TETRA TECH EC, INC.

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Mr. Eric Solorio
California Energy Commission
Docket No. 11-AFC-3
1516 9th St.
Sacramento, CA 95814

Cogentrix Quail Brush Generation Project - Docket Number 11-AFC-3, Plume Vertical Velocity Assessment

Docket Clerk:

Pursuant to the provisions of Title 20, California Code of Regulation, and on behalf of Quail Brush Genco, LLC, a wholly owned subsidiary of Cogentrix Energy, LLC, Tetra Tech hereby submits the *Plume Vertical Velocity Assessment for the Quail Brush Generation Project.* The Quail Brush generation Project is a 100 megawatt natural gas fired electric generation peaking facility to be located in the City of San Diego, California.

This Assessment was prepared to determine the potential for the reciprocating engine exhaust plumes to impact flight operations directly over the proposed project in accordance with the request from the California Energy Commission (CEC) at the December 2, 2011 public workshop. The vertical plume analysis is based on the guidelines for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in "*Guidelines for conducting plume rise assessments (CASA, 2004)*".

If you have any questions regarding this submittal, please contact Rick Neff at (704) 525-3800 or me at (303) 980.3653.

Sincerely,

Constance E. Farmer

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Plume Vertical Velocity Assessment for the Quail Brush Generation Project

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Background

The proposed Quail Brush Generation Project (QBGP) will be a nominal 100 MW facility intermediate peaking load utilizing eleven (11) Wartsila 20V34SG-C2 natural gas-fired internal reciprocating engines. Each engine has a 100 foot tall stack. The facility will be located on Sycamore Landfill Road in the City of San Diego, west of the City of Santee, California at UTM (NAD27) coordinates of 497321 meters East and 3634766 meters North. The facility elevation is 465 feet above mean sea level. Terrain around the project site varies in elevation from 600 feet up to 1200 feet (and higher) above mean sea level. Gillespie Field lies 3.08 miles southeast of the project site, while Miramar Naval Air Station lies 5.64 miles to the west-northwest. The project site and the Sycamore landfill area are not within the normal air traffic patterns of Gillespie Field. In order to understand the topography and terrain in the regional area surrounding the site, we recommend that the reader refer to the Visual Resources section of the AFC, i.e., Section 4.5, Figures 4.5-1 through 4.5-10.

An analysis of the potential for the reciprocating engine exhaust plumes to impact flight operations directly over the proposed project was made in accordance with a request from the California Energy Commission (CEC). Atmospheric Dynamics, Inc. has prepared a vertical plume velocity assessment for QBGP which utilizes the methods outlined in the Aviation Safety and Buoyant Plumes (Best et al 2003) paper. The analysis also includes a modified approach to the Best paper which has been utilized by the CEC on past Energy Commission Projects.

The vertical plume analysis is based on the guidelines for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in "*Guidelines for conducting plume rise assessments (CASA, 2004)"*. The aim of this assessment is to determine the height at which the *average* vertical plume velocity emitted from the power station natural gas reciprocating engines achieves the critical value of 4.3 m/s. (CASA Advisory Circular Sections 8.4) The CEC has used the 4.3 m/s velocity as a significance criteria on past power plant projects.

Vertical plume velocity guidelines

The assessment will conservatively determine the potential for turbulence generated by the plume-averaged vertical velocity of QBGP's exhaust plumes. The method uses worst-case assumptions of calm winds and neutral atmospheric conditions for the entire vertical extent of the plume to determine the worst-case impacts.

Since the development of a simple-cycle gas turbine power station at the end of a runway in Australia in the mid 1990s[a], the Australian Civil Aviation Safety Authority (CASA) has taken an active role in the review of the siting of facilities with the potential to affect aviation activities. Potential hazards that could affect the safety of aircraft include tall visible or invisible obstructions. Visible obstructions include structures such as tall stacks or communication towers. Invisible obstructions include industrial exhausts that generate significant turbulence due to high velocity and buoyancy. CASA has issued an Advisory Circular, (CASA 2004) that specifies the requirements and methodologies to be used to assess whether a new industrial plume is likely to have adverse implications for aviation safety.

The general CASA requirement is to determine the height at which the plume (or plumes) could generate atmospheric turbulence and to determine the dimensions of the plume in these circumstances. The frequency of in-plume vertical velocities at the lowest height an aircraft may travel over the site, and at other heights are also required. For large plumes that are remote from

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[a] It should be noted that this project consists of reciprocating internal-combustion engines (RICE) that have plume exhausts with much smaller volumetric flows and buoyancy fluxes than the turbine projects that elicited the initial interest of CASA.

airports, CASA requires an assessment that determines the size of a hazard zone to alert pilots to the potential hazard. Normally this analysis uses a sophisticated air dispersion model that determines plume vertical velocities and lateral/vertical extents based on wind fields generated from actual meteorological data. Rather than use such a refined technique, a conservative screening analysis based on calm wind field assumptions was used for this project.

For this assessment, the plume-averaged vertical velocities were calculated as a function of height under calm conditions. While the calculation output is provided in plume average velocity, there are some sections of the plume where the peak velocity could be up to two times higher than the average. It has been CASA's experience that these peak vertical velocities do not assess aviation safety risk appropriately. Past discussions between Katestone Environmental, who developed the vertical plume methodology used in this study and CASA have concluded that analysis of the average plume height and downwind distance is appropriate for these assessments (i.e., the use of plume-averaged vertical velocities is recommended by CASA). The established CASA significance criteria is for a averaged plume velocity to equal or exceed 4.3 m/s at altitudes where aircraft can operate.

Emission characteristics

The stack characteristics of the proposed QBGP engines are presented in Table 1. These stack parameters have relatively low buoyancy and volumetric flowrates as compared to turbine projects.

Table 1: Stack characteristics for the proposed power station.

