

## DOCKETED

<b>Docket Number:</b>	08-AFC-09C
<b>Project Title:</b>	Palmdale Energy Project (Formerly Palmdale Hybrid Power Plant) - Compliance
<b>TN #:</b>	214567
<b>Document Title:</b>	Palmdale Energy, LLC's Plume Vertical Velocity Assessment for the Air Cooled Condensers
<b>Description:</b>	N/A
<b>Filer:</b>	Marie Fleming
<b>Organization:</b>	DayZen LLC
<b>Submitter Role:</b>	Applicant Representative
<b>Submission Date:</b>	11/28/2016 11:29:16 AM
<b>Docketed Date:</b>	11/28/2016

# Plume Vertical Velocity Assessment for the Air Cooled Condensers

## Palmdale Energy Project

Palmdale, California

Submitted to  
**California Energy Commission**

Submitted by  
**Palmdale Energy, LLC**

Prepared by  
**Atmospheric Dynamics, Inc.**



**ATMOSPHERIC DYNAMICS, INC**  
Meteorological & Air Quality Modeling

**November 2016**

## Introduction

Palmdale Energy, LLC is proposing to develop the Palmdale Energy Project (PEP), located near the Palmdale Airport. The combined-cycle project will utilize two (2) Siemens SCC6-5000F natural gas-fired combustion turbine generators (CTG) and two (2) heat recovery steam generators (HRSG) with supplemental duct firing and a 32 cell air cooled condenser (ACC). The PEP site will be located on an approximately 50-acre undeveloped parcel west of the northwest corner of U.S. Air Force Plant 42. Based on updated ACC stack parameter data, provided by Siemens, a revised analysis of the ACC plume characteristics on vertical winds was prepared and compared to the California Energy Commission (CEC) significance criteria for the average vertical plume velocities as described below.

Atmospheric Dynamics, Inc. (ADI) prepared a screening level plume vertical velocity assessments which are based on the calm wind Spillane methodology outlined in the *“Aviation Safety and Buoyant Plumes”* paper (Peter Best, et. al., presented at the Clean Air Conference, Newcastle, New South Wales, Australia, 2003). This methodology is also recognized as a screening tool for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in *“AC 139-5(1) Plume Rise Assessments (CASA, 2012)”*.

In May, 2016 Palmdale Energy, LLC submitted the results of our ACC plume rise screening analysis. Recently an error in the ACC exit velocity data provided by our vendor was discovered and we are therefore submitting a revised analysis for CEC staff review.

The aim of this revised screening assessment is to conservatively determine the potential for turbulence generated by the ACC waste heat exhaust plumes. Part 139.370 of the Australian Civil Aviation Safety Regulations (1998, 2004) provides that CASA may determine that plume velocities in excess of 4.3 m/s is or will be a potential hazard to aircraft operations. The *Manual of Aviation Meteorology* (Australian Bureau of Meteorology 2003) defines severe turbulence as a vertical wind gust velocity in excess of 10.6 m/s. The assumed critical vertical velocity used as a CEC significance threshold is 5.3 meters per second\* (m/s) but it should be noted that the basis of the original CASA derived threshold of 4.3 m/s has been lost in antiquity and that CASA no longer relies on the 1998 and 2004 regulations that established this critical threshold other than to note that a more rigorous analysis, which includes site specific meteorology, should be used if the 4.3 m/s and 10.6 m/s screening thresholds are exceeded. The screening method uses absolute worst-case assumptions of calm winds and neutral atmospheric conditions for the entire vertical extent of the plume to determine these worst-case impacts. It should be noted that these results are extremely conservative in that these worst-case conditions typically only occur during a few hours each year.

The Spillane methodology is generally applied to a limited number of plume source geometry's (turbines, power plant boilers, etc.) with the stacks arranged linearly (in a single straight-line) and separated by distances that typically exceed the individual stack diameters. For this assessment, a conservative assumption was made in order to use the Spillane methodology on an atypical ACC plume configuration which is made up of 32 plumes or cells arranged on a two dimensional surface. Here, the methodology, as described below, assumed all operating ACC cells were merged into a single equivalent ACC cell with an effective diameter based on the combined diameters of all operating cells. In other words, a single large cell was assumed to initially describe the release parameters of the ACC.

