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Vertical Plume Velocity Assessment

Willow Rock Energy Storage Center

Rosamond, California

Submitted to

California Energy Commission

Submitted by

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Introduction

This report presents the evaluation of the Willow Rock Energy Storage Center (WRESC) project and its potential to generate vertical turbulence from exhaust plumes from the three (3) Koehler 2.5-megawatt (MW) diesel engines, four (4) air turbine exhaust stacks, and the three (3) air cooled heat exchangers which are part of the facility's Thermal Management System (TMS). This analysis can be used to assess whether there may be any potential effects on airport/aircraft operations from exhaust plumes associated with WRESC operations. This report is based upon an analysis prepared by Atmospheric Dynamics, Inc. in furtherance of the Supplemental Application for Certification (SAFC) for the WRESC pending before the California Energy Commission (CEC)

Using stack parameter data and employing the screening model previously used by the Commission, an analysis of the potential plume characteristics from the routine operations of the air turbines, diesel engines and heat exchangers on vertical winds was prepared and compared to the CEC Staff's exhaust plume velocity criteria of 5.3 meters per second (m/s) for the average vertical plume velocities as described below.

Atmospheric Dynamics, Inc. (ADI) prepared a screening level plume vertical velocity assessment which are based on the calm wind Spillane methodology outlined in the *"Aviation Safety and Buoyant Plumes"* paper (Peter Best, et. al., presented at the Clean Air Conference, Newcastle, New South Wales, Australia, 2003). This methodology is also recognized by Australian authorities as a screening tool for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in *"AC 139-5(1) Plume Rise Assessments (CASA, 2012)"*.

The aim of this screening model assessment is to conservatively determine the potential for turbulence generated by the turbines, diesel engines and heat exchanger waste heat exhaust plumes. Part 139.370 of the Australian Civil Aviation Safety Regulations (1998, 2004) provides that CASA may determine that plume velocities in excess of 4.3 m/s is or will be a potential hazard to aircraft operations. The *Manual of Aviation Meteorology* (Australian Bureau of Meteorology 2003) defines severe turbulence as a vertical wind gust velocity in excess of 10.6 m/s. The assumed critical vertical velocity used in the CEC Staff's analysis is 5.3 meters per second* (m/s).

It should be noted that the basis of the original CASA derived threshold of 4.3 m/s has been lost and that CASA no longer relies on the 1998 and 2004 regulations that established this critical threshold, other than to note that an additional more rigorous analysis, which includes site specific meteorology, should be used if the 4.3 m/s and 10.6 m/s screening thresholds are exceeded at altitudes that have the potential to affect aviation. Further, this screening method uses absolute worst-case assumptions of calm winds and neutral atmospheric conditions for the entire vertical extent of the plume to determine these worst-case impacts, even though these worst-case conditions typically only occur during a few hours each year. Accordingly, the screening tool presents results that are extremely "conservative" (i.e., the modeling tool tends to overstate potential impacts compared to actual impacts).

The Spillane methodology is generally applied to a limited number of plume source geometry's (turbines, power plant boilers, etc.) with the stacks arranged linearly (in a single straight-line) and separated by distances that typically exceed the individual stack diameters. For the turbines and diesel engines, this screening tool assumption was maintained. The minimum separation of the turbine exhaust stacks are 98 meters and while all four (4) turbines will be operational during the same time, the stack separation exceeds the calculated plume radius with height. For the diesel

*For the Puente Power Project (Docket#15-AFC-01, TN#213674, 9/15/2016), "CEC staff … concluded that an average velocity of 5.3 m/s is the appropriate velocity … [for a plume velocity threshold]." The CEC staff "Plume Background Threshold" attached to the docketed document concludes with "…[CEC] staff will use 10.6 m/s peak vertical plume velocity as the new threshold. The altitude at which a plume would have a peak vertical velocity of 10.6 m/s would be the same altitude at which a plume would have an average vertical velocity of half that, 5.3 m/s."

engines, only one (1) engine stack was modeled consistent with the normal operational testing schedule of the emergency generator engines where only one (1) engine is tested at any one time.

There will be three (3) heat exchangers utilized for the project design. On the plot plan submitted with the Supplemental Application for Certification (SAFC), all three heat exchangers are depicted as being identical in configuration. Hydrostor has further advanced the design of the heat exchangers since the submittal of the SAFC, and this thermal plume analysis reflects this most up-to-date heat exchanger information. The updated heat exchanger design will reflect a smaller, more compact configuration than depicted on the original plot plan, another conservative modeling assumption. All three heat exchangers will be located in the same general area within the facility but will occupy a smaller footprint. No other changes to the plot plan are necessitated by this minor design refinement. The updated heat exchanger design information relevant to this analysis is described below.

Two (2) of the heat exchangers in the southern portion of the project layout each consist of 36 fans, each with a 3.96-meter diameter, organized in a 12x3 arrangement. Each of these two (2) heat exchangers are separated by 36 meters. The single heat exchanger in the northern portion of the project layout consists of a single heat exchanger with 42 fans, each with a 3.96-meter diameter, organized in a 14 x 3 arrangement. For the heat exchangers, a conservative assumption was made in order to use the Spillane methodology. Here, the methodology, as described below, assumed that all 36 fans for each heat exchanger were merged into a single stack with an effective diameter based on the combined fan area of all 36 cells. In other words, a single stack was assumed to initially describe the release parameters of the combined heat exchanger fans, when in fact two exchangers are located to the south and one to the north, a conservative assumption for the screening tool inputs. During the winter months when fewer fans are operational, the effective diameter was then based on the anticipated number of operating fans given ambient temperature and metrological data. The effective stack diameter is an appropriate modeling assumption for each individual heat exchanger based on the close proximity and arrangement of the fans.

Screening Methodology and Vertical Plume Velocity Calculations

The Spillane methodology is based on worst-case calm wind neutral stability conditions to assess the average plume vertical velocity as a function of height. The methodology is based on well-verified laboratory and theoretical treatments of the rise and spread of a buoyant jet rise, both into a still ambient environment and into a light crosswind. This treatment covers in detail the initial dynamics of the plume as it exits the stack and the entrainment of ambient air into the plume as it rises directly above the stack. In addition to providing clarifications and algebraic solutions to the Spillane methodology, the 2003 Peter Best paper provides additional methodologies that also consider the enhancement of vertical velocities that may occur if the plumes from multiple identical stacks merge and form a higher buoyancy combined plume (referred to here as the enhanced Spillane methodology).

The vertical plume assessment will involve several stages of development. For individual plumes, the stages are:

(a) In the first stage very close to the stack exit, the high plume momentum will result in a short section in which the conditions at the center of the plume are relatively unaffected by ambient and plume buoyancy conditions. This jet phase extends from the stack exit to approximately a distance of 6.25 D above the stack (where D is the stack diameter) in calm conditions. At the end of this stage, the plume-averaged vertical velocity has decreased to half of the stack exit velocity, with a corresponding increase, or doubling, in effective plume diameter.

(b) In the second stage, the plume responds to differences between ambient and plume buoyancy conditions, with much cooler and less turbulent ambient air being entrained into the plume from the outside regions of the plume towards the plume centerline. The momentum and buoyancy of the plume significantly influences plume rise and subsequently the dilution of the stack exhaust to decrease plume vertical velocities. This dilution is very sensitive to ambient wind speed, so the calm wind conditions considered here are extremely conservative.

(c) In the third stage of plume development, plume rise is due entirely to the buoyancy of the plume and continues from some distance until there is an equalization of turbulence conditions within and outside the plume. This final rise is often only achieved at considerable heights/distances from the stack where the effective average vertical velocity is then close to zero. Since there is very little turbulence and near-zero vertical velocities, this stage of plume development is usually not considered for this type of analysis.

In the second stage of development, the analytical solution of the governing equations under these conditions is given by:

$$a = 0.16(z - z_v)$$
$$V = \{(Va)o^{3} + 0.12Fo\ [\ (z - zv)^2 - (6.25D - zv\ )^2]\}^{1/3} / a$$

Where the subscript 'o' refers to values of the parameters at the stack outlet and the variables are:

a	plume radius (m)
V	average vertical velocity (m/s)
z	height above stack top (m)
z_v	virtual source height (m)
D	stack diameter (m)
F_o	buoyancy flux evaluated at the stack outlet (m^4s^{-3})

These are the two primary equations governing the growth of a single plume in the second stage of development under neutral calm wind conditions. Additional equations governing the first stage of single plume development as well as the interaction of multiple plumes in the second stage of development are discussed in detail in the Best paper.