Stack/Parameters	**QBGP**	
	English	**Metric**
Wartsila 20V34SG-C2		
Height	100 feet	30.48 meters
Flowrate	36,530 ACFM	17.24 m^3/s
Velocity	48.46 ft/sec	14.771 m/s
Temperature	822ºF	712.04 Kelvin
Diameter	4.0 feet	1.212 meters

The eleven stacks are arranged in a straight line (6 in one set and 5 in another set), with a separation of 17.75 feet between adjacent stack centers. For this assessment, an ambient temperature of 52ºF (284.26K) was used, which represents the 10th percentile worst-case (coldest) temperature in the three years of meteorological data used in the AERMOD air quality modeling analyses.

Methodology

Katestone Environmental has developed a conservative method that uses worst-case calm wind conditions to assess the average plume vertical velocity as a function of height. The Katestone methodology is described in detail in Best et al 2003. Katestone Environmental has used this methodology throughout Australia.

The methodology used in this assessment has been based on well-verified laboratory and theoretical treatments of the rise and spread of a buoyant jet, both into a still ambient environment and into a light crosswind. This treatment (developed by Dr Kevin Spillane) covers in detail the initial dynamics of the plume as it exits the stack and the entrainment of ambient air into the plume as it rises directly above the stack. This method also considers the enhancement of vertical velocities that may occur if the plumes from multiple stacks merge and form a higher buoyancy combined plume.

The vertical plume assessment will involve several stages of development:

(a) In the first stage very close to the stack exit, the high plume momentum will result in a short section in which the conditions at the center of the plume are relatively unaffected by ambient and plume buoyancy conditions. This jet phase extends from the stack exit to approximately a distance of 6.25 D above the stack (where D is the stack diameter) in calm conditions. At the end of this stage, the plume-averaged vertical velocity has decreased to half of the stack exit velocity, with a corresponding increase in effective plume diameter.

(b) In the second stage, the plume responds to differences between ambient and plume buoyancy conditions, with much cooler and less turbulent ambient air being entrained into the plume from the outside regions of the plume towards the plume centerline. The momentum and buoyancy of the plume significantly influences plume rise and subsequently the dilution of the stack exhaust to decrease plume vertical velocities. This dilution is very sensitive to ambient wind speed, so the calm wind conditions considered here are conservative.

In the second stage for multiple stacks, there are three phases. The first phase extends from the end of the jet phase until plumes from adjacent stacks are touching (determined from the stack separation). This phase is governed by the equations given for a single stack. In the second phase, the plumes continue to merge from the height at which the plumes first touch until the height at which the plumes from the stacks on each end completely extends over the lateral extent of all the stacks. At this completely merged plume height, the methodology applies an enhancement factor to the plume vertical velocities and plume diameter. Plume velocities and diameters are linearly interpolated by height from the results obtained at the touching and fully merged heights. In the third phase, for heights above the fully merged plume height, the merged plume continues to expand (with reductions in plume-averaged vertical velocities) at the same rates as expected for an individual plume.

(c) In the third stage of plume development, plume rise is due entirely to the buoyancy of the plume and continues from some distance until there is an equalization of turbulence conditions within and outside the plume. This final rise is often only achieved at considerable heights/distances from the stack where the effective average vertical velocity is then close to zero. Since there is very little turbulence and near-zero vertical velocities, this stage of plume development is usually not considered for this type of analysis.

The CEC has modified this approach for multiple equivalent stacks by using a simplified method where the multiple stack combined plume velocity is based on the single stack plume velocity multiplied by the number of stacks raised to the 0.25 power. Staff notes that this methodology can predict somewhat lower velocity values than the full Spillane approach methodology presented in the Best paper (Best 2003). The single plume methodologies are identical.

Worst-case calm wind scenario

The equations governing the growth of an isolated plume and merged plumes under calm wind conditions in a neutral environment are given in the paper in Appendix A. An assessment assuming calm winds for the entire height of the plume is presented here to represent the worst-case. Results of the plume vertical velocities at various heights are presented in Appendix B and summarized in Table 2 based both the Spillane methodology and the CEC methodology.

CASA requires that the proponent of a facility with an exhaust plume that has a plume-averaged vertical velocity exceeding the limiting value of 4.3 m/s at the Obstacle Limitation Surface (or 110 meters above ground level anywhere else) to utilize more sophisticated methods to further assess the potential hazard posed by the plume to aircraft operations. For this conservative calm-wind analysis, both single plume and merged plumes velocities were evaluated at an elevation of 1000' above ground level since minimum flight altitudes in the vicinity of the site (due to the nearby landfill, the elevations of the hills, plus transmission tower obstructions) are 1050 to 1380 ft above ground level to the east and northeast of the site and 1125 to 1460 ft above ground level to the west and northwest of the site.

Table 2: Summary of vertical velocity at height for worst-case calm wind scenario

Heights	Average vertical plume velocities Spillane Methodology	Average vertical plume velocities CEC Methodology
1000 Feet AGL	1.52 m/s (single plume)	1.52 m/s (single plume)
1000 Feet AGL	2.94 m/s (merged plume)	2.76 m/s (merged plume)

Table 3: Summary of height for 4.3 m/s Screening Threshold

Plume-averaged Vertical Velocity	Height (feet) above ground level Spillane Methodology	Height (feet) above ground level CEC Methodology
4.3 m/s	154 feet (single plume)	154 feet (single plume)
4.3 m/s	154 feet (merged plume)	347 feet (merged plume)

The plume-average vertical velocity at 1000 feet above grade for both single and merged plumes are much less than the CASA screening threshold of 4.3 m/s. Using the Spillane methodology, the plume-averaged vertical velocities drops below the CASA screening threshold of 4.3 m/s occurs at 54 feet above the top of the 100 foot stack (or 154 feet above ground level). Using the CEC methodology, the plume velocity drops below the 4.3 m/s screening threshold at 247 feet above the top of the stack (or 347 feet above ground level).