\*For the Puente Power Project (Docket#15-AFC-01, TN#213674, 9/15/2016), “CEC staff ... concluded that an average velocity of 5.3 m/s is the appropriate velocity ... [for a plume velocity threshold].” The CEC staff “Plume Background Threshold” attached to the docketed document concludes with “...[CEC] staff will use 10.6 m/s peak vertical plume velocity as the new threshold. The altitude at which a plume would have a peak vertical velocity of 10.6 m/s would be the same altitude at which a plume would have an average vertical velocity of half that, 5.3 m/s.” Therefore, the May 2016 PEP ACC analyses have been revised to reflect the new significance criteria.



## Screening Methodology and Vertical Plume Velocity Calculations for ACC

The ACC is comprised of 32 individual cells, arranged along four rows of eight cells each in 4 x 8 matrix. Thus, the 32 cells or radiators are arranged along two axis of direction producing a two dimensional plane in both the x and y directions. ACC stack parameter data (plume velocity, plume temperature) was provided by Siemens and the ACC manufacturer. The ACC will utilize variable speed fans. Additionally, the number of fans that are operational are dependent upon ambient temperature and plant load. For all ambient conditions, plant operation was assumed to be at full load. Thus, during cold winter and annual average conditions, all 32 fans would be operational at lower fan speeds. During worst-case hot summer days, 30 fans would be operational at the maximum fan speed. This data is summarized in Table 1.

Case #	1	2	3
Ambient Temp (°F)*	23	64	98
Number of ACC Cells in Use*	32	32	30
Heat Rejection (MW)*	447.23	447.36	445.28
Exhaust Flow Rate (lb/hr)*	1.818E8	2.440E8	2.150E8
Cell Exit Temperature (°F)*	59.0	90.32	130.10
Cell Height (ft)*	130	130	130
Effective Cell Diameter (ft)	47.169	47.169	47.169
Effective Stack Diameter (ft)	266.83	266.83	258.36
Stack Exit Velocity (ft/s)*	13.16	18.60	18.67
*ACC stack data provided by Siemens			
** Calculated value based on the cell diameter of 47.17 feet to the square of the number of operating cells or for example, Case #1: $D_{eff} = 47.17 \cdot \sqrt{32}$			

The Spillane methodology was originally developed to treat multiple individual stacks that are arranged along a linear x or y direction, but not both directions at once, with stack separations much greater than the stack diameters, typical of boilers/turbines at large power plants. The thirty to thirty-two radiator cells (depending upon operating case number) are arranged in the 4 x 8 pattern. Therefore, the Spillane methodology was used for a single source with the effective stack diameter for the number of operating cells in use for each ambient temperature. For the cold day Case #1 and annual average Case #2, the effective single plume diameter would be based on 32 cells, while for the summer Case #3, the effective diameter would be based on 30 cells. The effective diameter for the single cell for each of the three ambient temperatures are presented in Table 1. The plume velocities were then calculated using the Spillane methodology for a single effective diameter.

## Results

Screening level vertical plume velocity assessments were made for the range of ambient temperatures with calm winds and neutral atmospheric conditions for the three cases presented in Table 1. The total heat rejection for the three ambient cases are similar to each other and are based on the plant at 100 percent load. The use of variable speed fans and the ability to cycle each fan based on the cooling needs of the plant will allow the ability to minimize plume exit temperature and velocity. The results based on the three ambient conditions are presented in Table 2 and the output from the calculation worksheet provided in Attachment A.



Because of the large effective stack diameters, the initial jet phase extends to a height of about 1,750 feet above grade level (ft-agl) for Case 3 to almost 1,800 ft-agl for Cases 1 and 2. Thus, the previously accepted critical height of 1,500 feet occurs in the jet phase, and the results in Table 2 were interpolated by height from the stack exit velocity to the height at the top of the jet (with a plume average velocity of one-half the exit velocity). After the jet phase, plume temperature buoyancy characteristics modeled in the Spillane methodology cause an increase in plume-average vertical velocities to a peak velocity at some distance above the jet, after which plume average vertical velocities again decrease. The heights of maximum plume vertical velocities occur around 2,800 ft-agl for all three cases as shown in Attachment A.