For multiple stacks in the enhanced Spillane methodology, the equations governing the second stage are calculated from the point when the plumes begin to merge until they are fully merged. The plume merging begins at the height where the plume diameters equal the stack (or fan) separations, and the plumes are fully merged at the height where the plume diameters are equal to 2d(N-1)/2 for three or more stacks or 2d for two stacks. At the fully merged height, the merged plume diameter and velocity is enhanced by the fourth root of the number of stacks. Above the fully merged plume height, the enhanced plume diameter and plume velocities follow the regular equations given for the second stage. Below the fully merged plume height for the merging phase, plume velocities are linearly interpolated by height from the single plume velocity at the height where the plumes begin to merge to the enhanced plume velocity at the fully merged plume height.

For the analysis, two ambient conditions were considered: 33.0°F, the minimum monthly mean of daily minimum temperatures, and 99.0°F, the maximum monthly mean of daily maximum temperatures for the blended data sets from Edwards Air Force Base and Mojave Air & Space Port (*"Climatology of the United States No. 81 – Monthly Station Normals of Temperature, Precipitation, and Heating and Cooling Degree Days 1971-2000 – California"*, February 2002, and *"Climatology of the United States No 20 – Monthly Station Climate Summaries, 1971-2000 – California"*, February 2004)

Vertical Plume Velocity Calculations for the Air Turbines

The WRESC is comprised of four individual air turbine stacks. Turbine stack parameter data (plume exit velocity, plume exit temperature and stack exit diameter) were provided by Kiewit, the project engineer. Four (4) turbine stacks will be constructed with the minimum lateral distance between the turbine stacks at 98 meters. Stack parameter data for the turbines are summarized in Table 1.

Table 1
Turbine Stack Characteristics for Vertical Plume Velocity Analysis

Ambient Temperature (°F)	33.0	99.0
Stack Diameter (m)	3.62	3.62
Exhaust Velocity (m/s)*	30.70	30.70
Exhaust Temperature (K)*	284.15	284.15
Stack Release Height (m)	30.5	30.5
Stack Buoyancy Flux (m^4/s^3)	36.25	-91.04*
*Negative buoyancy as stack temperature is less than ambient temperature for the summer case		

Screening level vertical plume velocity assessments were made for two ambient temperatures with calm winds and neutral atmospheric conditions for the cases presented in Table 1 which are based on the maximum velocity expected during normal operations.

The results based on the two ambient conditions are presented in Table 2, and the output from the calculation spreadsheet provided in Attachment A. The initial jet phase extends to a height of about 172.4 feet above grade level (ft-agl) for both cases. After the jet phase, plume temperature buoyancy characteristics modeled in the Spillane methodology cause a uniform decrease in plume-averaged vertical velocities, with the critical plume-averaged vertical velocity of 5.3 m/s occurring at about 320 ft-agl for the winter case and 306 ft-agl for the summer case.

Table 2
Turbine Vertical Plume Velocity Analysis Results for Reference Height

Ambient Temperature (°F)	33.0	99.0
Single Plume Results:		
Plume-Averaged Vertical Velocity at 500 feet-agl (m/s)	3.13	1.69
Height of 5.3 m/s Plume-Averaged Vertical Velocity (feet-agl)	319.5	305.8

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the turbines always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above about 320 ft-agl. The maximum plume radius where the plumes from two (2) or more turbine stacks would start to merge would be 48 meters, but at this stage of the plume growth, the vertical velocity is less than 2.0 m/s, well below the 5.3 m/s significance threshold. Based on this, plume merging from multiple turbine stacks was not assessed.

Vertical Plume Velocity Calculations for the 2.5 MW Diesel Engines

The WRESC is comprised of three (3) individual 2.5 MW diesel engines and one (1) small diesel emergency fire pump. The small diesel emergency generator was not assessed as it would have much smaller plume vertical velocities than the 2.5 MW engines. The 2.5 MW generator stack parameter data (plume exit velocity, plume exit temperature and stack exit diameter) were provided by Koehler. Only one (1) engine will be tested during any one hour. While the engines will be tested at minimum loads, the 100 percent load case was nevertheless utilized for the worst-case plume analysis. The stack data is summarized in Table 3.

Table 3
Koehler Diesel Stack Characteristics for Vertical Plume Velocity Analysis

Ambient Temperature (°F)*	33.0	99.0
Stack Diameter (m)	0.4453	0.4453
Exhaust Velocity (m/s)*	59.0	59.0
Exhaust Temperature (K)*	763.15	763.15
Stack Release Height (m)	22.93	22.93
Stack Buoyancy Flux (m^4/s^3)	18.4016	17.0233
*Stack data provided by Koehler at 100% load		

Screening level vertical plume velocity assessments were made for two ambient temperatures with calm winds and neutral atmospheric conditions for the cases presented in Table 3 which are based on 100 percent load.

The results based on the two ambient conditions are presented in Table 4 and the output from the calculation spreadsheet provided in Attachment A. The initial jet phase extends to a height of about 84 feet above grade level (ft-agl) for both cases. After the jet phase, plume temperature buoyancy characteristics modeled in the Spillane methodology cause a uniform decrease in plume-averaged vertical velocities, with the critical plume-averaged vertical velocity of 5.3 m/s occurring at about 114 ft-agl for the winter case and 115 ft-agl for the summer case.

Table 4
Diesel Engine Vertical Plume Velocity Analysis Results for Reference Height

Ambient Temperature (°F)	33.0	99.0
Single Plume Results:		
Plume-Averaged Vertical Velocity at 300 feet-agl (m/s)	2.03	1.99
Height of 5.3 m/s Plume-Averaged Vertical Velocity (feet-agl)	113.7	114.9

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the diesel engines always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above about 115 ft-agl.

Vertical Plume Velocity Calculations for the Heat Exchangers

The three (3) heat exchangers will be comprised of the following: two 36-cell systems each at 56.6 meters in length and 18.6 meters in width (in the southern portion of the facility arrangement) and a 16-cell system at 65.9 meters in length and 18.6 meters in width (in the northern portion of the facility arrangement). The 36 cell heat exchangers are arranged with 12 cells along the longer building length by three (3) along the shorter building width. There is a 36-meter separation between the two 36-cell heat exchangers. The northern 42-cell heat exchanger is arranged in a 14 x 3 pattern with 14 cells along the length and three (3) cells along with width. There are no other heat exchangers within several hundred meters of this system.

Based on the groupings of heat exchangers and the number of operational fans during the summer and winter months, the screening tool analyses were based on the following:

Two 36 cell heat exchangers

- 72 operational fans during the summer months
- 22 operational fans during the winter months
- 3.96 meter cell diameter

One 42 cell heat exchanger

- 42 operational fans during the summer months
- 8 operational fans during the winter month
- 3.96 meter cell diameter

The merging of plumes between adjacent heat exchangers was based on the two 36 heat exchanger arrangements for both summer and winter conditions. Heat exchanger stack parameter data (exit velocity and temperature) were provided by the applicant. An effective stack diameter was calculated based on the number of operating fans (cells).   The heat exchangers will utilize single speed fans and the number of fans that are operational are dependent upon ambient temperature and plant load. However, to be conservative, the fans/cells were assumed to be operating at full load during the summer and winter periods. This data are summarized in Table 5 for the same ambient temperatures used for the turbine and engine analyses.

