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Vertical Plume Velocity Assessment

NorthTown Backup Generating Facility

San Jose, California

Submitted to

California Energy Commission

Submitted by

GIC

Prepared by

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ATMOSPHERIC DYNAMICS, INC
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Introduction

This report presents the evaluation of the Northtown Backup Generating Facility (NTBGF) source generated plumes from the routine testing of the 42 Caterpillar diesel engines and 36 rooftop closed loop cooling towers on the effects on airport/aircraft operations. The Normal Y. Mineta San Jose International Airport is located approximately 0.7 kilometers miles east west-southwest of the NTBGF. This report is based upon an analysis prepared by Atmospheric Dynamics, Inc. in accordance with the California Energy Commission (CEC) application requirements for a Small Power Plant Exemption (SPPE) pursuant to the power plant siting regulations. This analysis is but one part of a larger analysis, which seeks an SPPE Decision from the CEC.

Based on the stack parameter data, an analysis of the potential plume characteristics from the routine operation the diesel engines and rooftop cooling towers on vertical winds was prepared and compared to the California Energy Commission (CEC) significance criteria of 5.3 meters per second (m/s) for the average vertical plume velocities as described below.

Atmospheric Dynamics, Inc. (ADI) prepared a screening level plume vertical velocity assessment which are based on the calm wind Spillane methodology outlined in the *"Aviation Safety and Buoyant Plumes"* paper (Peter Best, et. al., presented at the Clean Air Conference, Newcastle, New South Wales, Australia, 2003). This methodology is also recognized as a screening tool for aviation safety set out by the Australian Civil Aviation Safety Authority (CASA) and presented in *"AC 139-5(1) Plume Rise Assessments (CASA, 2012)".*

The aim of this screening assessment is to conservatively determine the potential for turbulence generated by the diesel engines and rooftop cooling tower waste heat exhaust plumes. Part 139.370 of the Australian Civil Aviation Safety Regulations (1998, 2004) provides that CASA may determine that plume velocities in excess of 4.3 m/s is or will be a potential hazard to aircraft operations. The *Manual of Aviation Meteorology* (Australian Bureau of Meteorology 2003) defines severe turbulence as a vertical wind gust velocity in excess of 10.6 m/s. The assumed critical vertical velocity used as a CEC significance threshold is 5.3 meters per second* (m/s) but it should be noted that the basis of the original CASA derived threshold of 4.3 m/s has been lost in antiquity and that CASA no longer relies on the 1998 and 2004 regulations that established this critical threshold other than to note that a more rigorous analysis, which includes site specific meteorology, should be used if the 4.3 m/s and 10.6 m/s screening thresholds are exceeded. The screening method uses absolute worst-case assumptions of calm winds and neutral atmospheric conditions for the entire vertical extent of the plume to determine these worst-case impacts. It should be noted that these results are extremely conservative in that these worst-case conditions typically only occur during a few hours each year.

The Spillane methodology is generally applied to a limited number of plume source geometry's (turbines, power plant boilers, etc.) with the stacks arranged linearly (in a single straight-line) and separated by distances that typically exceed the individual stack diameters. For the diesel engines, this assumption was maintained. Only one engine stack was modeled consistent with the normal operational testing schedule of the emergency generator engines. For the rooftop closed circuit cooling towers, a conservative assumption was made in order to use the Spillane methodology on an atypical cooling tower plume configuration, which is made up of 18 three (3) cell towers on each building arranged in two (2) parallel rows. Here, the methodology, as described below, assumed that all three (3) cells for each side by side cooling tower were merged into a single stack with an effective diameter based on the combined area of all six (6) cooling

*For the Puente Power Project (Docket#15-AFC-01, TN#213674, 9/15/2016), "CEC staff … concluded that an average velocity of 5.3 m/s is the appropriate velocity … [for a plume velocity threshold]." The CEC staff "Plume Background Threshold" attached to the docketed document concludes with "…[CEC] staff will use 10.6 m/s peak vertical plume velocity as the new threshold. The altitude at which a plume would have a peak vertical velocity of 10.6 m/s would be the same altitude at which a plume would have an average vertical velocity of half that, 5.3 m/s."

tower cells. In other words, a single stack was assumed to initially describe the release parameters of the combined cooling tower cells in each of the 18 individual cooling towers. The effective plume diameter is appropriate for each individual cooling tower based on the close proximity and arrangement of the six (6) individual cells.