<b>Table 2 ACC Vertical Plume Velocity Analysis Results for Reference Height</b>			
<b>Case #</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>Ambient Temp (°F)</b>	<b>23</b>	<b>64</b>	<b>98</b>
<b>Single Plume Results (m/s):</b>			
<b>at 1,500-feet agl (Within the Jet Phase)</b>	<b>2.36</b>	<b>3.34</b>	<b>3.28</b>
<b>Maximum Velocity above 1500-foot agl</b>	<b>4.72</b>	<b>4.71</b>	<b>4.87</b>

From these results and for each ambient condition, the vertical plume velocities are less than the threshold value of 5.3 m/s for all heights through 1,500 feet-agl and above for the ACC. The heights at which plume-averaged vertical velocities exceed 5.3 m/s only occur during the jet phase for Cases 2 and 3 and occur at a height of 240 ft-agl as shown in Attachment A. For Case 1, the jet phase velocity is always less than 5.3 m/s. These cases also represent the worst-case conditions of calm winds at all levels of a neutral atmosphere.

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the ACC always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above 1,500 ft-agl. Even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear when realistically, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.



**Attachment A**  
**Spillane Method Plume Velocity Calculations**



SINGLE Plume Average Vertical Velocities for PEP ACC with Effective Diameter for 32 cells at 23F (Winter)					
"Aviation Safety and Buoyant Plumes," Peter Best, et. al.					
"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia," Dr. K.T. Spillane					
<b>Ambient Conditions:</b>		Constants: Assume neutral conditions (dθ/dz=0 or θ <sub>s</sub> =θ <sub>a</sub> )			
Ambient Potential Temp θ <sub>a</sub>	268.15 Kelvins	23.0 °F	0.3048 meters/feet		
<b>Plume Exit Conditions:</b>		Gravity g 9.81 m/s <sup>2</sup>			
Stack Height h <sub>s</sub>	39.62 meters	130.0 feet	λ	1.11	
Stack Diameter D	81.3297 meters	266.83 feet	λ <sub>0</sub>	-1.0	
Stack Velocity V <sub>exit</sub>	4.01 m/s	13.16 ft/sec	πV <sub>exit</sub> D <sup>2</sup> /4		
Volumetric Flow	20,832.08 cu.m/sec	44,140,674 ACFM	Sect.2/¶1		
Stack Potential Temp θ <sub>s</sub>	288.15 Kelvins	59.00 °F	gV <sub>exit</sub> D <sup>2</sup> (1-θ <sub>s</sub> /θ <sub>a</sub> )/4 = Vol.Flow(g/π)(1-θ <sub>s</sub> /θ <sub>a</sub> )		
Initial Stack Buoyancy Flux F <sub>0</sub>	4,515.06 m <sup>4</sup> /s <sup>3</sup>		Sect.2/¶1		
Plume Buoyancy Flux F	N/A m <sup>4</sup> /s <sup>3</sup>		λ <sup>2</sup> gVa <sup>2</sup> (1-θ <sub>s</sub> /θ <sub>a</sub> ) for a,V,θ <sub>s</sub> at plume height (see below)		
<b>Conditions at End (Top) of Jet Phase:</b>					
Height above Stack z <sub>jet</sub>	508.311 meters*	1667.7 feet*	z <sub>jet</sub> = 6.25D, meters*=meters above stack top		Sect.3/¶1
Height above Ground z <sub>jet</sub> +h <sub>s</sub>	547.931 meters	1797.7 feet			
Vertical Velocity V <sub>jet</sub>	2.005 m/s	6.58 ft/sec	V <sub>jet</sub> = 0.5V <sub>exit</sub> = V <sub>exit</sub> /2		"
Plume Top-Hat Diameter 2a <sub>jet</sub>	162.659 meters	533.7 feet	2a <sub>jet</sub> = 2D		Conservation of momentum
<b>Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase</b>					
<b>Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:</b>					
Plume Top-Hat Radius a	Solutions in Table Below		0.16(z-z <sub>v</sub> ), or linear increase with height		Sect.2/Eq.6
Virtual Source Height z <sub>v</sub>	17.958 meters*	58.9 feet*	6.25D[1-(θ <sub>s</sub> /θ <sub>a</sub> ) <sup>1/2</sup> ], meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z <sub>v</sub> +h <sub>s</sub>	57.578 meters	188.9 feet	where (θ <sub>s</sub> /θ <sub>a</sub> ) <sup>1/2</sup> = (θ <sub>s</sub> /θ <sub>a</sub> ) <sup>1/2</sup> = 0.9647		