In reality, even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in Table 2 and Table 3, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear. In reality, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Appendix B presents the detailed calculation results for the single plume and merged plume conditions using both approaches.

Conclusion

The results of these modeling analyses demonstrate that the plume velocity drops below the 4.3 m/s significance criteria at an altitude of 154 or 347 feet above ground level. To encounter this velocity, a fixed or rotary wing aircraft would need to fly within 54 or 247 feet of the top of the exhaust stacks. These altitudes are well below the obstacle limitation heights in the area. Thus, the project is not expected to impact flight operations in the region.

Appendix A
Calculation Methodology

Clean Air Conference, Newcastle, New South Wales, Australia, 2003

AVIATION SAFETY AND BUOYANT PLUMES

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Summary

Very buoyant plumes generally experience good dispersion but can, in some circumstances, affect aviation safety. Large in-plume vertical velocities can occur in calm conditions with minimal wind shear. Recent civil aviation guidelines seek to restrict the horizontal or vertical extent where average in-plume vertical velocities exceed a threshold that can threaten aircraft performance or structural stability. Key plume calculation procedures require adequate predictions or measurements of vertical profiles of wind and turbulence parameters. The TAPM scheme proves useful but requires additional features for complex source geometry. A hybrid approach overcomes most of these limitations, whilst treating the initial plume development in more detail. Design issues for typical stack configurations are discussed.

Keywords: Plume velocities, stacks, cooling towers, flares, safety

1. Introduction

Over the past 25 years, considerable laboratory, field and theoretical work has been undertaken on the dispersion of very buoyant plumes from industrial sources. Such sources have traditionally included single or multi-flue stacks for major power stations, cooling towers and gas turbine generating plants where large volume flows, together with high exit temperatures, produce some of the highest buoyancy fluxes for normal power station configurations. With the increasing emphasis on gas and similar alternatives for power generation and the recent consideration of stack-in-tower configurations for locations where dry cooling is preferred, highly buoyant plumes are becoming the rule. In addition, industrial flares or unintended releases from pressurised pipelines can yield plumes with large momentum and/or buoyancy fluxes and may have structures approximating line or area sources. Recent dispersion analyses (Weil et al 2001) have shown that very buoyant plumes can readily interact with the overlying inversion and have plume spread dominated by buoyancy for most of the near-field. Plume rise and spread descriptions may need to be revisited.

High buoyancy plumes can, however, give rise to other problems that may require addressing in environmental impact assessments. High buoyancy plumes rise quickly and have significant in-plume vertical velocities. Should the facility be close to local airfields or aviation transport routes, any aircraft encountering the buoyant plumes may experience sufficient vertical uplift and turbulence to cause some temporary disruption to the manoeuvrability of aircraft, especially light commercial (rather than jet) aircraft.

There are no publicly-available field studies that document the decline of in-plume velocities with plume travel time for a variety of conditions necessary to produce validated modelling schemes. Various experimental and theoretical work was conducted around open-cycle and combined-cycle gas turbines at Kuala Lumpur, with field measurements taken for stack-top windspeeds in the range 2-8 m/s (but not for calm conditions). The Cessna aircraft used (Flinders Institute for Atmospheric and Marine Sciences) was fitted out to measure turbulence and air quality parameters as well as aircraft variables. The unpublished results showed a strong decrease of in-plume vertical velocities with windspeed and height, core vertical velocities a factor of approximately 2 greater than plume-averaged values and significant influences on aircraft handling for near-instantaneous (~ 1 sec) exposures to strong plume velocities, especially if encountered by surprise.

The importance of vertical motion in causing aviation problems is better documented by the number of light aircraft incidents reported during strong convection in Australia (Spillane and Hess 1988). During extreme events, naturally-occurring vertical velocities can reach 8 m/s.

The current studies were conducted for an environmental impact assessment of a 700 MW open cycle gas-fired turbine near an army aviation centre at Oakey in southern Queensland. Previous studies by Spillane (1980) on moist plumes were adapted to treat buoyant plumes from closely located sources in calm and low windspeed neutral conditions (Katestone

Scientific 1997). At the time, there was no model recommended by the Civil Aviation Safety Authority of Australia (CASA) and, indeed, very little guidance internationally as to the manner in which available velocity thresholds should be interpreted. Representations were made and generally accepted that the threshold vertical velocity of 4.3 m/s recommended by Australia and New Zealand authorities should be viewed as a plume-average rather than plume centreline criterion.

Critical (but extreme) aviation conditions are expected to be very light winds and neutral stability to heights of 500 m or more. For most assessment sites, there is unlikely to be a substantial database of near-surface and upper-level wind and temperature information to estimate the frequency of occurrence of such rare cases. Recognising this, CASA recently recommended the use of the CSIRO TAPM model for producing long-term databases of such profiles at any location within Australia and for providing a publicly-available method of calculating plume vertical velocities in the near-field of a single plume source (CASA 2003). The TAPM treatment of plume rise (Hurley and Manins 1995) uses coupled non-linear first-order differential equations for the plume volume G, buoyancy F and momentum M fluxes that are generalisations of the original Briggs (1975) plume rise formulation, based on the work of Glendening et al (1984) for stable atmospheres with complex structures. The TAPM scheme does not include any influence of source-altered flow fields or moisture content. It is also strictly valid only for single sources, with multiple sources being treated only via use of a plume enhancement factor, a relatively coarse device for describing near-field plume dynamics. For cooling tower sources, moisture emissions, the confluence of adjacent plumes and the influence of suction occurring due to tower bypass flow can be important (Rezacova and Sokol, 2000). This paper restricts attention to essentially dry plumes with no interactions with distorted flow fields.

Aviation safety risk assessments require the evaluation of concurrence of adverse vertical velocities with the presence of aircraft in the vicinity of the plume and a spectrum of aircraft types and pilot skill. Ideally, a generalised scheme should facilitate the prediction of likely pilot response to such events but publicly-available schemes are not yet available. As for many air quality problems, the main difficulties are assessing the relevance of traditional techniques to the forecasting of extreme conditions and determining the reliability of such assessments based on existing knowledge.