Screening Methodology and Vertical Plume Velocity Calculations

The Spillane methodology is based on worst-case calm wind neutral stability conditions to assess the average plume vertical velocity as a function of height. The methodology is based on well-verified laboratory and theoretical treatments of the rise and spread of a buoyant jet, both into a still ambient environment and into a light crosswind. This treatment covers in detail the initial dynamics of the plume as it exits the stack and the entrainment of ambient air into the plume as it rises directly above the stack. In addition to providing clarifications and algebraic solutions to the Spillane methodology, the 2003 Peter Best paper provides additional methodologies that also consider the enhancement of vertical velocities that may occur if the plumes from multiple identical stacks merge and form a higher buoyancy combined plume (referred to here as the enhanced Spillane methodology).

The vertical plume assessment will involve several stages of development. For individual plumes, the stages are:

(a) In the first stage very close to the stack exit, the high plume momentum will result in a short section in which the conditions at the center of the plume are relatively unaffected by ambient and plume buoyancy conditions. This jet phase extends from the stack exit to approximately a distance of 6.25 D above the stack (where D is the stack diameter) in calm conditions. At the end of this stage, the plume-averaged vertical velocity has decreased to half of the stack exit velocity, with a corresponding increase, or doubling, in effective plume diameter.

(b) In the second stage, the plume responds to differences between ambient and plume buoyancy conditions, with much cooler and less turbulent ambient air being entrained into the plume from the outside regions of the plume towards the plume centerline. The momentum and buoyancy of the plume significantly influences plume rise and subsequently the dilution of the stack exhaust to decrease plume vertical velocities. This dilution is very sensitive to ambient wind speed, so the calm wind conditions considered here are extremely conservative.

(c) In the third stage of plume development, plume rise is due entirely to the buoyancy of the plume and continues from some distance until there is an equalization of turbulence conditions within and outside the plume. This final rise is often only achieved at considerable heights/distances from the stack where the effective average vertical velocity is then close to zero. Since there is very little turbulence and near-zero vertical velocities, this stage of plume development is usually not considered for this type of analysis.

In the second stage of development, the analytical solution of the governing equations under these conditions is given by:

$$a = 0.16(z - z_v)$$

$$V = \{(Va)_o^{3} + 0.12F_o\,[\,(z - z_v)^2 - (6.25D - z_v)^2]\}^{1/3} / a$$

Where the subscript ‘o’ refers to values of the parameters at the stack outlet and the variables are:

a	plume radius (m)
V	average vertical velocity (m/s)
z	height above stack top (m)
z_v	virtual source height (m)
D	stack diameter (m)
F_o	buoyancy flux evaluated at the stack outlet (m^4s^{-3})

These are the two primary equations governing the growth of a single plume in the second stage of development under neutral calm wind conditions. Additional equations governing the first stage of single plume development as well as the interaction of multiple plumes in the second stage of development are discussed in detail in the Best paper.

For multiple stacks in the enhanced Spillane methodology, the equations governing the second stage are calculated from the point when the plumes begin to merge until they are fully merged. The plume merging begins at the height where the plume diameters equal the stack separations and the plumes are fully merged at the height where the plume diameters are equal to 2d(N-1)/2 for three or more stacks or 2d for two stacks. At the fully merged height, the merged plume diameter and velocity is enhanced by the fourth root of the number of stacks. Above the fully merged plume height, the enhanced plume diameter and plume velocities follow the regular equations given for the second stage. Below the fully merged plume height for the merging phase, plume velocities are linearly interpolated by height from the single plume velocity at the height where the plumes begin to merge to the enhanced plume velocity at the fully merged plume height.