Vertical Velocity V	Solutions in Table Below		{(Va) <sub>0</sub> <sup>3</sup> + 0.12F <sub>0</sub> [(z-z <sub>v</sub> ) <sup>2</sup> - (6.25D-z <sub>v</sub> ) <sup>2</sup> ] <sup>1/3</sup> / a		
Product (Va) <sub>0</sub>	157.305 m <sup>2</sup> /s		V <sub>exit</sub> D/2(θ <sub>s</sub> /θ <sub>a</sub> ) <sup>1/2</sup>		
<b>Solve for plume-averaged vertical velocity at height 1,500.0 feet</b> 457.2 meters above ground (z+h <sub>s</sub> )					
Gives the following Height above Stack z'	417.580 meters*	1370.0 feet*	<b>Height Entered &lt; Top of Jet - Cannot Be Solved</b>		
Plume Top-Hat Diameter 2a'	#N/A meters	#N/A feet	2a' = 2*0.16(z'-z <sub>v</sub> )		Sect.2/Eq.6
Vertical Velocity V	#N/A m/s	#N/A ft/sec	V = {(Va) <sub>0</sub> <sup>3</sup> + 0.12F <sub>0</sub> [(z-z <sub>v</sub> ) <sup>2</sup> - (6.25D-z <sub>v</sub> ) <sup>2</sup> ] <sup>1/3</sup> / (2a'/2)}		
<b>Solve for Height of CASC critical vertical velocity V<sub>crit</sub> 5.30 m/s plume-averaged vertical velocity</b> <b>Critical VV &lt; Top of Jet</b>					
Find Height above Stack z <sub>crit</sub>	#N/A meters	#N/A feet	Solve for x=(z-z <sub>v</sub> ) simultaneously in both eqs. (i.e., Va and a)		
Height above Ground z <sub>crit</sub> +h <sub>s</sub>	#N/A meters	#N/A feet	for V=4.3 m/s using the cubic equation ax <sup>3</sup> +bx <sup>2</sup> +cx+d=0, where a=1, c=0, and b=(-0.12F <sub>0</sub> )/(4.3 <sup>3</sup> 0.16 <sup>3</sup> )= -888.4987		
<b>Interpolated Height of critical vertical velocity in Jet Phase:</b> and d=[0.12F <sub>0</sub> (6.25D-z <sub>v</sub> ) <sup>2</sup> -(Va) <sub>0</sub> <sup>3</sup> ]/(4.3 <sup>3</sup> 0.16 <sup>3</sup> )= 207252730					
Find Height above Stack z <sub>crit</sub>	#N/A meters	#N/A feet	<a href="http://www.1728.org/cubic.htm">http://www.1728.org/cubic.htm</a>		
Height above Ground z <sub>crit</sub> +h <sub>s</sub>	#N/A meters	#N/A feet	gives the real solution x = z-z <sub>v</sub> = -400.9163		
or z(m) = -382.959					
<b>Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase:</b> z(ft) = -1256.4					
Height (feet)	(meters)	Plume Radius(m)	Vert. Vel(m/s)	Plume Temp(K)	
above ground	above stack				
Stack. Rel. Ht = 130.0	0.00	40.665	4.01		Jet Phase/Interpolated
150.0	6.10	41.153	3.99		
200.0	21.34	42.372	3.93		Jet Phase < 5.3m/s everywhere
300.0	51.82	44.810	3.81		
400.0	82.30	47.248	3.69		
500.0	112.78	49.687	3.57		
600.0	143.26	52.125	3.44		
700.0	173.74	54.564	3.32		
800.0	204.22	57.002	3.20		
900.0	234.70	59.440	3.08		
1000.0	265.18	61.879	2.96		
1100.0	295.66	64.317	2.84		
1200.0	326.14	66.756	2.72		
1300.0	356.62	69.194	2.60		
1400.0	387.10	71.632	2.48		
1500.0	417.58	74.071	2.36		
1600.0	448.06	76.509	2.24		
1700.0	478.54	78.947	2.12		
1744.7	492.16	80.037	2.07		
<b>Top of jet = 1797.7 508.32 81.330 2.01 Spillane Equations:</b>					
1800.0	509.02	78.570	2.06	276.01	V <sub>plume</sub> =(Va) <sub>0</sub> <sup>3</sup> +0.12F <sub>0</sub> [(z-z <sub>v</sub> ) <sup>2</sup> -(6.25D-z <sub>v</sub> ) <sup>2</sup> ] <sup>1/3</sup> / a
1900.0	539.50	83.447	3.31	272.50	a = 0.16(z-z <sub>v</sub> )
2000.0	569.98	88.324	3.83	271.50	θ <sub>s</sub> =θ <sub>a</sub> (1+(1-(θ <sub>s</sub> /θ <sub>a</sub> ))*V <sub>exit</sub> D <sup>2</sup> /(4V <sub>plume</sub> *a <sup>2</sup> *λ <sup>2</sup> ))
2100.0	600.46	93.200	4.14	270.94	
2200.0	630.94	98.077	4.34	270.55	
2400.0	691.90	107.831	4.57	270.03	
2600.0	752.86	117.584	4.68	269.70	
2800.0	813.82	127.338	4.72	269.46	Max<5.3 m/s
3000.0	874.78	137.092	4.72	269.28	
3200.0	935.74	146.845	4.71	269.14	
3400.0	996.70	156.599	4.68	269.02	
3600.0	1057.66	166.352	4.64	268.93	
3800.0	1118.62	176.106	4.60	268.85	
4000.0	1179.58	185.860	4.55	268.79	
4200.0	1240.54	195.613	4.50	268.73	
4400.0	1301.50	205.367	4.46	268.68	
4600.0	1362.46	215.120	4.41	268.64	
4800.0	1423.42	224.874	4.36	268.60	
5000.0	1484.38	234.628	4.32	268.57	