The present paper outlines the available plume calculation methodologies for the Spillane and TAPM approaches, addresses the modifications necessary for multiple sources and assesses the utility of the various schemes for dispersion and meteorological modelling in providing initial and detailed assessments. The high buoyancy of the plumes diminishes the utility of various design alternatives such as increasing stack separation, reducing exit velocity and changing the orientation of discharge. Practical measures are discussed.

2. General considerations

For the generic stack problem, we choose the case of multiple but identical sources of high initial exit velocity and temperature but low enough water vapour content to neglect latent heat considerations. In light winds, influences of the aerodynamic wakes or other effects of stack or cooling tower structures can be neglected. The initial stage (exit conditions) is assumed to be a plume emanating from a stack of height h_s and diameter D, with plume exit velocity either uniform over the cross-section (with a value V_{exit}) or, more likely, a non-uniform velocity profile with plume average velocity V_{exit}. The exit virtual potential temperature θ_s, volume flow $\pi D^2 V_{exit}/4$ and initial buoyancy flux $F_o = gV_{exit} D^2 (1 - \theta_a/\theta_s) / 4$ are readily calculated, with θ_a denoting ambient conditions. The ambient airspeed at stack top is denoted u_e with $K_o = V_{exit}/u_e$ being the initial plume to ambient velocity ratio.

An outline is given in the following sections of the Spillane and TAPM plume dynamics modules for single plumes (retaining their respective notations). The physical interpretation of the processes is outlined in Section 3 with the additional considerations needed for multiple plumes.

2.1 Spillane methodology

The plume radius a, orientation ϕ and velocity V are followed along the plume trajectory. Five equations are solved numerically for the normalised vertical velocity $K = V/u_e$:

Radial growth of a forced-plume bending in a wind:

$$\frac{da}{ds} = \beta_n \cos\phi / K + \beta_e \left|1 - \frac{\sin\phi}{K}\right| \qquad (1)$$

Rate of entrainment, E, into the plume:

$$2E/V = \left(\frac{da}{ds} + (\lambda^2 \cos\phi)/2F_r^2\right)/(1 - \sin\phi/2K) \qquad (2)$$

Momentum flux, Va, (longitudinal)

$$\frac{d(Va)}{ds} = 2E - V\frac{da}{ds} \qquad (3)$$

Trajectory curvature; transverse momentum flux

$$\frac{d\phi}{ds} = \left(2\,Eau_e \cos\phi - (F\sin\phi)/2.25V\right)/(Va)^2 \qquad (4)$$

Flux of heat:

$$\frac{d\left(Va^2\Delta\theta/\theta\right)}{ds} = 0, \text{ in a neutral environment} \qquad (5)$$

where the notation is as follows:

a = plume top-hat radius;
s = distance along plume trajectory;
ϕ = angle of plume centre line to vertical ;
$K = V/u_e$;
V = plume-averaged speed.
$\beta_n = 0.40$; $\beta_e = 0.16$; $\lambda = 1.11$;
F_r^2 = Froude No = $V^2/(ag\Delta\theta/\theta)$
F = flux of buoyancy = $\lambda^2 a^2 Vg\ \Delta\theta/\theta$; $\Delta\theta = \theta_p - \theta_e$ and suffices p and e for plume and environment.
θ = virtual potential temperature.

Initial conditions for ϕ, V, a and z are set for the end of the momentum rise stage (for a single plume) or at the end of the merged plume stage (for multiple plumes). An along-plume distance step of $\Delta s = 20$ m is used, and the appropriate value of $u_e(z)$ adopted for non-uniform profiles.

For the case of calm conditions, analytic solutions are possible, one for the product Va at any height, the other a linear increase of $a = 0.16\ (z - z_v)$ where the virtual source height (above stacktop) $z_v = 6.25\ D\ [1 - (\theta_e / \theta_s)^{1/2}]$. For $z > 6.25\ D > z_v$ we have:

$$(Va)^3 = (Va)_o^3 + 0.12\,F_o\left[(z - z_v)^2 - (6.25D - z_v)^2\right]$$

$$\text{where } (Va)_o = V_{exit}\,D/2(\theta_e/\theta_s)^{1/2} \tag{6}$$

2.2 CSIRO TAPM methodology

The TAPM mean plume rise estimation takes the Glendening et al (1984) approach but assumes that the horizontal plume velocity instantaneously takes up the ambient horizontal velocity at stack height. Cartesian co-ordinates are adopted. The differential equation for plume volume flux G:

$$\frac{dG}{dt} = 2R\ w_p\left(\alpha\, w_p + \beta\, u_e\right) \tag{7}$$

neglects a third term due to ambient turbulence entrainment. $w_p = \frac{dz_p}{dt}$ is the plume vertical velocity, $\alpha = 0.1$ and $\beta = 0.6$ are vertical and bent-over entrainment coefficients and R is the plume radius. For the buoyancy flux F, it assumes:

$$\frac{dF}{dt} = -\frac{sM}{u_p}\left(A\,u_a + w_p\right) \tag{8}$$

where $s^2 = \frac{g}{\theta_a}\frac{\partial\theta_a}{\partial z}$ gives the ambient buoyancy frequency ($s = 0$ in neutral conditions), $u_p^2 = u_e^2 + w_p^2$, $A = 1/2.25$ and M is determined by $\frac{dM}{dt} = F$ ($= F_o$ in neutral conditions). By definition,

$$G = \frac{\theta_e}{\theta_p}u_p\,R^2,\ F = gu_p\,R^2\,\frac{\Delta\theta}{\theta_p},\ u_pR^2 = G + F/g,$$

$$w_p = M/G \tag{9}$$

Initial conditions are set with G, F and M evaluated with $w_p = V_{exit}$, $R = R_s = D/2$ but with the initial integration having

$$R = R_o = R_s\left(V_{exit}/\left(u_a^2 + V_{exit}^2\right)^{1/2}\right)^{1/2} \tag{10}$$

The plume rise height is terminated when $F = 0$ and plume and ambient dissipation rates are equal. The plume dimensions are based on $R = 0.4\ (z - h_s)$ or equivalent prescriptions.