Vertical Plume Velocity Calculations for the Diesel Engines

The NTBGF is comprised of 40 individual large and two (2) small diesel emergency generator stacks. The small diesel emergency generator was not assessed as it would have smaller plume vertical velocities. Generator stack parameter data (plume exit velocity, plume exit temperature and stack exit diameter) were provided by Caterpillar. Only one (1) engine will be tested during any one hour. While the engines will be tested at minimum loads, the 100 percent load case was utilized for the worst-case plume analysis. For the engine analysis, two ambient conditions were considered: 41.0°F, the minimum monthly mean of daily minimum temperatures, and 84.3°F, the maximum monthly mean of daily maximum temperatures for the San Jose Airport (*“Climatology of the United States No. 81 – Monthly Station Normals of Temperature, Precipitation, and Heating and Cooling Degree Days 1971-2000 – California”*, February 2002, and *“Climatology of the United States No 20 – Monthly Station Climate Summaries, 1971-2000 – California”*, February 2004). These data is summarized in Table 1.

Table 1
Caterpillar Diesel Stack Characteristics for Vertical Plume Velocity Analysis

Case #	1	2
Ambient Temperature (°F)*	41.0	41.0
Stack Diameter (m)	0.7112	0.7112
Exhaust Velocity (m/s)*	30.44	30.44
Exhaust Temperature (K)*	733.15	733.15
Stack Release Height (m)	7.72	7.72
Stack Buoyancy Flux (m^4/s^3)	22.195	23.439
*Stack data provided by Caterpillar at 100% load		

Screening level vertical plume velocity assessments were made for two ambient temperatures with calm winds and neutral atmospheric conditions for the cases presented in Table 1 which are based on 100 percent load. The results based on the two ambient conditions are presented in Table 2 and the output from the calculation spreadsheet provided in Attachment A.

The initial jet phase extends to a height of about 39.9 feet above grade level (ft-agl) for both cases. After the jet phase, plume temperature buoyancy characteristics modeled in the Spillane methodology cause a uniform decrease in plume-averaged vertical velocities, with the critical plume-averaged vertical velocity of 5.3 m/s occurring at about 62.3 ft-agl for both cases

Table 2
Diesel Engine Vertical Plume Velocity Analysis Results for Reference Height

Case #	1	2
Ambient Temperature (°F)	41.0	84.3
Single Plume Results:		
Plume-Averaged Vertical Velocity at 200 feet-agl (m/s)	2.40	2.36
Height of 5.3 m/s Plume-Averaged Vertical Velocity (feet-agl)	62.3	62.6

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the diesel engine always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above about 63 ft-agl. Even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities, so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear when realistically, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Vertical Plume Velocity Calculations for the Rooftop Cooling Towers

Each data center has 18 rooftop cooling towers, each of which are each comprised of three (3) individual cells, with a cell fan diameter of 66 inches. The 18 cooling towers are arranged in a 9 x 2 configuration along the longer building length (averaging 4 feet between adjacent towers). Based on the groupings of cooling towers, the single and merged plumes were based on the combined effective stack of six (6) individual cells (each tower is made up of three cells) by merging plumes along the center in between the two cooling towers (1 merged stack for each cooling tower and 9 merged stacks for the combined plume assessment). Cooling tower stack parameter data (exit velocity and temperature) were provided by the applicant. An effective stack diameter for all six (6) cells was utilized for each merged cooling tower. To utilize conservative modeling methods, all towers/cells were assumed to be operating at full load. These data are summarized in Table 3 for the same ambient temperatures used for the engine analysis.

Table 3
Chiller Stack Characteristics for Vertical Plume Velocity Analysis

Case #	1	2
Ambient Temperature (°F)*	41.0	84.3
Effective Stack Diameter (m)**	4.11	4.11
Exhaust Velocity (m/s)*	13.11	13.11
Exhaust Temperature (K)*	313.32	313.32
Stack Release Height (m)	24.54	24.54
Stack Buoyancy Flux (m^4/s^3)	20.85	19.25

*Chiller stack data provided by the applicant
** Calculated value based on the cell diameter of 66 inches multiplied by the square of the number of operating cells, or $D_{eff} = 66*\sqrt{6}$

The Spillane methodology was originally developed to treat multiple individual stacks that are arranged along a linear x or y direction, but not both directions at once, with stack separations much greater than the stack diameters, typical of boilers/turbines at large power plants. As noted above, the 18 cooling towers are generally arranged in a 9 x 2 pattern. Therefore, the enhanced Spillane methodology was based on calculating the total merging height for the largest linear direction of tower placements (which is nine cooling tower cells spaced 11.7 feet apart (based on the distance between the calculated equivalent diameter cells) along the longer length of the building). The grouping of 9 (3x18) towers were considered in the calculation of vertical velocity plume enhancement (both at and above the totally merged height, and for the interpolation down to the plume touching height. The effective single stack diameter of each merged tower was based on the merged six (6) cell diameter.