SINGLE Plume Average Vertical Velocities for PEP ACC with Effective Diameter for 32 cells at 64F (Annual-Average)					
"Aviation Safety and Buoyant Plumes," Peter Best, et. al.					
"The Evaluation of Maximum Uplift Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia," Dr. K.T. Spillane					
<b>Ambient Conditions:</b>		<b>Constants:</b> Assume neutral conditions (dB/dz=0 or $\theta_a = \theta_b$ )			
Ambient Potential Temp $\theta_a$	290.93 Kelvins	64.0 °F	0.3048 meters/feet		
<b>Plume Exit Conditions:</b>		Gravity g 9.81 m/s <sup>2</sup>			
Stack Height $h_s$	39.62 meters	130.0 feet	$\lambda$	1.11	
Stack Diameter D	81.3297 meters	266.83 feet	$\lambda_0$	-1.0	
Stack Velocity $V_{exit}$	5.67 m/s	18.60 ft/sec	$\pi V_{exit} D^2 / 4$		
Volumetric Flow	29,455.83 cu.m/sec	62,413.371 ACFM	Sect.2/¶1		
Stack Potential Temp $\theta_s$	305.55 Kelvins	90.32 °F	$g V_{exit} D^2 (1-\theta_s/\theta_a) / 4 = \text{Vol.Flow}(g/m)(1-\theta_s/\theta_a)$		
Initial Stack Buoyancy Flux $F_0$	4,401.04 m <sup>4</sup> /s <sup>3</sup>		Sect.2/¶1		
Plume Buoyancy Flux F	N/A m <sup>4</sup> /s <sup>3</sup>		$\lambda^2 g V a^2 (1-\theta_s/\theta_a)$ for a, V, $\theta_p$ at plume height (see below)		
<b>Conditions at End (Top) of Jet Phase:</b>					
Height above Stack $z_{jet}$	508.311 meters*	1667.7 feet*	$z_{jet} = 6.25D$ , meters*=meters above stack top	Sect.3/¶1	
Height above Ground $z_{jet}+h_s$	547.931 meters	1797.7 feet	"		
Vertical Velocity $V_{jet}$	2.835 m/s	9.30 ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"	
Plume Top-Hat Diameter $2a_{jet}$	162.659 meters	533.7 feet	$2a_{jet} = 2D$	Conservation of momentum	"
<b>Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase</b>					
<b>Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:</b>					
Plume Top-Hat Radius a	Solutions in Table Below		0.16(z-z <sub>v</sub> ), or linear increase with height		Sect.2/Eq.6
Virtual Source Height z <sub>v</sub>	12.310 meters*	40.4 feet*	6.25D[1-( $\theta_s/\theta_a$ ) <sup>1/2</sup> ], meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z <sub>v</sub> +h <sub>s</sub>	51.930 meters	170.4 feet	where ( $\theta_s/\theta_a$ ) <sup>1/2</sup> = ( $\theta_s/\theta_a$ ) <sup>1/2</sup> = 0.9758		
Vertical Velocity V	Solutions in Table Below		$((Va)_0^3 + 0.12F_0 [(z-z_v)^2 - (6.25D-z_v)^2])^{1/3} / a$		Sect.2.1(6)
Product (Va) <sub>0</sub>	224.986 m <sup>2</sup> /s		$V_{exit} D / (2(\theta_s/\theta_a)^{1/2})$		
<b>Solve for plume-averaged vertical velocity at height 1,500.0 feet 457.2 meters above ground (z'+h<sub>s</sub>)</b>					
Gives the following Height above Stack z'	417.580 meters*	1370.0 feet*	<b>Height Entered &lt; Top of Jet - Cannot Be Solved</b>		
Plume Top-Hat Diameter 2a'	#N/A meters	#N/A feet	$2a' = 2^*0.16(z'-z_v)$		Sect.2/Eq.6
Vertical Velocity V	#N/A m/s	#N/A ft/sec	$V = ((Va)_0^3 + 0.12F_0 [(z-z_v)^2 - (6.25D-z_v)^2])^{1/3} / (2a'/2)$		Sect.2/Eq.6