3. Treatment of multiple plumes

For N multiple, identical sources with stack separation d, Table 1 summarises the expected multi-stage plume development as well as Figure 1. The first stage is the rapid (almost vertical) rise of the individual plumes due to their momentum. The external surface of the plume entrains air as it rises (and the vertical velocities are reduced). The end of the momentum-dominated phase occurs when this entrainment reaches the plume core, the plume centreline has a vertical velocity equal to V_{exit} and the velocity profile will be essentially Gaussian. The peak (core) vertical velocity is therefore V_{exit} but the plume average value is $0.5\ V_{exit}$. Conservation of momentum therefore requires the plume width to have effectively doubled from its initial value a_o.

In this first phase, the plume travels a height of 6.25 D in calm conditions and $0.4\ K_o a_o$ for K_o reasonably large (based on laboratory experiments). Davidson (1994) has also shown that an analytic form for plume rise in a uniform wind has an initial component of 6.2 D exp $(-3.3/K_o)$.

In the second stage, the plume dynamics and trajectories respond to ambient conditions, with much cooler air being entrained into the stack plume. The buoyancy of the plumes has significant influences on the rise as this air mixes into the plume and provides dilution of the exhaust. This dilution is very sensitive to ambient wind speed. For multiple plumes from closely-spaced stacks, this leads almost immediately to a height at which two plumes first touch each other (and plume merging commences) when the effective plume radius is equal to half the stack separation (this is exact in calm winds and approximately correct for light winds). Total merging is assumed to occur when the single plume radius equals stack separation. Conservation of buoyancy flux and Froude number (a reasonable assumption for coherent plumes) leads to a conclusion that the plume radius and vertical velocity will be increased overall by a factor of $2^{0.25} = 1.189$ by the merging of 2 adjacent plumes.

For more than two stacks, the situation is more complex. In calm conditions, the combined plumes from pairs of stacks will coalesce shortly after to form a coherent plume, assumed to be complete before the single plume radius, a^{sp}, is ½ d (N-1) At this height, the combined plume velocity V_m and radius a_m are $N^{0.25}$

greater than for a single plume. For non-calm conditions, a simplified treatment shows that total merging is likely to occur soon after the merging of two adjacent plumes, for winds at right angles to the line of separation of the stack. For winds at smaller angles ω to the line of stacks, the process is more sequential and the effective stack separation can be reduced by a factor proportional to $\cos \omega$.

In the third stage of plume development, plume rise is due entirely to the buoyancy of the (merged) plume and continues until there is an equalisation of turbulent conditions within and outside the plume. The effective average vertical velocity is then close to zero. The third stage of plume development can then be treated as that of a single merged plume (with different initial conditions for a, V and ϕ) passing through different atmospheric layers with varying horizontal velocity u_e. The Katestone software uses a simple successive substitution method to determine a, E (the entrainment), V and ϕ in that order. These equations are valid up to a critical value of ϕ_c ($\phi_c < \pi/2$) at which either the assumptions become invalid or plume rise should be effectively terminated.

These equations can be used in the second stage prior to plume touching and in the third stage once merging has been completed. Plume height is calculated by aggregating $\Delta s \cos \phi$, centreline displacement by aggregating $\Delta s \sin \phi$. For each Δs, the appropriate ambient windspeed is determined by linear interpolation (or power law curve fitting of available meteorological profile measurements or predictions).

A fourth stage can occur if the coherent plume reaches the base of the overlying inversion (height Z_i). Some of the plume will punch through the inversion base, albeit with reduced vertical velocity. The remainder will be effectively trapped within the inversion layer with essentially zero vertical velocity. Weil et al (2001) show that the penetration in convective conditions depends on $F_*^{2/3}$ where $F_* = F/(u_e w_*^2 Z_i)$ and w_* is the convective velocity scale. There is as yet little guidance on plume dimensions and vertical velocity for the penetrative component.

Table 1: **Key parameters for the various stages of development for merging plumes.**

Stage	Average plume velocity		Plume width	Plume height	Plume angle	Comments
	Vertical	Horizontal				
Stack exit	V_{exit}	0	a_o	h_s	0°	
End of jet phase	$0.5\ V_{exit}$	$u_e(z) + V \sin \phi_o$	$2a_o$	$h_s + z_o$	ϕ_o	$z_o = K_o a_o < 6.25D$
Plumes first touch	$V_t \cos \phi_t$	$u_e(z) + V_t \sin \phi_t$	a_t	z_t	ϕ_t	$V_t < 0.5\ V_{exit}$
End of plume merging	$V_m \cos \phi_m$	$u_e(z) + V_m \sin \phi_m$	a_m	z_m	ϕ_m	$a_m \approx N^{1/4} a^{sp}$ $V_m \approx N^{1/4} V^{sp}$
Coherent merged plume	$V \cos \phi$	$u_e(z) + V \sin \phi$	a	z	ϕ	$V < V_m\ a > a_m$
Maximum plume rise	0	$u_e(z) + V \sin \phi$	a_c	z_c	ϕ_c	$\phi_c < 90°$
Inversion interaction	Low	Shear-affected	Enhanced	$> Z_i$	Variable	(Weil et al 2001)