Table 5
Heat Exchanger Stack Characteristics for Vertical Plume Velocity Analysis

Ambient Temperature (°F)*	33.0	99.0
12 x 3 Heat Exchanger		
Effective Stack Diameter (m)**	13.142	23.774
Exhaust Velocity (m/s)*	9.91	9.91
Exhaust Temperature (K)*	312.04	327.04
Stack Release Height (m)	9.91	9.91
Stack Buoyancy Flux (m^4/s^3)	466.75	633.84
14 x 3 Heat Exchanger		
Effective Stack Diameter (m)**	11.21	25.68
Exhaust Velocity (m/s)*	7.83	7.83
Exhaust Temperature (K)*	312.04	318.15
Stack Release Height (m)	8.36	8.36
Stack Buoyancy Flux (m^4/s^3)	296.29	309.68

*Heat exchanger stack data provided by the applicant. Velocity based on ACFM per fan multiplied by the number of operating fans.
** As an example the calculated value based on the cell diameter multiplied by the square of the number of operating cells, or for Case 2 Summer for the 12x3 heat exchanger, D_{eff} = 3.96"*$\sqrt{36}$ = 23.774

The Spillane methodology was originally developed to treat multiple individual stacks that are arranged along a linear x or y direction, but not both directions at once, with stack separations much greater than the stack diameters, typical of boilers/turbines at large power plants. As noted above, the heat exchangers are arranged in a 12 x 3 and 14 x 3 pattern. For the individual heat exchangers, all operating cells were merged into a single effective stack, thus each temperature case with different operational fans have a different effective stack diameter. In other words, each individual heat exchanger was modeled as a single merged stack based on the number of operational cells. For the merging of plumes between the two 12 x 3 heat exchangers, the enhanced Spillane methodology was based on calculating the total merging height for the linear y direction (separation) of 36 meters between each of the 12 x 3 heat exchangers. Thus, the merged plume analysis was based on the two effective stack diameters for winter and summer conditions. The largest grouping of 72 cells (12 x 3 x 2 heat exchangers) during the summer were considered in the calculation of vertical velocity plume enhancement (both at and above the totally merged height, and for the interpolation down to the plume touching height with the effective single stack diameter of each heat exchanger based on the combined 36 cells each. The winter case used a grouping of 22 cells (11 from each heat exchanger) with the effective diameter based on the combined 11 cells for each unit.

Screening level vertical plume velocity assessments were made for the same ambient temperatures with calm winds and neutral atmospheric conditions as was done for the emergency generator engines. The results are presented in Table 6 and the output from the calculation spreadsheets are provided in Attachment A.

For the 12 x 3 heat exchangers, the initial jet phase extends to a height of about 302 ft-agl for the winter case and 520 ft-agl for the summer case. The critical plume-averaged vertical velocity of 5.3 m/s occurs in the jet phase at about 253 ft-agl for winter and 431.5 ft-agl for summer. The plumes touch (begin to merge) at about 414 ft-agl and are fully merged at about 788 ft-agl for both cases. Under the enhanced Spillane methodology, the merged plume-averaged vertical velocities never approach 5.3 m/s (either above the totally merged height or when interpolated down to the touching height).

For the 14 x 3 heat exchanger, the initial jet phase extends to a height of about 257.2 ft-agl for the winter case and 554 ft-agl for the summer case. The critical plume-averaged vertical velocity of 5.3 m/s occurs in the jet phase at about 176 ft-agl for winter and at 368 ft-agl for summer. Plume merging with adjacent heat exchangers was not assumed as noted above.

Table 6
Heat Exchanger Vertical Plume Velocity Analysis Results for Reference Height

Ambient Temperature (°F)	33.0	99.0
12 x 3 Heat Exchanger		
Single Plume Results:		
Height of 5.3 m/s Plume-Averaged Vertical Velocity (Within the Jet Phase, feet-agl)	253.0	431.5
Merged Plume Results:		
Plume-Averaged Vertical Velocity at 600 feet-agl (m/s)	4.20	4.27
14 x 3 Heat Exchanger		
Single Plume Results:		
Height of 5.3 m/s Plume-Averaged Vertical Velocity (Within the Jet Phase, feet-agl)	124.8	376.1
Merged Plume Results:		
Plume-Averaged Vertical Velocity at 600 feet-agl (m/s)	NA	NA

From these results and for each ambient condition, the vertical plume velocities are less than the threshold value of 5.3 m/s for all heights above about 432 ft-agl for either type of the heat exchangers. The heights at which plume-averaged vertical velocities exceed 5.3 m/s only occur during the jet phase for all heat exchanger cases.

Conclusion

These modeled cases all represent worst-case conditions of calm winds at all vertical levels of a neutral atmosphere. These results indicate that mechanical and thermal turbulence levels due to the exhaust flow from the turbines, diesel engines or heat exchangers will always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above 432 ft-agl. Additionally, the plume radius for all assessments results in small plume diameters. The calculated vertical plume velocities of the WRESC conclude that none are expected to generate severe turbulence at altitudes for normal aircraft operations.

It should be further noted that even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities, so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. The calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following principal reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear when realistically, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Attachment A
Spillane Method Plume Velocity Calculations

SINGLE Plume Average Vertical Velocities for Single WRESC Turbine Exhaust - Summer Max*

***"Aviation Safety and Buoyant Plumes,"* Peter Best, et. al.**

"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume

***from a Gas-Turbine Power Station at Oakey, Queensland, Australia,"* Dr. K.T. Spillane**

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	273.71	Kelvins	33.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Maximum Stack Height h_s	30.48	meters	100	feet-inches	λ	1.11		
Stack Diameter D	3.6200	meters	142.5	inches	λ_o	~1.0		
Stack Velocity V_{exit}	30.70	m/s	100.72	ft/sec				
Volumetric Flow	315.97	cu.m/sec	669,497	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	284.15	Kelvins	52	°F				
Initial Stack Buoyancy Flux F_o	36.2508	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	22.625	meters*	74.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	53.105	meters	**174.2**	feet				"
Vertical Velocity V_{jet}	15.350	m/s	50.36	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	7.240	meters	23.8	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	0.420	meters*	1.4	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	30.900	meters	101.4	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9815
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	54.537	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height		**500.0**	**feet**	152.4	meters above ground (z'+h_s)			
Gives the following Height above Stack z'	121.920	meters*	400.0	feet*				
Plume Top-Hat Diameter 2a'	38.880	meters	127.6	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**3.125**	**m/s**	10.25	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**				**Critical VV > Top of Jet (Spillane)**	
Find Height above Stack z_{crit}	66.916	meters	219.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	97.396	meters	**319.5**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=		-7.1336
Interpolated Height of critical vertical velocity in Jet Phase:					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=			-262485.30
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet			gives the real solution x = z-zv =	66.4961
							or z(m/above stack) =	66.916
							z(ft/above ground) =	319.5

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 100.0	***0.00***	**1.810**	**30.70**			
110.0	3.05	2.054	28.63		**Jet Phase Eqs:**	**10 foot Intervals**
120.0	6.10	2.316	26.58		Linearly interpolated from Stack Rel.Ht to Top of Jet	
130.0	9.14	2.569	24.53		**Spillane Equations:**	
140.0	12.19	2.822	22.47		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
150.0	15.24	3.075	20.41		$a = 0.16(z-z_v)$	**20 foot Intervals**
160.0	18.29	3.328	18.35		$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
170.0	21.34	3.581	16.29			
Top of jet = 174.2	***22.62***	***3.620***	***15.35***			
180.0	24.38	3.834	14.23	277.63		
200.0	30.48	4.810	11.38	276.83		**Max<5.30 m/s**
220.0	36.58	5.785	9.50	276.29		
240.0	42.67	6.760	8.16	275.91		
260.0	48.77	7.736	7.16	275.62		
280.0	54.86	8.711	6.40	275.40		**50 foot Intervals**
300.0	60.96	9.686	5.79	275.22		
Spillane 5.3 m/s Height = 319.5	**66.92**	**10.639**	**5.30**	275.08		
350.0	76.20	12.125	4.70	274.90		
369.5	82.16	13.078	4.39	274.80		
389.5	88.25	14.053	4.12	274.72		
409.5	94.35	15.028	3.88	274.65		
429.5	100.44	16.004	3.68	274.58		**100 foot Intervals**
449.5	106.54	16.979	3.50	274.52		
469.5	112.64	17.955	3.34	274.47		
489.5	118.73	18.930	3.19	274.43		
509.5	124.83	19.905	3.07	274.39		
529.5	130.92	20.881	2.95	274.35		
549.5	137.02	21.856	2.85	274.31		
569.5	143.12	22.831	2.75	274.28		

*Summer Max = Monthly Mean of Maximum Daily Temperatures for 1971-2000 (Highest in July)

NOAA Sources: Climatography of the United States No.81 *"Monthly Station Normals of Temperatures, Precipitation, and Heating and Cooling Degree Days, 1971-2000 California"* and Climatography of the United States No. 20 *"Monthly Station Climate Summaries, 1971-2000 California"*

SINGLE Plume Average Vertical Velocities for Single WRESC Turbine Exhaust - Summer Max*

"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.

