}$
Note: The exit velocities from the ACC are always less than the critical velocity threshold of 4.3 m/s.

The Spillane methodology was originally developed to treat multiple individual stacks that are arranged along a linear x or y direction but not both directions at once. The ten to thirty-two 11-meter diameter radiator cells (depending upon operating case number) are arranged in the 4 x 8 are separated by between 13.1 meters in the x direction (east-west) and 14.5 meters in the y direction (north-south) in a center-to-center distance. Thus, the stack separations are about the same as the stack diameter. The ability of the Spillane method is limited to a single projection along a single axis and is not suitable for the ACC geometry. Therefore, the Spillane methodology for a single cell, based on the effective plume diameter was calculated for the number of cells in use for each ambient temperature. So for the cold day Case #1, the effective single plume diameter would be based on ten cells while during the annual temperature Case #2, the effective diameter would be based on 16 cells and during the summer Case #3 operation, the effective diameter would be based on 32 cells. The effective diameter for the single cell for each of the three ambient temperatures are presented in Table 1. The plume velocities were then calculated using the Spillane methodology for a single effective diameter or merged cell.

Results

Screening level vertical plume velocity assessments were made for the range of ambient temperatures with calm winds and neutral atmospheric conditions for the three cases presented in Table 1: 10 cells, 16 cells and 32 cells, each case with the calculated effective diameters as shown in the table.

The ACC exit temperature for the three ambient cases are similar to each other and are based on the plant at 100 percent load. Thus, the total heat rejection is similar for each case. However, the effective stack diameters (based on the number of cells in use) and exit velocities decrease significantly for cooler ambient temperatures as the ability to transfer waste heat to the atmosphere is increased for cooler atmospheric temperatures. Thus, the use of variable speed fans and the ability to cycle each fan based on the cooling needs of the plant allow for the decreased number of fans that are operational during cooler weather (and the decreased fan speed as well). The Spillane method was used for each of the three ambient conditions with the results presented in Table 2 and the output from the calculation worksheet provided in Attachment A.

Table 2 ACC Vertical Plume Velocity Analysis Results for Reference Height			
Case #	1	2	3
Ambient Temp (°F)	23	64	98
Single Plume Results (m/s):			
at 1500-feet agl	3.21	3.54	2.94
Maximum Velocity above 1500-feet agl	3.21	3.56	4.06

From these results and for each ambient condition, the vertical plume velocities are less than the significance value of 4.3 m/s for all heights through 1500 feet-agl and above for the ACC. These cases also represent the worst-case conditions of calm winds at all levels of a neutral atmosphere.

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the ACC always remain in the light turbulence category and below the significance level of 4.3 m/s at all heights above 1500' agl. In reality, even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear. In reality, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Attachment A
Spillane Method Plume Velocity Calculations

SINGLE Plume Average Vertical Velocities for Effective Diameter of 10 ACC Cells at 23F - Palmdale Energy Project								
	"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	268.15	Kelvins	23.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	Single Cell Values
Stack Height h_s	39.62	meters	130.0	feet	λ	1.11	10	#Cells
Effective Stack Diameter D	34.79	meters	114.12	feet	λ_o	~1.0	11.00	Diam(m)
Stack Velocity V_{exit}	0.97	m/s	3.18	ft/sec				
Volumetric Flow	921.82	cu.m/sec	1,953,077	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	336.54	Kelvins	146.1	°F				
Initial Stack Buoyancy Flux F_o	584.95	m^4/s^3	123.1	°F Delta.T	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow(g/π)($1-\theta_a/\theta_s$)			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	217.407	meters*	713.3	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	257.027	meters	**843.3**	feet				"
Vertical Velocity V_{jet}	0.485	m/s	1.59	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	69.570	meters	228.2	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a		**Solutions in Table Below**			$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	23.343	meters*	76.6	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	62.963	meters	206.6	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.8926
Vertical Velocity V		**Solutions in Table Below**			$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a			Sect.2.1(6)
Product $(Va)_o$	15.059	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height		**1,500.0**	**feet**	457.2	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	417.580	meters*	1370.0	feet*				
Plume Top-Hat Diameter 2a'	126.156	meters	413.9	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**3.206**	**m/s**	10.52	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**2.15**	**m/s plume-averaged vertical velocity**					
Find Height above Stack z_{crit}	1725.314	meters	5660.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	1,764.934	meters	**5790.5**	feet	for $V=V_{crit}$(m/s) using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=		-1724.3600
					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=			64856436
								http://www.1728.org/cubic.htm
							gives the real solution x = z-zv =	1701.9703
							or z(m) =	1725.314
Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase:							z(ft) =	5660.5
Height (feet)	**(meters)**	**Plume**	**Vert.**	**Plume**				
above ground	**above stack**	**Radius(m)**	**Vel(m/s)**	**Temp(K)**				
Top of jet = 843.3	***217.42***	***34.785***	***0.49***			**Spillane Equations:**		
1450.0	402.34	60.639	3.22	269.25		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a		
1500.0	417.58	63.078	3.21	269.17		$a = 0.16(z-z_v)$		
1550.0	432.82	65.516	3.19	269.10		$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$		
1600.0	448.06	67.955	3.17	269.04				
1650.0	463.30	70.393	3.15	268.98				
1700.0	478.54	72.831	3.14	268.93				
1750.0	493.78	75.270	3.12	268.89				
1800.0	509.02	77.708	3.10	268.84				
1850.0	524.26	80.147	3.08	268.81				
1900.0	539.50	82.585	3.06	268.77				
1950.0	554.74	85.023	3.03	268.74				
2000.0	569.98	87.462	3.01	268.71				
2100.0	600.46	92.339	2.98	268.66				
2200.0	630.94	97.215	2.94	268.62				
2300.0	661.42	102.092	2.90	268.58				
2400.0	691.90	106.969	2.86	268.55				
2500.0	722.38	111.846	2.83	268.52				
2600.0	752.86	116.723	2.79	268.49				
2700.0	783.34	121.599	2.76	268.47				
2800.0	813.82	126.476	2.73	268.45				
2900.0	844.30	131.353	2.70	268.43				
3000.0	874.78	136.230	2.67	268.41				
3500.0	1027.18	160.614	2.54	268.35				
4000.0	1179.58	184.998	2.43	268.31				
4500.0	1331.98	209.382	2.34	268.28				
5000.0	1484.38	233.766	2.26	268.26				