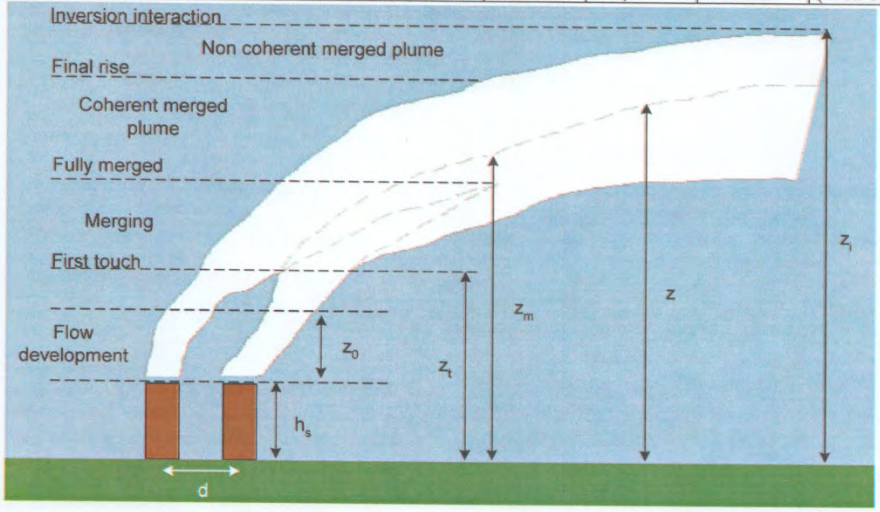

Figure 1: Schematic of plumes merging.

4. Illustrative examples

The simplest cases assume identical sources with stack separation d operating in a neutral and unbounded atmosphere with uniform conditions. For the Spillane approach, Table 2 gives the resulting plume-average vertical velocities for the cases with $V_{exit} = 38.9$ m/s, $h_s = 35$ m, $F = 2300$ m^4/s^3 and N = 1 and separately N = 2 with d = 25 m.

The heights experiencing threshold exceedances are dramatically reduced going from calm to light winds. The TAPM approach for single plumes gives similar results if some allowance is made for an initial displacement offset z_0 (Figure 2).

Table 2: Plume average vertical velocities (m/s) for uniform calm and light wind conditions in a neutral atmosphere

Height	Calm		$u_e = 1.5$ m/s		$u_e = 3$ m/s	
	Single	**Double**	**Single**	**Double**	**Single**	**Double**
100	12.2	12.2	9.0	9.3	6.9	8.3
200	7.8	9.2	5.5	7.0	3.6	5.1
300	6.5	8.0	4.4	5.8	2.6	3.9
500	5.3	6.6	3.2	4.5		2.8
700	4.8	6.0	2.6	3.7		2.2
1000	4.1	5.2				

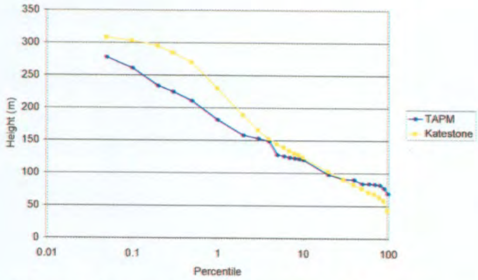

Figure 2: Comparison of methodologies for plume height calculations for a 5 year period.

5. Meteorological modelling

Meteorological inputs are critical for a reasonable treatment of risk, especially for near-calm conditions at stack-top and above. Unfortunately, it is these very conditions under which near-surface measurements (together with stability-dependent profile laws) or TAPM-like prediction methodologies are likely to be poor indicators of actual conditions, at least for inland sites (Jackson et al 2003). Presumably this quandary lead CASA to recommend the TAPM approach. If measurements are available from a nearby 30-100 m tower, we would recommend their use unless TAPM results are carefully tuned to the appropriate surface conditions.

Recent project work near Williamtown Airport gave a comparison of five years of hourly TAPM results with available balloon and 30 m tower measurements. The main conclusions were:

- Moderate interannual variability in the actual and predicted occurrence of light winds at 30 m and above.
- TAPM tends to underpredict the frequency of occurrence of very light winds (< 1 m/s) compared to tower observations (typically 1.2 - 3.5% compared to 5.7 - 14.9%).
- For available balloon profiles, TAPM overpredicted the frequency of very light winds at 600 m and 900 m agl.
- Very few measurements are available in the crucial 100-500 m height range.

6. Synthetic approaches

The Spillane approach has been adapted to take in the TAPM wind profile conditions. Figure 3 compares the cumulative probability distributions for critical heights (where the in-plume average velocity drops below 4.3 m/s) obtained by using either the TAPM wind predictions or the interpolated measured winds, for the case of two 35 m high, 54 m separated combined-cycle units of total capacity over 800 MW. Close agreement is obtained.

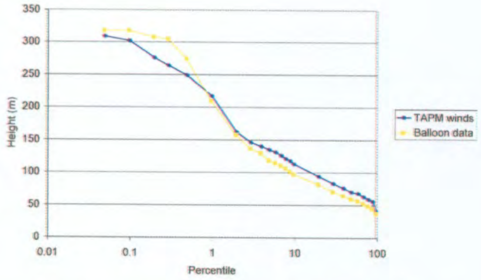

Figure 3: Comparison of Spillane plume height calculations for TAPM - generated and measured winds.

7. Design options

Decreasing the exit velocity will reduce the initial flow development length but plume buoyancy is the key factor in the magnitude of the vertical velocity. Similarly any reduction in stack height gives little benefit to aviation safety concerns and may risk poor plume dispersion in high-wind conditions (due to building wake influences). Increasing the stack separation does delay the time when plumes merge but with little overall practical benefit (Figure 4). Horizontally-pointing stack exits will reduce initial momentum but again buoyancy is dominant.

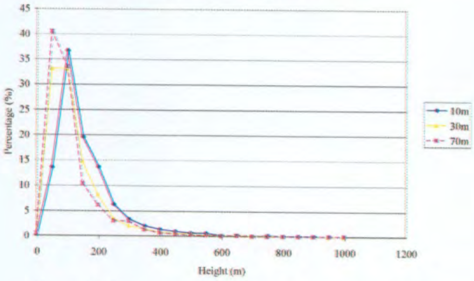

Figure 4: Frequency of critical height for varying stack configurations.