"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	310.37	Kelvins	99.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Maximum Stack Height h_s	30.48	meters	100	feet-inches	λ	1.11		
Stack Diameter D	3.6200	meters	142.5	inches	λ_o	~1.0		
Stack Velocity V_{exit}	30.70	m/s	100.72	ft/sec				
Volumetric Flow	315.97	cu.m/sec	669,497	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	284.15	Kelvins	52	°F				
Initial Stack Buoyancy Flux F_o	-91.0436	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	22.625	meters*	74.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	53.105	meters	**174.2**	feet				"
Vertical Velocity V_{jet}	15.350	m/s	50.36	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	7.240	meters	23.8	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	-1.021	meters*	-3.3	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	29.459	meters	96.7	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	1.0451
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	58.074	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height		**500.0**	**feet**	152.4	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	121.920	meters*	400.0	feet*				
Plume Top-Hat Diameter 2a'	39.341	meters	129.1	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**1.692**	**m/s**	5.55	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV > Top of Jet (Spillane)**
Find Height above Stack z_{crit}	62.681	meters	205.6	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	93.161	meters	**305.6**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=		17.9161
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=		-331204.12
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet			gives the real solution x = z-zv =	63.7022
							or z(m/above stack) =	62.681
							z(ft/above ground) =	305.6

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 100.0	***0.00***	**1.810**	**30.70**			
110.0	3.05	2.054	28.63		**Jet Phase Eqs:**	**10 foot Intervals**
120.0	6.10	2.374	26.59		Linearly interpolated from Stack Rel.Ht to Top of Jet	
130.0	9.14	2.656	24.54		**Spillane Equations:**	
140.0	12.19	2.937	22.48		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
150.0	15.24	3.219	20.43		$a = 0.16(z-z_v)$	**20 foot Intervals**
160.0	18.29	3.501	18.37		$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
170.0	21.34	3.783	16.32			
Top of jet = 174.2	***22.62***	***3.620***	***15.35***			
180.0	24.38	4.065	14.26	300.45		
200.0	30.48	5.040	11.43	302.32		**Max<5.30 m/s**
220.0	36.58	6.015	9.50	303.57		
240.0	42.67	6.991	8.09	304.46		
260.0	48.77	7.966	7.02	305.12		
280.0	54.86	8.942	6.17	305.63		**50 foot Intervals**
300.0	60.96	9.917	5.47	306.03		
Spillane 5.3 m/s Height = 305.6	**62.68**	**10.192**	**5.30**	306.12		
350.0	76.20	12.355	4.17	306.70		
355.6	77.92	12.631	4.05	306.75		
375.6	84.02	13.606	3.65	306.91		
395.6	90.11	14.581	3.30	307.04		
415.6	96.21	15.557	2.97	307.12		**100 foot Intervals**
435.6	102.31	16.532	2.66	307.16		
455.6	108.40	17.508	2.37	307.15		
475.6	114.50	18.483	2.07	307.07		
495.6	120.59	19.458	1.76	306.87		
515.6	126.69	20.434	1.41	306.39		
535.6	132.79	21.409	0.87	304.48		

SINGLE Plume Average Vertical Velocities for Single WRESC Large Emer.Gen Engine, 100% Load, and Maximum Stack Height - Winter Min*								
	"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	273.71	Kelvins	33.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Maximum Stack Height h_s	22.93	meters	75 3/12	feet-inches	λ	1.11		
Stack Diameter D	0.4453	meters	17.5	inches	λ_o	~1.0		
Stack Velocity V_{exit}	59.00	m/s	193.56	ft/sec				
Volumetric Flow	9.19	cu.m/sec	19,467	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	763.15	Kelvins	914	°F				
Initial Stack Buoyancy Flux F_o	18.4016	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	2.783	meters*	9.1	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	25.713	meters	**84.4**	feet				"
Vertical Velocity V_{jet}	29.500	m/s	96.78	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	0.891	meters	2.9	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	1.116	meters*	3.7	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	24.046	meters	78.9	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.5989
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a			Sect.2.1(6)
Product $(Va)_o$	7.867	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height	**300.0**	**feet**		91.44	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	68.510	meters*	224.8	feet*				
Plume Top-Hat Diameter 2a'	21.566	meters	70.8	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**2.031**	**m/s**	6.66	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV > Top of Jet (Spillane)**	
Find Height above Stack z_{crit}	11.733	meters	38.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	34.663	meters	**113.7**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=		-3.6212
Interpolated Height of critical vertical velocity in Jet Phase:					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=			-788.38
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet		gives the real solution x = z-zv =		10.6163
							or z(m/above stack) =	11.733
							z(ft/above ground) =	113.7
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet)	**(meters)**	**Plume**	**SingleStk**	**Plume**				
above ground	**above stack**	**Radius(m)**	**VertVel(m/s)**	**Temp(K)**				
Stack.Rel.Ht = 75.2	***0.00***	**0.223**	**59.00**					
80.0	1.45	0.338	43.65				**Jet Phase Eqs:**	**5 foot Intervals**
85.0	2.98	0.460	27.57				Linearly interpolated from Stack Rel.Ht to Top of Jet	
Top of jet = 84.4	***2.80***	***0.445***	***29.50***				**Spillane Equations:**	
90.0	4.50	0.542	14.71	370.24			$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
100.0	7.55	1.029	8.06	322.47			$a = 0.16(z-z_v)$	**10 foot Intervals**
110.0	10.60	1.517	5.79	304.96			$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Spillane 5.3 m/s Height = 113.7	**11.73**	**1.699**	**5.30**	300.96				
120.0	13.65	2.005	4.68	295.85				
130.0	16.69	2.492	4.03	290.34				
140.0	19.74	2.980	3.61	286.70				**Max<5.30 m/s**
150.0	22.79	3.468	3.31	284.17				
160.0	25.84	3.955	3.09	282.32				
170.0	28.89	4.443	2.92	280.94				
220.0	44.13	6.882	2.41	277.36				**50 foot Intervals**
270.0	59.37	9.320	2.14	275.95				
320.0	74.61	11.758	1.97	275.24				
370.0	89.85	14.197	1.84	274.83				
420.0	105.09	16.635	1.74	274.57				
470.0	120.33	19.074	1.66	274.40				
520.0	135.57	21.512	1.60	274.27				
620.0	166.05	26.389	1.49	274.11				**100 foot Intervals**
720.0	196.53	31.266	1.41	274.01				
820.0	227.01	36.142	1.34	273.95				
920.0	257.49	41.019	1.28	273.90				
1020.0	287.97	45.896	1.24	273.87				
1120.0	318.45	50.773	1.19	273.85				
1220.0	348.93	55.650	1.16	273.83				
1320.0	379.41	60.526	1.13	273.81				
1420.0	409.89	65.403	1.10	273.80				
1520.0	440.37	70.280	1.07	273.79				
1620.0	470.85	75.157	1.05	273.78				
1720.0	501.33	80.034	1.03	273.77				
1820.0	531.81	84.910	1.01	273.77				
1920.0	562.29	89.787	0.99	273.76				
2020.0	592.77	94.664	0.97	273.76				