SINGLE Plume Average Vertical Velocities for Effective Diameter of 16 ACC Cells at 64F - Palmdale Energy Project

"***Aviation Safety and Buoyant Plumes***," **Peter Best, et. al.**

"***The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia***," **Dr. K.T. Spillane**

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	290.93	Kelvins	64.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	Single Cell Values
Stack Height h_s	39.62	meters	130.0	feet	λ	1.11	16	#Cells
Effective Stack Diameter D	44.00	meters	144.36	feet	λ_o	~1.0	11.00	Diam(m)
Stack Velocity V_{exit}	1.60	m/s	5.24	ft/sec				
Volumetric Flow	2,432.85	cu.m/sec	5,145,957	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	336.04	Kelvins	145.2	°F				
Initial Stack Buoyancy Flux F_o	1,019.80	m^4/s^3	81.2	°F Delta.T	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	275.000	meters*	902.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	314.620	meters	**1032.2**	feet				"
Vertical Velocity V_{jet}	0.800	m/s	2.62	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	88.000	meters	288.7	feet	$2a_{jet}$ = 2D		Conservation of momentum	"

Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase

Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:

Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	19.123	meters*	62.7	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	58.743	meters	192.7	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}=$	0.9305
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o\,[\,(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	Sect.2.1(6)
Product $(Va)_o$	32.752	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$	

Solve for plume-averaged vertical velocity at height	**1,500.0**	**feet**	457.2	meters above ground ($z'+h_s$)		
Gives the following Height above Stack z'	417.580	meters*	1370.0	feet*		
Plume Top-Hat Diameter 2a'	127.506	meters	418.3	feet	$2a'=2*0.16(z'-z_v)$	Sect.2/Eq.6
Vertical Velocity V	**3.536**	**m/s**	11.60	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$	Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**2.15**	**m/s plume-averaged vertical velocity**				
Find Height above Stack z_{crit}	3003.348	meters	9853.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)	
Height above Ground $z_{crit}+h_s$	3,042.968	meters	**9983.5**	feet	for $V=V_{crit}$(m/s) using the cubic equation $ax^3+bx^2+cx+d=0$, where	
					a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)=$	-3006.2294
					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)=$	195964112
					http://www.1728.org/cubic.htm	
					gives the real solution x = z-zv =	2984.2248
					or z(m) =	3003.348

Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase: z(ft) = 9853.5

Height (feet) above ground	(meters) above stack	Plume Radius(m)	Vert. Vel(m/s)	Plume Temp(K)	
Top of jet = 1032.2	***274.99***	***44.000***	***0.80***		**Spillane Equations:**
1450.0	402.34	61.315	3.51	292.79	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a
1500.0	417.58	63.753	3.54	292.64	$a = 0.16(z-z_v)$
1550.0	432.82	66.192	3.55	292.51	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$
1600.0	448.06	68.630	3.56	292.40	
1650.0	463.30	71.068	3.56	292.30	
1700.0	478.54	73.507	3.56	292.21	
1750.0	493.78	75.945	3.55	292.13	
1800.0	509.02	78.384	3.54	292.06	
1850.0	524.26	80.822	3.53	291.99	
1900.0	539.50	83.260	3.52	291.94	
1950.0	554.74	85.699	3.51	291.88	
2000.0	569.98	88.137	3.49	291.83	
2100.0	600.46	93.014	3.46	291.75	
2200.0	630.94	97.891	3.43	291.68	
2300.0	661.42	102.768	3.40	291.61	
2400.0	691.90	107.644	3.36	291.56	
2500.0	722.38	112.521	3.33	291.51	
2600.0	752.86	117.398	3.30	291.47	
2700.0	783.34	122.275	3.26	291.43	
2800.0	813.82	127.152	3.23	291.40	
2900.0	844.30	132.028	3.20	291.37	
3000.0	874.78	136.905	3.17	291.34	
3500.0	1027.18	161.289	3.03	291.24	
4000.0	1179.58	185.673	2.90	291.18	
4500.0	1331.98	210.057	2.80	291.13	
5000.0	1484.38	234.441	2.70	291.10	