The reduction of plume buoyancy by using heat recovery results in a very significant reduction of critical heights but open-cycle operation usually has to be considered in any risk assessment. For critical cases, it appears better to take advantage of the relatively small zone of influence on vertical velocities and the usual requirement of CASA to identify stack locations for low-flying aircraft. A notice to aircrew together with real-time indication of site operations may be effective in most situations.

8. Conclusions

Methodologies now exist for major point sources and point to the dominating role of initial plume buoyancy. Detailed measurements are required for light-wind conditions and are readily taken by experienced research aircrews. TAPM methodologies are reasonable for single plumes but inappropriate for multiple plumes. For key sites, remote sensing equipment is required to gather reliable wind statistics in the critical 100-500 m range. Theoretical advances are needed to treat inversion penetration in very light-wind conditions and to extend the methods to moist plumes and different source geometries.

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Appendix B
Results

Quail Brush Engines - Plume Averaged Vertical Velocities for Individual Stack								
	"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.							
	"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume*							
		***from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane**						
Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)		
10th percentile Ambient Potential Temp θ_a	284.26	Kelvins	52.0	°F		0.3048	meters/feet	
Plume Exit Conditions:					Gravity g	9.81	m/s^2	
Stack Height h_s	30.48	meters	100.0	feet	λ	1.11		
Stack Diameter D	1.2192	meters	4.0	feet	λ_o	~1.0		
Stack Velocity V_{exit}	14.771	m/s	48.46	ft/sec				
Volumetric Flow	17.24	cu.m/sec	36,530	ACFM	$\pi V_{exit}D^2/4$			Sect.2/¶1
Stack Potential Temp θ_s	712.039	Kelvins	822	°F				
Initial Stack Buoyancy Flux F_o	32.35	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$			Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a, V, θ_p at plume height (see below)			
Conditions at End (Top) of Jet Phase:								
Height above Stack z_{jet}	7.620	meters*	25.0	feet*	z_{jet} = 6.25D, meters*=meters above stack top			Sect.3/¶1
Height above Ground $z_{jet}+h_s$	38.100	meters	125.0	feet				"
Vertical Velocity V_{jet}	7.386	m/s	24.23	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$			"
Plume Top-Hat Diameter $2a_{jet}$	2.438	meters	8.0	feet	$2a_{jet}$ = 2D		Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase								
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:								
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height			Sect.2/Eq.6
Virtual Source Height z_v	2.805	meters*	9.2	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top			Sect.2/Eq.6
Height above Ground z_v+h_s	33.285	meters	109.2	feet			where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6318
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o\,[\,(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}\,/\,a$			Sect.2.1(6)
Product $(Va)_o$	5.689	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$			
Solve for plume-averaged vertical velocity at height	**1,000.0**	**feet**		304.8	meters above ground ($z'+h_s$)			
Gives the following Height above Stack z'	274.320	meters*	900.0	feet*				
Plume Top-Hat Diameter 2a'	86.885	meters	285.1	feet	$2a'=2*0.16(z'-z_v)$			Sect.2/Eq.6
Vertical Velocity V	**1.517**	**m/s**	4.98	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$			Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**4.30**	**m/s plume-averaged vertical velocity**						
Find Height above Stack z_{crit}	16.311	meters	53.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)			
Height above Ground $z_{crit}+h_s$	46.791	meters	**153.5**	**feet**	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where			
							a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=	-11.9207
							and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=	-289.1
								http://www.akiti.ca/Quad3Deg.html
							gives the real solution x = z-zv =	13.506
							or z(m) =	16.311
							z(ft) =	53.5

Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**Vert. Vel(m/s)**	**Plume Temp(K)**	**Elev Increments(m)**	
Top of jet = 125.0	***7.62***	***1.219***	***7.39***			
130.0	9.14	1.014	6.21	403.33	10.0	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}\,/\,a$
140.0	12.19	1.502	5.05	351.06		$a = 0.16(z-z_v)$
150.0	15.24	1.990	4.45	327.45		$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$
160.0	18.29	2.477	4.07	314.72		
Begin Merging (touch) = 164.7	***19.71***	***2.705***	***3.93***	***310.70***		
200.0	30.48	4.428	3.28	296.09	100.0	
300.0	60.96	9.305	2.54	287.72		
400.0	91.44	14.182	2.21	285.98		
500.0	121.92	19.058	2.00	285.31		
600.0	152.40	23.935	1.85	284.98		
End Merging (full/mp) = 663.9	***171.88***	***27.051***	***1.78***	***284.85***		
800.0	213.36	33.689	1.65	284.67	200.0	
1000.0	274.32	43.442	1.52	284.53		
1200.0	335.28	53.196	1.42	284.45		
1400.0	396.24	62.950	1.34	284.40		
1600.0	457.20	72.703	1.28	284.37		
1800.0	518.16	82.457	1.23	284.35		
2000.0	579.12	92.210	1.18	284.34		
2200.0	640.08	101.964	1.14	284.32		
2400.0	701.04	111.718	1.11	284.32		
2600.0	762.00	121.471	1.08	284.31		
2800.0	822.96	131.225	1.05	284.30		
3000.0	883.92	140.978	1.02	284.30		

Quail Brush Engines - Plume Averaged Vertical Velocities for Merged Plumes from Eleven (11) Stacks

"Aviation Safety and Buoyant Plumes," **Peter Best, et. al.**

"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged
Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia," **Dr. K.T. Spillane**