SINGLE Plume Average Vertical Velocities for Single WRESC Large Emer.Gen Engine, 100% Load, and Maximum Stack Height - Summer Max*								
	"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	310.37	Kelvins	99.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Maximum Stack Height h_s	22.93	meters	75 3/12	feet-inches	λ	1.11		
Stack Diameter D	0.4453	meters	17.5	inches	λ_o	~1.0		
Stack Velocity V_{exit}	59.00	m/s	193.56	ft/sec				
Volumetric Flow	9.19	cu.m/sec	19,467	ACFM	$\pi V_{exit} D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	763.15	Kelvins	914	°F				
Initial Stack Buoyancy Flux F_o	17.0233	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	2.783	meters*	9.1	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	25.713	meters	**84.4**	feet				"
Vertical Velocity V_{jet}	29.500	m/s	96.78	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	0.891	meters	2.9	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	1.008	meters*	3.3	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	23.938	meters	78.5	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6377
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a			Sect.2.1(6)
Product $(Va)_o$	8.377	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height	**300.0**	**feet**		91.44	meters above ground $(z'+h_s)$			
Gives the following Height above Stack z'	68.510	meters*	224.8	feet*				
Plume Top-Hat Diameter 2a'	21.601	meters	70.9	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**1.987**	**m/s**	6.52	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV > Top of Jet (Spillane)**	
Find Height above Stack z_{crit}	12.103	meters	39.7	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	35.033	meters	**114.9**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where			
							a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=	-3.3499
Interpolated Height of critical vertical velocity in Jet Phase:							and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=	-953.45
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet			gives the real solution x = z-zv =	11.0951
							or z(m/above stack) =	12.103
							z(ft/above ground) =	114.9
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet)	**(meters)**	**Plume**	**SingleStk**	**Plume**				
above ground	**above stack**	**Radius(m)**	**VertVel(m/s)**	**Temp(K)**				
Stack.Rel.Ht = 75.2	***0.00***	**0.223**	**59.00**					
80.0	1.45	0.338	43.65				**Jet Phase Eqs:**	**5 foot Intervals**
85.0	2.98	0.460	27.57				Linearly interpolated from Stack Rel.Ht to Top of Jet	
Top of jet = 84.4	***2.80***	***0.445***	***29.50***				**Spillane Equations:**	
90.0	4.50	0.559	15.14	402.76			$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
100.0	7.55	1.047	8.36	358.13			$a = 0.16(z-z_v)$	**10 foot Intervals**
110.0	10.60	1.534	5.97	341.46			$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Spillane 5.3 m/s Height = 114.9	**12.10**	**1.775**	**5.30**	336.54				
120.0	13.65	2.022	4.79	332.70				
130.0	16.69	2.510	4.09	327.33				
140.0	19.74	2.997	3.64	323.74				**Max<5.30 m/s**
150.0	22.79	3.485	3.32	321.21				
160.0	25.84	3.973	3.08	319.35				
170.0	28.89	4.460	2.90	317.94				
220.0	44.13	6.899	2.37	314.24				**50 foot Intervals**
270.0	59.37	9.337	2.10	312.76				
320.0	74.61	11.776	1.92	312.01				
370.0	89.85	14.214	1.80	311.57				
420.0	105.09	16.652	1.70	311.30				
470.0	120.33	19.091	1.62	311.11				
520.0	135.57	21.529	1.56	310.98				
620.0	166.05	26.406	1.45	310.80				**100 foot Intervals**
720.0	196.53	31.283	1.37	310.70				
820.0	227.01	36.160	1.30	310.63				
920.0	257.49	41.036	1.25	310.58				
1020.0	287.97	45.913	1.20	310.54				
1120.0	318.45	50.790	1.16	310.52				
1220.0	348.93	55.667	1.13	310.50				
1320.0	379.41	60.544	1.10	310.48				
1420.0	409.89	65.420	1.07	310.47				
1520.0	440.37	70.297	1.04	310.45				
1620.0	470.85	75.174	1.02	310.45				
1720.0	501.33	80.051	1.00	310.44				
1820.0	531.81	84.928	0.98	310.43				
1920.0	562.29	89.804	0.96	310.43				
2020.0	592.77	94.681	0.94	310.42				

SINGLE/Approximated Plume Average Vertical Velocities for WRESC TMS - Winter Min*								
Based on 11 cells/heat exchanger. Calc' eff.diam for each heat exchanger with each fan at 13' ID (234,400 ACFM total for each fan). 11 fans	**"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.**							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	273.71	Kelvins	33.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Stack Height h_s	9.91	meters	32.50	feet	λ	1.11		
Merged Stack Diameter D	13.1417	meters	517.4	inches	λ_o	~1.0		
Stack Velocity V_{exit}	8.97	m/s	29.43	ft/sec	$4Vol/(60\pi D^2)$			
Individual Heat Exchanger Volumetric Flow	1,216.87	cu.m/sec	2,578,400	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	312.04	Kelvins	102.0	°F				
Initial Stack Buoyancy Flux F_o	466.7460	m^4/s^3	69.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Number of Heat Exchangers n	1			1.000	Multiple Stack Multiplication Factor ($n^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	82.136	meters*	269.5	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	92.042	meters	**302.0**	feet				"
Vertical Velocity V_{jet}	4.486	m/s	14.72	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	26.283	meters	86.2	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	5.210	meters*	17.1	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	15.116	meters	49.6	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9366
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	55.209	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Single Heat Exchanger Results:								
Solve for plume-averaged vertical velocity at height		**600.0**	**feet**	182.88	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	172.974	meters*	567.5	feet*				
Plume Top-Hat Diameter 2a'	53.685	meters	176.1	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**4.181**	**m/s**	13.72	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}{}^3 0.16^3)$=		-91.84896
Interpolated Height of critical vertical velocity in Jet Phase:					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}{}^3 0.16^3)$=			267567.83
Find Height above Stack z_{crit}	67.223	meters	220.5	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	77.129	meters	**253.0**	feet			gives the real solution x = z-zv =	-44.3269
							or z(m/above stack) =	-39.117
							z(ft/above ground) =	-95.8
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet)	**(meters)**	**Plume**	**SingleStk**	**Plume**				
above ground	**above stack**	**Radius(m)**	**VertVel(m/s)**	**Temp(K)**				
Stack.Rel.Ht = 32.5	**0.00**	**6.571**	**8.97**					
40.0	2.29	6.754	8.85				**Jet Phase Eqs:**	**20 ft Intervals**
60.0	8.38	7.241	8.51				Linearly interpolated from Stack Rel.Ht to Top of Jet	
80.0	14.48	7.729	8.18				**Spillane Equations:**	
100.0	20.57	8.217	7.85					
120.0	26.67	8.704	7.51				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
140.0	32.77	9.192	7.18				$a = 0.16(z-z_v)$	
160.0	38.86	9.680	6.85				$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
180.0	44.96	10.167	6.52				**CEC Staff Equation:**	
200.0	51.05	10.655	6.18				$V_{mp}=n^{0.25}V_{sp}$	
220.0	57.15	11.143	5.85				**Brigg's Equation:**	
Single Jet 5.3 m/s Height = 253.0	**67.22**	**11.949**	**5.30**				$V_{Brigg's} = (2/3) \times 1.6^{(3/2)} \times F_{mp}{}^{(1/2)} \times u^{(-1/2)} \times z^{(-1/2)}$	
260.0	69.34	12.118	5.18				where $F_{mp} = nF_{sp}$	
280.0	75.44	12.606	4.85					
Top of Single jet = 302.0	**82.14**	**13.142**	**4.49**					
350.0	96.77	14.650	4.60	287.35				**50 ft Intervals**
400.0	112.01	17.089	4.57	284.41			**Max<5.3 m/s**	
450.0	127.25	19.527	4.48	281.63				
500.0	142.49	21.965	4.38	279.89				
550.0	157.73	24.404	4.28	278.71				
600.0	172.97	26.842	4.18	277.86				
700.0	203.45	31.719	4.00	277.22				
800.0	233.93	36.596	3.84	276.34				
900.0	264.41	41.473	3.70	275.77				**100 ft Intervals**
1000.0	294.89	46.349	3.57	275.37				
1100.0	325.37	51.226	3.46	275.09				
1200.0	355.85	56.103	3.36	274.87				
1300.0	386.33	60.980	3.28	274.71				
1500.0	447.29	70.733	3.12	274.58				
2000.0	599.69	95.117	2.84	274.39				
2500.0	752.09	119.501	2.63	274.12				
3000.0	904.49	143.885	2.47	273.99				**500 ft Intervals**
3500.0	1056.89	168.269	2.35	273.92				
4000.0	1209.29	192.653	2.25	273.87				
4500.0	1361.69	217.037	2.16	273.84				