SINGLE Plume Average Vertical Velocities for Effective Diameter of 32 ACC Cells at 98F - Palmdale Energy Project

"***Aviation Safety and Buoyant Plumes***," **Peter Best, et. al.**

"***The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia***," **Dr. K.T. Spillane**

Ambient Conditions:				**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	309.82	Kelvins	98.0	°F	0.3048 meters/feet	
Plume Exit Conditions:				Gravity g	9.81 m/s^2	Single Cell Values
Stack Height h_s	39.62	meters	130.0	feet	λ 1.11	32 #Cells
Effective Stack Diameter D	62.23	meters	204.15	feet	λ_o ~1.0	11.00 Diam(m)
Stack Velocity V_{exit}	3.30	m/s	10.83	ft/sec		
Volumetric Flow	10,035.50	cu.m/sec	21,270,042	ACFM	$\pi V_{exit} D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	333.21	Kelvins	140.1	°F		
Initial Stack Buoyancy Flux F_o	2,199.74	m^4/s^3	42.1	°F Delta.T	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	388.909	meters*	1275.9	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	428.529	meters	**1405.9**	feet		"
Vertical Velocity V_{jet}	1.650	m/s	5.41	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	124.451	meters	408.3	feet	$2a_{jet}$ = 2D Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	13.898	meters*	45.6	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	53.518	meters	175.6	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9643
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	Sect.2.1(6)
Product $(Va)_o$	99.003	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$	
Solve for plume-averaged vertical velocity at height	**1,500.0**	**feet**	457.2		meters above ground ($z'+h_s$)	
Gives the following Height above Stack z'	417.580	meters*	1370.0	feet*		
Plume Top-Hat Diameter 2a'	129.178	meters	423.8	feet	$2a'=2*0.16(z'-z_v)$	Sect.2/Eq.6
Vertical Velocity V	**2.942**	**m/s**	9.65	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$	Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**2.15**	**m/s plume-averaged vertical velocity**				
Find Height above Stack z_{crit}	6477.137	meters	21250.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)	
Height above Ground $z_{crit}+h_s$	6,516.757	meters	**21380.4**	feet	for $V=V_{crit}$(m/s) using the cubic equation $ax^3+bx^2+cx+d=0$, where	
					a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=	-6484.4989
					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=	888095619
					http://www.1728.org/cubic.htm	
					gives the real solution x = z-zv =	6463.2391
					or z(m) =	6477.137

Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase: z(ft) = 21250.5

Height (feet) above ground	(meters) above stack	Plume Radius(m)	Vert. Vel(m/s)	Plume Temp(K)	
Top of jet = 1405.9	***388.90***	***62.225***	***1.65***		**Spillane Equations:**
1450.0	402.34	62.151	2.48	315.70	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$
1500.0	417.58	64.589	2.94	314.41	$a = 0.16(z-z_v)$
1550.0	432.82	67.027	3.23	313.70	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$
1600.0	448.06	69.466	3.44	313.22	
1650.0	463.30	71.904	3.59	312.86	
1700.0	478.54	74.343	3.70	312.58	
1750.0	493.78	76.781	3.79	312.34	
1800.0	509.02	79.219	3.86	312.15	
1850.0	524.26	81.658	3.91	311.98	
1900.0	539.50	84.096	3.95	311.84	
1950.0	554.74	86.535	3.99	311.71	
2000.0	569.98	88.973	4.01	311.60	
2100.0	600.46	93.850	4.04	311.40	
2200.0	630.94	98.727	4.06	311.25	
2300.0	661.42	103.603	4.06	311.11	
2400.0	691.90	108.480	4.06	311.00	
2500.0	722.38	113.357	4.04	310.91	
2600.0	752.86	118.234	4.03	310.82	
2700.0	783.34	123.111	4.01	310.75	
2800.0	813.82	127.987	3.99	310.68	
2900.0	844.30	132.864	3.96	310.63	
3000.0	874.78	137.741	3.94	310.57	
3500.0	1027.18	162.125	3.81	310.38	
4000.0	1179.58	186.509	3.68	310.26	
4500.0	1331.98	210.893	3.56	310.18	
5000.0	1484.38	235.27[illegible]	3.45	310.12	