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
10th percentile Ambient Potential Temp θ_a	284.26	Kelvins	52.0	°F		0.3048 meters/feet	
Plume Exit Conditions:					Gravity g	9.81 m/s^2	
Stack Height h_s	30.48	meters	**100.0**	feet	λ	1.11	
Stack Diameter D	1.2192	meters	4.0	feet	λ_o	~1.0	
Number of Stacks N (≥3)	11		ok		**Base calcs on multiple plume treatment in Peter Best Paper**		
Stack Separation d	5.41	meters	17.75	feet		**plume velocities/diameters increased by $N^{0.25}$**	
Stack Velocity V_{exit}	**14.771**	m/s	48.46	ft/sec			
Volumetric Flow	17.24	cu.m/sec	36,530	ACFM	$\pi V_{exit}D^2/4$		Sect.2/¶1
Stack Potential Temp θ_s	712.039	Kelvins	822	°F			
Initial Stack Buoyancy Flux F_o	32.35	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol Flow$(g/\pi)(1-\theta_a/\theta_s)$		Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)		
Conditions at End (Top) of Jet Phase:							
Height above Stack z_{jet}	7.620	meters*	25.0	feet*	z_{jet} = 6.25D, meters*=meters above stack top		Sect.3/¶1
Height above Ground $z_{jet}+h_s$	38.100	meters	**125.0**	feet			"
Vertical Velocity V_{jet}	**7.386**	m/s	24.23	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$		"
Plume Top-Hat Diameter $2a_{jet}$	2.438	meters	8.0	feet	$2a_{jet}$ = 2D	Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases							
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:							
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height		Sect.2/Eq.6
Virtual Source Height z_v	2.805	meters*	9.20	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z_v+h_s	33.285	meters	109.20	feet		where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6318
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o\,[\,(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a		Sect.2.1(6)
Product $(Va)_o$	5.689	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$		
Plume Merging - Based on Single Plume Calculations where:							Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	5.410	meters	17.75	feet	$2a_{touch}$=d, (or a_{touch}=d/2)		
Height above Stack z_{touch}	19.712	meters*	64.67	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top		
Height above Ground $z_{touch}+h_s$	50.192	meters	**164.67**	feet			
Vertical Velocity V_{touch}	**3.933**	m/s	12.90	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o\,[\,(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a		
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	54.100	meters	177.5	feet	$2a_{full}$=2d(N-1)/2, (or a_{full}=d(N-1)/2)		
Height above Stack z_{full}	171.868	meters*	563.9	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top		
Height above Ground $z_{full}+h_s$	202.348	meters	**663.9**	feet			
Vertical Velocity V_{full}	1.777	m/s	5.8	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o\,[\,(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a_{full}		
Product $(V^3a)_{full}$	152	m^4/s^3					
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations:							
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height		
Revised Merged Plume **Radius** a_m	49.262	meters	161.6	feet	where $a_m = N^{0.25}a_{full}$ where Total Merging Occurs		
Revised Merged Plume Velocity V_m	**3.236**	m/s	10.62	ft/sec	and $V_m = N^{0.25}V_{full}$ where Total Merging Occurs		
Revised Virtual Source Height z_{full}	171.868	meters*	563.9	feet*	Height above stack where Total Merging Occurs (shown above)		
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{N(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation		
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$		
Multiple Plume Calculations						for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height	1,000.0	**feet**	304.8		meters above ground $(z+h_s)$		
Gives the following Height above Stack z	274.320	meters*	900.0	feet*	REGULAR EQNS		
Plume Top-Hat Radius a	65.655	meters	215.4	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**		
Vertical Velocity V	**2.941**	**m/s**	9.65	ft/sec	$V=\{N(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**		
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**		
					V'=single plume values **if $z<z_{touch}$**		
Solve for Height of CASC critical vertical velocity V_{crit}	4.30	**m/s**			BEFORE TOUCHING-USE SINGLE PLUME VALUES		
Find Height above Stack z_{crit}	16.311	meters	53.5	feet	$z_{crit} = z_{full} + \{[N(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$** (after merged)		
Height above Ground $z_{crit}+h_s$	46.791	meters	**153.5**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_m<V_{crit}<V_{touch}$** (during merge)		
					z_{crit}=single plume sol'n if **$V_{crit}>V_{touch}$** (before merge)		

Table of Plume-averaged Vertical Velocities:

Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**Vert. Vel(m/s)**	**CEC Staff Calc(m/s)** $V_m=N^{0.25}V_s$	
Top of jet = 125.0	***7.62***	***1.219***	***7.39***		*Single Plume Eqns*
130.0	9.14	1.014	6.21	11.31	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a
140.0	12.19	1.502	5.05	9.20	$a = 0.16(z-z_v)$
150.0	15.24	1.990	4.45	8.11	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$
160.0	18.29	2.477	4.07	7.41	
Begin Merging (touch) = 164.7	***19.71***	***2.705***	***3.93***	7.16	*Interpolated Layer Eqns*
200.0	30.48	#N/A	3.88	5.98	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$
250.0	45.72	#N/A	3.81	5.13	
300.0	60.96	#N/A	3.74	4.63	
350.0	76.20	#N/A	3.67	4.28	
400.0	91.44	#N/A	3.60	4.02	
End Merging (full/mp) = 663.9	***171.88***	***49.262***	***3.24***	3.24	*Merged Plume Eqns*
800.0	213.36	55.901	3.10	3.01	$V=\{N(V^3a)_{full}/a\}^{1/3}$
1000.0	274.32	65.655	2.94	2.76	$a=a_m+0.16(z-z_{full})$
1200.0	335.28	75.408	2.81	2.58	
1400.0	396.24	85.162	2.70	2.44	
1600.0	457.20	94.916	2.60	2.33	
1800.0	518.16	104.669	2.52	2.23	
2000.0	579.12	114.423	2.44	2.15	
2200.0	640.08	124.176	2.38	2.08	
2400.0	701.04	133.930	2.32	2.02	
2600.0	762.00	143.684	2.26	1.96	
2800.0	822.96	153.437	2.22	1.91	
3000.0	883.92	163.191	2.17	1.87	