MERGED (along length) Plume Average Vertical Velocities for WRESC TMS using CEC Staff Methodology - Winter Min*

***"Aviation Safety and Buoyant Plumes,"* Peter Best, et. al.**

***"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia,"* Dr. K.T. Spillane**

Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	273.71	Kelvins	33.0	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	9.91	meters	32 6/12	feet-inches	λ 1.11	
Individual Stack Diameter D	13.141706	meters	517.4	inches	λ_o ~1.0	
Stack Velocity V_{exit}	8.97	m/s	29.43	ft/sec	$4Vol/(60\pi D^2)$	
Individual Volumetric Flow	1,216.87	cu.m/sec	2,578,400	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	312.04	Kelvins	102.0	°F		
Initial Stack Buoyancy Flux F_o	466.75	m^4/s^3	69.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Total Number of Stacks n	1					
Average Adjacent Stack Separation d	36.00	meters	118.1	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**	
Number of Stacks along Orientation N	2				**plume velocities increased by $N^{0.25}$ at the height where plumes**	
					fully merged (interp. below ht, single merged stack above ht)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	82.136	meters*	269.5	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	92.042	meters	**302.0**	feet		"
Vertical Velocity V_{jet}	**4.486**	m/s	14.72	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	26.283	meters	86.2	feet	$2a_{jet}$ = 2D — Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	5.210	meters*	17.1	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	15.116	meters	49.6	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9366
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o[(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	Sect.2.1(6)
Product $(Va)_o$	55.209	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$	
Plume Merging - Based on Single Plume Calculations where:						Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	36.000	meters	118.1	feet	$2a_{touch}$=d, (or a_{touch}=d/2)	
Height above Stack z_{touch}	117.710	meters*	386.2	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{touch}+h_s$	127.616	meters	**418.7**	feet		
Vertical Velocity V_{touch}	**4.540**	m/s	14.9	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o[(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**72.000**	meters	236.2	feet	$2a_{full}$=2d(N-1)/2, (or a_{full}=d(N-1)/2) **FOR 2 STACKS, $2a_{full}$=2d**	
Height above Stack z_{full}	230.210	meters*	755.3	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{full}+h_s$	240.116	meters	**787.8**	feet		
Vertical Velocity V_{full}	3.855	m/s	12.7	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o[(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$	
Product $(V^3a)_{full}$	2,062	m^4/s^3				
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):						
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height	
Revised Merged Plume **Radius** a_m	36.000	meters	118.1	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs	
Revised Merged Plume Velocity V_m	**3.855**	m/s	12.65	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs	
Revised Virtual Source Height z_{full}	230.210	meters*	755.3	feet*	Height above stack where Total Merging Occurs (shown above)	
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation	
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$	
Multiple Plume Calculations					for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height	**600.0**	**feet**	182.88		meters above ground ($z+h_s$)	
Gives the following Height above Stack z	172.974	meters*	567.5	feet*	LESS THAN TOP OF MERGING PHASE-INTERPOLATE	
Plume Top-Hat Radius a	#N/A	meters	#N/A	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**	
Vertical Velocity V	**4.203**	**m/s**	13.79	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**	
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**	
					V'=single plume values **if $z<z_{touch}$**	
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s**			BEFORE TOUCHING	**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	JET	meters	JET	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**	
Height above Ground $z_{crit}+h_s$	JET	meters	**JET**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**	

Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:

Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**Vert. Vel(m/s)**	*Single Plume Eqns (see Single Plume spreadsheet)*	
				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
				$a = 0.16(z-z_v)$	
Begin Merging (touch) = 418.7	***117.71***	***18.000***	***4.54***	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
440.0	124.21	#N/A	4.50	*Interpolated Layer Eqns*	**40 ft Intervals**
480.0	136.40	#N/A	4.43	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
520.0	148.59	#N/A	4.35		
560.0	160.78	#N/A	4.28		
600.0	172.97	#N/A	4.20		
640.0	185.17	#N/A	4.13		
680.0	197.36	#N/A	4.05		
720.0	209.55	#N/A	3.98		
760.0	221.74	#N/A	3.91	*Merged Plume Eqns*	
750.0	218.69	#N/A	3.92	$V=\{n(V^3a)_{full}/a\}^{1/3}$	**50 ft Intervals**
End Merging (full/mp) = 787.8	230.22	***36.000***	***3.85***	$a=a_m+0.16(z-z_{full})$	
800.0	233.93	36.596	3.83		
850.0	249.17	39.034	3.75		
900.0	264.41	41.473	3.68		
1000.0	294.89	46.349	3.54		**100 ft Intervals**
1100.0	325.37	51.226	3.43		
1200.0	355.85	56.103	3.32		
1300.0	386.33	60.980	3.23		
1400.0	416.81	65.857	3.15		
1500.0	447.29	70.733	3.08		
1600.0	477.77	75.610	3.01		
1700.0	508.25	80.487	2.95		
1800.0	538.73	85.364	2.89		**500 ft Intervals**
2000.0	599.69	95.117	2.79		
2500.0	752.09	119.501	2.58		
3000.0	904.49	143.885	2.43		
3500.0	1056.89	168.269	2.31		
4000.0	1209.29	192.653	2.20		
4500.0	1361.69	217.037	2.12		
5000.0	1514.09	241.421	2.04		

SINGLE/Approximated Plume Average Vertical Velocities for WRESC TMS - Summer Max*								
Based on 36 cells/heat exchanger. Calc' eff.diam for each heat exchanger with each fan at 13' ID (234,400 ACFM total for each fan). 36 fans	**"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.**							
	"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	310.37	Kelvins	99.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Stack Height h_s	9.91	meters	32.50	feet	λ	1.11		
Merged Stack Diameter D	23.7744	meters	936.0	inches	λ_o	~1.0		
Stack Velocity V_{exit}	8.97	m/s	29.43	ft/sec	$4Vol/(60\pi D^2)$			
Individual Heat Exchanger Volumetric Flow	3,982.48	cu.m/sec	8,438,400	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	327.04	Kelvins	129.0	°F				
Initial Stack Buoyancy Flux F_o	633.8407	m^4/s^3	30.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Number of Heat Exchangers n	1			1.000	Multiple Stack Multiplication Factor ($n^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	148.590	meters*	487.5	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	158.496	meters	**520.0**	feet				"
Vertical Velocity V_{jet}	4.486	m/s	14.72	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	47.549	meters	156.0	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	3.836	meters*	12.6	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	13.742	meters	45.1	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9742
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	103.888	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Heat Exchanger Results:								
Solve for plume-averaged vertical velocity at height		**600.0**	**feet**	182.88	meters above ground $(z'+h_s)$			
Gives the following Height above Stack z'	172.974	meters*	567.5	feet*				
Plume Top-Hat Diameter 2a'	54.124	meters	177.6	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**4.413**	**m/s**	14.48	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=		-124.73083
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=		774874.76
Find Height above Stack z_{crit}	121.610	meters	399.0	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	131.516	meters	**431.5**	feet			gives the real solution x = z-zv =	-64.0648
							or z(m/above stack) =	-60.229
							z(ft/above ground) =	-165.1
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet)	**(meters)**	**Plume**	**SingleStk**	**Plume**				
above ground	**above stack**	**Radius(m)**	**VertVel(m/s)**	**Temp(K)**				
Stack.Rel.Ht = 32.5	**0.00**	**11.887**	**8.97**					
40.0	2.29	12.070	8.90				**Jet Phase Eqs:**	**20 ft Intervals**
60.0	8.38	12.558	8.72				Linearly interpolated from Stack Rel.Ht to Top of Jet	
80.0	14.48	13.045	8.53				**Spillane Equations:**	
100.0	20.57	13.533	8.35					
120.0	26.67	14.021	8.17				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
140.0	32.77	14.508	7.98				$a = 0.16(z-z_v)$	
160.0	38.86	14.996	7.80				$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
180.0	44.96	15.484	7.61				**CEC Staff Equation:**	
200.0	51.05	15.972	7.43				$V_{mp}=n^{0.25}V_{sp}$	
220.0	57.15	16.459	7.25				**Brigg's Equation:**	
240.0	63.25	16.947	7.06				$V_{Brigg's} = (2/3) \times 1.6^{(3/2)} \times F_{mp}^{(1/2)} \times u^{(-1/2)} \times z^{(-1/2)}$	
260.0	69.34	17.435	6.88				where $F_{mp} = nF_{sp}$	
280.0	75.44	17.922	6.69					
300.0	81.53	18.410	6.51					
320.0	87.63	18.898	6.33					**50 ft Intervals**
340.0	93.73	19.385	6.14				**Max<5.3 m/s**	
360.0	99.82	19.873	5.96					
380.0	105.92	20.361	5.77					
400.0	112.01	20.848	5.59					
420.0	118.11	21.336	5.41					
Single Jet 5.3 m/s Height = 431.5	**121.61**	**21.616**	**5.30**					
440.0	124.21	21.824	5.22					
460.0	130.30	22.311	5.04					**100 ft Intervals**
480.0	136.40	22.799	4.85					
500.0	142.49	23.287	4.67					
Top of Single jet = 520.0	**148.59**	**23.774**	**4.49**					
600.0	172.97	27.062	4.41	316.79				
1000.0	294.89	46.569	3.90	315.41				
1500.0	447.29	70.953	3.44	312.30				
2000.0	599.69	95.337	3.13	311.31				
2500.0	752.09	119.721	2.91	310.94				**500 ft Intervals**
3000.0	904.49	144.105	2.74	310.76				
3500.0	1056.89	168.489	2.60	310.66				
4000.0	1209.29	192.873	2.48	310.59				