<b>Solve for Height of CASC critical vertical velocity V<sub>crit</sub> 5.30 m/s plume-averaged vertical velocity Critical VV &lt; Top of Jet</b>					
Find Height above Stack z <sub>crit</sub>	#N/A meters	#N/A feet	Solve for x=(z-z <sub>v</sub> ) simultaneously in both eqs. (i.e., Va and a)		
Height above Ground z <sub>crit</sub> +h <sub>s</sub>	#N/A meters	#N/A feet	for V=4.3 m/s using the cubic equation ax <sup>3</sup> +bx <sup>2</sup> +cx+d=0, where		
a=1, c=0, and b=(-0.12F <sub>0</sub> )/(4.3 <sup>3</sup> 0.16 <sup>3</sup> )= -866.0624					
and d=[0.12F <sub>0</sub> (6.25D-z <sub>v</sub> ) <sup>2</sup> -(Va) <sub>0</sub> <sup>3</sup> ]/(4.3 <sup>3</sup> 0.16 <sup>3</sup> )= 194390031					
<a href="http://www.1728.org/cubic.htm">http://www.1728.org/cubic.htm</a>					
Find Height above Stack z <sub>crit</sub>	66.340 meters	217.7 feet	gives the real solution x = z-z <sub>v</sub> = -392.9380		
Height above Ground z <sub>crit</sub> +h <sub>s</sub>	105.960 meters	347.6 feet	or z(m) = -380.628		
z(ft) = -1248.8					
<b>Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase:</b>					
	Height (feet)	(meters)	Plume Radius(m)	Vert. Vel(m/s)	Plume Temp(K)
	above ground	above stack			
	<b>Stack.Rel.Ht = 130.0</b>	<b>0.00</b>	<b>40.665</b>	<b>5.67</b>	<b>Jet Phase/Interpolated</b>
	150.0	6.10	41.153	5.64	
	200.0	21.34	42.372	5.55	<b>Jet Phase &gt; 5.3m/s up to ~350'agl</b>
	300.0	51.82	44.810	5.38	
	400.0	82.30	47.248	5.21	
	500.0	112.78	49.687	5.04	
	600.0	143.26	52.125	4.87	
	700.0	173.74	54.564	4.70	
	800.0	204.22	57.002	4.53	
	900.0	234.70	59.440	4.36	
	1000.0	265.18	61.879	4.19	
	1100.0	295.66	64.317	4.02	
	1200.0	326.14	66.756	3.85	
	1300.0	356.62	69.194	3.68	
	1400.0	387.10	71.632	3.51	
	1500.0	417.58	74.071	3.34	
	1600.0	448.06	76.509	3.17	
	1700.0	478.54	78.947	3.00	
	1744.7	492.16	80.037	2.93	
	<b>Top of jet = 1797.7</b>	<b>508.32</b>	<b>81.330</b>	<b>2.84</b>	<b>Spillane Equations:</b>
	1800.0	509.02	79.474	2.86	296.79 $V_{plume} = ((Va)_0^3 + 0.12F_0 [(z-z_v)^2 - (6.25D-z_v)^2])^{1/3} / a$
	1900.0	539.50	84.350	3.61	295.05 a = 0.16(z-z <sub>v</sub> )
	2000.0	569.98	89.227	4.01	294.25 $\theta_p = \theta_s (1 + (1 - \theta_s/\theta_a)) * (V_{exit} D^2 / (4V_{plume}^2 a^2 \lambda^2))$
	2100.0	600.46	94.104	4.25	293.74
	2200.0	630.94	98.981	4.42	293.38
	2400.0	691.90	108.734	4.60	292.88
	2600.0	752.86	118.488	4.69	292.54
	2800.0	813.82	128.242	4.71	292.30 <b>Max&lt;5.3 m/s</b>
	3000.0	874.78	137.995	4.71	292.11
	3200.0	935.74	147.749	4.69	291.97
	3400.0	996.70	157.502	4.65	291.85
	3600.0	1057.66	167.256	4.61	291.75
	3800.0	1118.62	177.010	4.57	291.67
	4000.0	1179.58	186.763	4.52	291.60
	4200.0	1240.54	196.517	4.47	291.54
	4400.0	1301.50	206.270	4.42	291.49
	4600.0	1362.46	216.024	4.37	291.45
	4800.0	1423.42	225.778	4.33	291.41
	5000.0	1484.38	235.531	4.28	291.38