MERGED (along length) Plume Average Vertical Velocities for WRESC TMS - Summer Max*								
	"Aviation Safety and Buoyant Plumes," Peter Best, et. al.							
	"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged							
		Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia," Dr. K.T. Spillane						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	310.37	Kelvins	99.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Stack Height h_s	9.91	meters	32 6/12	feet-inches	λ	1.11		
Individual Stack Diameter D	23.7744	meters	936.0	inches	λ_o	~1.0		
Stack Velocity V_{exit}	8.97	m/s	29.43	ft/sec	$4Vol/(60\pi D^2)$			
Individual Volumetric Flow	3,982.48	cu.m/sec	8,438,400	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	327.04	Kelvins	129.0	°F				
Initial Stack Buoyancy Flux F_o	633.84	m^4/s^3	30.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Total Number of Stacks n	1							
Average Adjacent Stack Separation d	36.00	meters	118.1	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**			
Number of Stacks along Orientation N	2				**plume velocities increased by $N^{0.25}$ at the height where plumes**			
					fully merged (interp. below ht, single merged stack above ht)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	148.590	meters*	487.5	feet*	$z_{jet} = 6.25D$, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	158.496	meters	**520.0**	feet				"
Vertical Velocity V_{jet}	**4.486**	m/s	14.72	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	47.549	meters	156.0	feet	$2a_{jet} = 2D$		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	3.836	meters*	12.6	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	13.742	meters	45.1	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9742
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			Sect.2.1(6)
Product $(Va)_o$	103.888	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$			
Plume Merging - Based on Single Plume Calculations where:								Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	36.000	meters	118.1	feet	$2a_{touch}=d$, (or $a_{touch}=d/2$)			
Height above Stack z_{touch}	116.336	meters*	381.7	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top			
Height above Ground $z_{touch}+h_s$	126.242	meters	**414.2**	feet				
Vertical Velocity V_{touch}	**4.380**	m/s	14.4	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$			
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	72.000	meters	236.2	feet	$2a_{full}=2d(N-1)/2$, (or $a_{full}=d(N-1)/2$) **FOR 2 STACKS, $2a_{full}=2d$**			
Height above Stack z_{full}	228.836	meters*	750.8	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top			
Height above Ground $z_{full}+h_s$	238.742	meters	**783.3**	feet				
Vertical Velocity V_{full}	4.168	m/s	13.7	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o [(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$			
Product $(V^3a)_{full}$	2,607	m^4/s^3						
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):								
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height			
Revised Merged Plume **Radius** a_m	36.000	meters	118.1	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs			
Revised Merged Plume Velocity V_m	**4.168**	m/s	13.67	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs			
Revised Virtual Source Height z_{full}	228.836	meters*	750.8	feet*	Height above stack where Total Merging Occurs (shown above)			
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation			
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$			
Multiple Plume Calculations							for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height		**600.0**	**feet**	182.88	meters above ground $(z+h_s)$			
Gives the following Height above Stack z	172.974	meters*	567.5	feet*	LESS THAN TOP OF MERGING PHASE-INTERPOLATE			
Plume Top-Hat Radius a	#N/A	meters	#N/A	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**			
Vertical Velocity V	**4.273**	**m/s**	14.02	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**			
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**			
					V'=single plume values **if $z<z_{touch}$**			
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s**		BEFORE TOUCHING		**Critical VV < Top of Jet**	
Find Height above Stack z_{crit}	JET	meters	JET	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**			
Height above Ground $z_{crit}+h_s$	JET	meters	**JET**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**			
Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:					*Single Plume Eqns (see Single Plume spreadsheet)*			
Height (feet)	**(meters)**	**Plume**	**Vert.**		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$			
above ground	**above stack**	**Radius(m)**	**Vel(m/s)**		$a = 0.16(z-z_v)$			
Begin Merging (touch) = 414.2	***116.34***	***18.000***	***4.38***		$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$			
440.0	124.21	#N/A	4.37		*Interpolated Layer Eqns*			**40 ft Intervals**
480.0	136.40	#N/A	4.34		$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$			
520.0	148.59	#N/A	4.32					
560.0	160.78	#N/A	4.30					
600.0	172.97	#N/A	4.27					
640.0	185.17	#N/A	4.25					
680.0	197.36	#N/A	4.23					
720.0	209.55	#N/A	4.20					
760.0	221.74	#N/A	4.18		*Merged Plume Eqns*			
End Merging (full/mp) = 783.3	***228.84***	***36.000***	***4.17***		$V=\{n(V^3a)_{full}/a\}^{1/3}$			**50 ft Intervals**
800.0	233.93	36.816	4.14		$a=a_m+0.16(z-z_{full})$			
850.0	249.17	39.254	4.05					
900.0	264.41	41.692	3.97					
950.0	279.65	44.131	3.89					
1000.0	294.89	46.569	3.83					**100 ft Intervals**
1100.0	325.37	51.446	3.70					
1200.0	355.85	56.323	3.59					
1300.0	386.33	61.200	3.49					
1400.0	416.81	66.076	3.40					
1500.0	447.29	70.953	3.32					
1600.0	477.77	75.830	3.25					
1700.0	508.25	80.707	3.18					
1800.0	538.73	85.584	3.12					**500 ft Intervals**
2000.0	599.69	95.337	3.01					
2500.0	752.09	119.721	2.79					
3000.0	904.49	144.105	2.62					
3500.0	1056.89	168.489	2.49					
4000.0	1209.29	192.873	2.38					
4500.0	1361.69	217.257	2.29					
5000.0	1514.09	241.641	2.21					

SINGLE/Approximated Plume Average Vertical Velocities for WRESC AT-ACHE - Summer Max*								
Based on 1 cell/heat exchanger. Calc' eff.diam for each heat exchanger with each fan at 13' ID (204,600 ACFM total for each fan). 42 fans	**"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.**							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	273.71	Kelvins	33.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Stack Height h_s	8.36	meters	27.42	feet	λ	1.11		
Merged Stack Diameter D	11.2075	meters	441.2	inches	λ_o	~1.0		
Stack Velocity V_{exit}	7.83	m/s	25.69	ft/sec	$4Vol/(60\pi D^2)$			
Individual Heat Exchanger Volumetric Flow	772.48	cu.m/sec	1,636,800	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	312.04	Kelvins	102.0	°F				
Initial Stack Buoyancy Flux F_o	296.2961	m^4/s^3	69.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Number of Heat Exchangers n	1			1.000	Multiple Stack Multiplication Factor ($n^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	70.047	meters*	229.8	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	78.404	meters	**257.2**	feet				"
Vertical Velocity V_{jet}	3.915	m/s	12.85	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	22.415	meters	73.5	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	4.443	meters*	14.6	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	12.801	meters	42.0	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9366
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a			Sect.2.1(6)
Product $(Va)_o$	41.096	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Single Heat Exchanger Results:								
Solve for plume-averaged vertical velocity at height		**600.0**	**feet**	182.88	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	174.522	meters*	572.6	feet*				
Plume Top-Hat Diameter 2a'	54.425	meters	178.6	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**3.606**	**m/s**	11.83	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=		-58.30685
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=		137127.08
Find Height above Stack z_{crit}	45.271	meters	148.5	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	53.629	meters	**175.9**	feet		gives the real solution x = z-zv =		-37.7777
							or z(m/above stack) =	-33.335
							z(ft/above ground) =	-81.9
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet)	**(meters)**	**Plume**	**SingleStk**	**Plume**				
above ground	**above stack**	**Radius(m)**	**VertVel(m/s)**	**Temp(K)**				
Stack.Rel.Ht = 27.4	**0.00**	**5.604**	**7.83**					
40.0	3.83	5.910	7.62				**Jet Phase Eqs:**	**20 ft Intervals**
60.0	9.93	6.398	7.28				Linearly interpolated from Stack Rel.Ht to Top of Jet	
80.0	16.03	6.886	6.93				**Spillane Equations:**	
100.0	22.12	7.374	6.59					
120.0	28.22	7.861	6.25				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
140.0	34.31	8.349	5.91				$a = 0.16(z-z_v)$	
160.0	40.41	8.837	5.57				$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Single Jet 5.3 m/s Height = 175.9	**45.27**	**9.225**	**5.30**				**CEC Staff Equation:**	
180.0	46.51	11.711	6.89				$V_{mp}=n^{0.25}V_{sp}$	
200.0	52.60	12.512	6.76				**Brigg's Equation:**	
220.0	58.70	13.313	6.64				$V_{Brigg's}$ = (2/3) x $1.6^{(3/2)}$ x $F_{mp}^{(1/2)}$ x $u^{(-1/2)}$ x $z^{(-1/2)}$	
240.0	64.79	14.113	6.52				where $F_{mp} = nF_{sp}$	
Top of Single jet = 257.2	**70.05**	**11.207**	**3.92**					
300.0	83.08	12.582	4.09	287.35				
350.0	98.32	15.021	4.08	284.07				**50 ft Intervals**
400.0	113.56	17.459	4.00	281.00			**Max<5.3 m/s**	
450.0	128.80	19.898	3.90	279.22				
500.0	144.04	22.336	3.80	278.06				
550.0	159.28	24.774	3.70	277.25				
600.0	174.52	27.213	3.61	276.67				
650.0	189.76	29.651	3.52	276.22				
700.0	205.00	32.090	3.44	275.88				
750.0	220.24	34.528	3.37	275.60				**100 ft Intervals**
800.0	235.48	36.966	3.30	275.38				
850.0	250.72	39.405	3.24	275.20				
900.0	265.96	41.843	3.18	275.05				
950.0	281.20	44.282	3.12	274.92				
1000.0	296.44	46.720	3.07	274.81				
1050.0	311.68	49.158	3.02	274.71				
1100.0	326.92	51.597	2.97	274.63				
1150.0	342.16	54.035	2.93	274.56				**500 ft Intervals**
1200.0	357.40	56.474	2.89	274.49				
1250.0	372.64	58.912	2.85	274.44				
1300.0	387.88	61.350	2.81	274.39				

SINGLE/Approximated Plume Average Vertical Velocities for WRESC AT-ACHE - Summer Max*								
Based on 1 cell/heat exchanger. Calc' eff.diam for each heat exchanger with each fan at 13' ID (204,600 ACFM total for each fan). 42 fans	**"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.**							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
Ambient Potential Temp θ_a	310.37	Kelvins	99.0	°F		0.3048	meters/feet	
Plume Exit Conditions:								
Stack Height h_s	8.36	meters	27.42	feet	Gravity g	9.81	m/s^2	
Merged Stack Diameter D	25.6791	meters	1011.0	inches	λ	1.11		
Stack Velocity V_{exit}	7.83	m/s	25.69	ft/sec	$4Vol/(60\pi D^2)$	λ_o ~1.0		
Individual Heat Exchanger Volumetric Flow	4,055.54	cu.m/sec	8,593,200	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	318.15	Kelvins	113.0	°F				
Initial Stack Buoyancy Flux F_o	309.6816	m^4/s^3	14.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)			
Number of Heat Exchangers n	1			1.000	Multiple Stack Multiplication Factor ($n^{0.25}$)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	160.495	meters*	526.6	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	168.852	meters	**554.0**	feet				"
Vertical Velocity V_{jet}	3.915	m/s	12.85	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	51.358	meters	168.5	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	1.975	meters*	6.5	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	10.332	meters	33.9	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9877
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a			Sect.2.1(6)
Product $(Va)_o$	99.305	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Single Heat Exchanger Results:								
Solve for plume-averaged vertical velocity at height		**600.0**	**feet**	182.88	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	174.522	meters*	572.6	feet*				
Plume Top-Hat Diameter 2a'	55.215	meters	181.2	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**3.797**	**m/s**	12.46	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV < Top of Jet**
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where			
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=		-60.94093
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=		-74564.33
Find Height above Stack z_{crit}	103.735	meters	340.3	feet				http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	112.092	meters	**367.8**	feet			gives the real solution x = z-zv =	74.4084
							or z(m/above stack) =	76.383
							z(ft/above ground) =	278.0
Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:								
Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**SingleStk VertVel(m/s)**	**Plume Temp(K)**				
Stack.Rel.Ht = 27.4	**0.00**	**12.840**	**7.83**					
40.0	3.83	13.146	7.74				**Jet Phase Eqs:**	**20 ft Intervals**
60.0	9.93	13.634	7.59				Linearly interpolated from Stack Rel.Ht to Top of Jet	
80.0	16.03	14.122	7.44				**Spillane Equations:**	
100.0	22.12	14.609	7.29					
120.0	28.22	15.097	7.14				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
140.0	34.31	15.585	6.99				$a = 0.16(z-z_v)$	
160.0	40.41	16.072	6.84				$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
180.0	46.51	16.560	6.70				**CEC Staff Equation:**	
200.0	52.60	17.048	6.55				$V_{mp}=n^{0.25}V_{sp}$	
220.0	58.70	17.535	6.40				**Brigg's Equation:**	
240.0	64.79	18.023	6.25				$V_{Brigg's}$ = (2/3) x $1.6^{(3/2)}$ x $F_{mp}^{(1/2)}$ x $u^{(-1/2)}$ x $z^{(-1/2)}$	
260.0	70.89	18.511	6.10				where $F_{mp} = nF_{sp}$	
280.0	76.99	18.998	5.95					
300.0	83.08	19.486	5.80					
320.0	89.18	19.974	5.66					**50 ft Intervals**
340.0	95.27	20.462	5.51				**Max<5.3 m/s**	
Single Jet 5.3 m/s Height = 367.8	**103.73**	**21.138**	**5.30**					
380.0	107.47	21.437	5.21					
400.0	113.56	21.925	5.06					
420.0	119.66	22.412	4.91					
440.0	125.75	22.900	4.76					
460.0	131.85	23.388	4.61					
Top of Single jet = 554.0	**160.49**	**25.679**	**3.92**					**100 ft Intervals**
600.0	174.52	27.608	3.80	313.45				
700.0	205.00	32.484	3.58	313.12				
800.0	235.48	37.361	3.41	312.47				
900.0	265.96	42.238	3.27	312.04				
1000.0	296.44	47.115	3.15	311.73				
1500.0	448.84	71.499	2.73	311.51				
2000.0	601.24	95.883	2.48	310.94				
2500.0	753.64	120.267	2.30	310.72				**500 ft Intervals**
3000.0	906.04	144.651	2.16	310.61				
3500.0	1058.44	169.035	2.05	310.55				
4000.0	1210.84	193.419	1.96	310.51				