SINGLE Plume Average Vertical Velocities for PEP ACC with Effective Diameter for 30 cells at 98F (Summer)					
"Aviation Safety and Buoyant Plumes," Peter Best, et al.					
"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia," Dr. K.T. Spillane					
<b>Ambient Conditions</b>		Constants: Assume neutral conditions ( $\delta\theta/\delta z=0$ or $\theta_s=\theta_e$ )			
Ambient Potential Temp $\theta_s$	309.82 Kelvins	98.0 °F	0.3048 meters/feet		
<b>Plume Exit Conditions</b>		Gravity g 9.81 m/s <sup>2</sup>			
Stack Height $h_s$	39.62 meters	130.0 feet	$\lambda$	1.11	
Stack Diameter D	78.7472 meters	258.36 feet	$\lambda_0$	~1.0	
Stack Velocity $V_{exit}$	5.69 m/s	18.67 f/sec	$\pi V_{exit} D^2/4$		
Volumetric Flow	27,712.29 cu.m/sec	58,719.014 ACFM	Sect.2¶1		
Stack Potential Temp $\theta_s$	327.65 Kelvins	130.10 °F	$g V_{exit} D^2 (1-\theta_s/\theta_e)/4 = Vol.Flow(g/m^3)(1-\theta_s/\theta_e)$		
Initial Stack Buoyancy Flux $F_0$	4,709.04 m <sup>3</sup> /s <sup>2</sup>		Sect.2¶1		
Plume Buoyancy Flux F	N/A m <sup>3</sup> /s <sup>2</sup>		$\lambda^2 g V_{exit}^2 (1-\theta_s/\theta_e)$ for a, V, $\theta_s$ at plume height (see below)		
<b>Conditions at End (Top) of Jet Phase:</b>					
Height above Stack $z_{jet}$	492.170 meters*	1614.7 feet*	$z_{jet} = 6.25D$ , meters*=meters above stack top		Sect.3¶1
Height above Ground $z_{jet}+h_s$	531.790 meters	1744.7 feet			
Vertical Velocity $V_{jet}$	2.845 m/s	9.33 f/sec	$V_{jet} = 0.5 V_{exit} = V_{exit}/2$		"
Plume Top-Hat Diameter $2a_{jet}$	157.494 meters	516.7 feet	$2a_{jet} = 2D$		Conservation of momentum
<b>Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase</b>					
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:					
Plume Top-Hat Radius a	Solutions in Table Below		0.16(z-z <sub>v</sub> ), or linear increase with height		Sect.2/Eq.6
Virtual Source Height z <sub>v</sub>	13.579 meters*	44.5 feet*	6.25D[1-( $\theta_s/\theta_e$ ) <sup>1/2</sup> ], meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z <sub>v</sub> +h <sub>s</sub>	53.199 meters	174.5 feet	where ( $\theta_s/\theta_e$ ) <sup>1/2</sup> = ( $\theta_s/\theta_e$ ) <sup>1/2</sup> = 0.9724		
Vertical Velocity V	Solutions in Table Below		$\{[(Va)^3 + 0.12F_0 [(z-z_v)^2 - (6.25D-z_v)^2]]^{1/3}\} / a$		Sect.2.1(6)
Product (Va) <sub>e</sub>	217.855 m <sup>3</sup> /s		$V_{exit} D/2(\theta_s/\theta_e)^{1/2}$		
Solve for plume-averaged vertical velocity at height 1,500.0 feet 457.2 meters above ground (z'+h <sub>s</sub> )					
Gives the following Height above Stack z'	417.580 meters*	1370.0 feet*	Height Entered < Top of Jet - Cannot Be Solved		
Plume Top-Hat Diameter 2a'	#N/A meters	#N/A feet	$2a' = 2*0.16(z'-z_v)$		Sect.2/Eq.6
Vertical Velocity V	#N/A m/s	#N/A f/sec	$V = \{[(Va)^3 + 0.12F_0 [(z-z_v)^2 - (6.25D-z_v)^2]]^{1/3}\} / (2a' / 2)$		Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V <sub>crit</sub> 5.30 m/s plume-averaged vertical velocity Critical VV < Top of Jet					
Find Height above Stack z <sub>crit</sub>	#N/A meters	#N/A feet	Solve for x=(z-z <sub>v</sub> ) simultaneously in both eqs. (i.e., Va and a)		
Height above Ground z <sub>crit</sub> +h <sub>s</sub>	#N/A meters	#N/A feet	for V=4.3 m/s using the cubic equation ax <sup>3</sup> +bx <sup>2</sup> +cx+d=0, where		
Interpolated Height of critical vertical velocity in Jet Phase:					
Find Height above Stack z <sub>crit</sub>	67.468 meters	221.4 feet	a=1, c=0, and b=-[0.12F <sub>0</sub> ]/(4.3 <sup>3</sup> ·0.16 <sup>3</sup> )= -926.6727		
Height above Ground z <sub>crit</sub> +h <sub>s</sub>	107.088 meters	351.3 feet	and d=[0.12F <sub>0</sub> (6.25D-z <sub>v</sub> ) <sup>2</sup> -V <sub>crit</sub> <sup>3</sup> ]/(4.3 <sup>3</sup> ·0.16 <sup>3</sup> )= 195298353		
gives the real solution x = z-z <sub>v</sub> = -385.7550					
or z(m) = -372.176					
Table of Plume Top-Hat Diameters (2a) and Plume-averaged Vertical Velocities starting at end of jet phase: z(ft) = -1221.1					
Height (feet)	(meters)	Plume Radius(m)	Vert. Vel(m/s)	Plume Temp(K)	
Stack Rel.Ht = 130.0	0.00	39.374	5.69		Jet Phase/interpolated
150.0	6.40	39.861	5.65		
200.0	21.34	41.081	5.57		Jet Phase > 5.3m/s up to ~350'agl
300.0	51.82	43.519	5.39		
400.0	82.30	45.957	5.21		
500.0	112.78	48.396	5.04		
600.0	143.26	50.834	4.86		
700.0	173.74	53.273	4.69		
800.0	204.22	55.711	4.51		
900.0	234.70	58.150	4.33		
1000.0	265.18	60.588	4.16		
1100.0	295.66	63.027	3.98		
1200.0	326.14	65.465	3.80		
1300.0	356.62	67.903	3.63		
1400.0	387.10	70.342	3.45		
1500.0	417.58	72.780	3.28		
1600.0	448.06	75.219	3.10		
1700.0	478.54	77.657	2.92		
Top of jet = 1744.7	492.16	78.747	2.85		
1797.7	508.32	79.158	3.38	315.51	Spillane Equations
1800.0	509.02	79.271	3.40	315.47	$V_{plume} = \{[(Va)^3 + 0.12F_0 [(z-z_v)^2 - (6.25D-z_v)^2]]^{1/3}\} / a$
1900.0	539.50	84.147	3.97	314.12	a = 0.16(z-z <sub>v</sub> )
2000.0	569.98	89.024	4.29	313.37	$\theta_s = \theta_e (1 + (1 - (\theta_s/\theta_e)) / (V_{exit} D^2 / (4 V_{plume}^2 a^2 \lambda^2)))$
2100.0	600.46	93.901	4.50	312.86	
2200.0	630.94	98.778	4.64	312.49	
2400.0	691.90	108.531	4.79	311.96	
2600.0	752.86	118.285	4.86	311.60	
2800.0	813.82	128.039	4.87	311.33	Max < 5.3 m/s
3000.0	874.78	137.792	4.86	311.13	
3200.0	935.74	147.546	4.83	310.97	
3400.0	996.70	157.299	4.79	310.84	
3600.0	1057.66	167.053	4.74	310.73	
3800.0	1118.62	176.807	4.69	310.64	
4000.0	1179.58	186.560	4.64	310.57	
4200.0	1240.54	196.314	4.59	310.50	
4400.0	1301.50	206.067	4.54	310.45	
4600.0	1362.46	215.821	4.49	310.40	
4800.0	1423.42	225.575	4.44	310.35	
5000.0	1484.38	235.328	4.39	310.32	