Screening level vertical plume velocity assessments were made for the same ambient temperatures with calm winds and neutral atmospheric conditions as was done for the emergency generator engines. These cases represent worst-case conditions of calm winds at all levels of a neutral atmosphere. The results are presented in Table 4 and the output from the calculation spreadsheets are provided in Attachment A.

The initial jet phase extends to a height of about 164.7 ft-agl for both cases. The critical plume-averaged vertical velocity of 5.3 m/s occurs at the end of the jet phase at about 186 ft-agl for both cases. The plumes touch (begin to merge) at about 118.7 ft-agl and are fully merged at about 374.9 ft-agl for both cases. Under the enhanced Spillane methodology, the merged plume-averaged vertical velocities approach 5.3 m/s at 343.9 ft-agl.

Table 4 Chiller Vertical Plume Velocity Analysis Results for Reference Height		
Case #	**1**	**2**
Ambient Temperature (°F)	41.0	84.3
Single Plume Results:		
Height of 5.3 m/s Plume-Averaged Vertical Velocity (Within the Jet Phase, feet-agl)	186.0	185.9
Merged Plume Results:		
Height of 5.3 m/s Plume-Averaged Vertical Velocity (Within the Merging Phase, feet-agl)	343.9	342.8
Plume-Averaged Vertical Velocity at 500 feet-agl (m/s)	3.75	3.70

These screening results indicate that mechanical and thermal turbulence levels due to the flow from the cooling towers always remain in the light turbulence category and below the significance level of 5.3 m/s at all heights above about 344 ft-agl. Even light wind speeds can dramatically decrease the predicted plume-averaged vertical velocities so the above results are very conservative indications of adverse conditions. The important factor for a given location is the appropriateness of available information for estimating true wind and temperature profiles throughout a typical year. Theoretical calculations, as shown in the tables above, are likely to overestimate the expected vertical velocities, for the following reasons:

- The wind profile is assumed constant with height with no occurrence of wind-shear when realistically, there is a considerable variation with height, especially in light winds;
- Worst-case scenarios are based on very light-wind, near-neutral atmospheric conditions with maximum loading.

Attachment A
Spillane Method Plume Velocity Calculations

SINGLE Plume Average Vertical Velocities for Large Emer.Gen Engine, 100% Load, and Maximum Stack Height - Winter Min*

"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.

"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane

Ambient Conditions:

Constants: Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)

Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F		0.3048 meters/feet
Plume Exit Conditions:					Gravity g	9.81 m/s^2
Maximum Stack Height h_s	7.72	meters	25 4/12	feet-inches	λ	1.11
Stack Diameter D	0.7112	meters	28	inches	λ_o	~1.0
Stack Velocity V_{exit}	30.44	m/s	99.86	ft/sec		
Volumetric Flow	12.09	cu.m/sec	25,620	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	733.37	Kelvins	860	°F		
Initial Stack Buoyancy Flux F_o	23.4388	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)	

Conditions at End (Top) of Jet Phase:

Height above Stack z_{jet}	4.445	meters*	14.6	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	12.165	meters	**39.9**	feet		"
Vertical Velocity V_{jet}	15.220	m/s	49.93	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	1.422	meters	4.7	feet	$2a_{jet}$ = 2D; Conservation of momentum	"

Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase

Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:

Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	1.708	meters*	5.6	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	9.428	meters	30.9	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6159
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	Sect.2.1(6)
Product $(Va)_o$	6.666	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$	

Solve for plume-averaged vertical velocity at height			**200.0**	**feet**	60.96 meters above ground ($z'+h_s$)	
Gives the following Height above Stack z'	53.240	meters*	174.7	feet*		
Plume Top-Hat Diameter 2a'	16.490	meters	54.1	feet	$2a'=2*0.16(z'-z_v)$	Sect.2/Eq.6
Vertical Velocity V	**2.400**	**m/s**	7.87	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$	Sect.2/Eq.6

Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s plume-averaged vertical velocity**			**Critical VV > Top of Jet (Spillane)**
Find Height above Stack z_{crit}	11.262	meters	36.9	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)	
Height above Ground $z_{crit}+h_s$	18.982	meters	**62.3**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where	
					a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=	-4.6124
Interpolated Height of critical vertical velocity in Jet Phase:					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=	-451.18
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet		http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	gives the real solution x = z-zv =	9.5546
					or z(m/above stack) =	11.262
					z(ft/above ground) =	62.3

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**SingleStk VertVel(m/s)**	**Plume Temp(K)**		
Stack.Rel.Ht = 25.3	**0.00**	**0.356**	**30.44**			
30.0	1.42	0.470	25.56		**Jet Phase Eqs:**	**5 foot Intervals**
35.0	2.95	0.592	20.34		Linearly interpolated from Stack Rel.Ht to Top of Jet	
Top of jet = 39.9	**4.44**	**0.711**	**15.22**		**Spillane Equations:**	
40.0	4.47	0.442	15.08	461.00	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
50.0	7.52	0.930	7.72	358.93	$a = 0.16(z-z_v)$	**10 foot Intervals**
60.0	10.57	1.418	5.58	326.22	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Spillane 5.3 m/s Height = 62.3	**11.26**	**1.529**	**5.30**	321.70		
70.0	13.62	1.905	4.60	310.44		
80.0	16.66	2.393	4.04	301.46		
90.0	19.71	2.881	3.68	295.83		**Max<5.30 m/s**
100.0	22.76	3.368	3.41	292.07		
110.0	25.81	3.856	3.22	289.43		
120.0	28.86	4.344	3.06	287.49		
160.0	41.05	6.294	2.65	283.29		**40 foot Intervals**
200.0	53.24	8.245	2.40	281.46		
240.0	65.43	10.196	2.23	280.48		
280.0	77.62	12.147	2.10	279.89		
320.0	89.82	14.097	1.99	279.51		
360.0	102.01	16.048	1.90	279.25		
400.0	114.20	17.999	1.83	279.06		
500.0	144.68	22.876	1.69	278.76		**100 foot Intervals**
600.0	175.16	27.752	1.58	278.59		
700.0	205.64	32.629	1.50	278.49		
800.0	236.12	37.506	1.43	278.42		
900.0	266.60	42.383	1.37	278.37		
1000.0	297.08	47.260	1.33	278.33		
1100.0	327.56	52.136	1.28	278.30		
1200.0	358.04	57.013	1.24	278.28		
1300.0	388.52	61.890	1.21	278.27		
1400.0	419.00	66.767	1.18	278.25		
1500.0	449.48	71.644	1.15	278.24		
1600.0	479.96	76.520	1.13	278.23		
1700.0	510.44	81.397	1.11	278.22		
1800.0	540.92	86.274	1.08	278.22		
1900.0	571.40	91.151	1.06	278.21		

*Winter Min = Monthly Mean of Minimum Daily Temperatures for 1971-2000 (Lowest in December)

NOAA Sources: Climatography of the United States No.81 *"Monthly Station Normals of Temperatures, Precipitation, and Heating and Cooling Degree Days, 1971-2000 California"* and Climatography of the United States No. 20 *"Monthly Station Climate Summaries, 1971-2000 California"*

SINGLE Plume Average Vertical Velocities for Single Large Emer.Gen Engine, 100% Load, and Maximum Stack Height - Summer Max*

"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.

"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F	0.3048	meters/feet	
Plume Exit Conditions:					Gravity g 9.81	m/s^2	
Maximum Stack Height h_s	7.72	meters	25 4/12	feet-inches	λ 1.11		
Stack Diameter D	0.7112	meters	28	inches	λ_o ~1.0		
Stack Velocity V_{exit}	30.44	m/s	99.86	ft/sec			
Volumetric Flow	12.09	cu.m/sec	25,620	ACFM	$\pi V_{exit}D^2/4$		Sect.2/¶1
Stack Potential Temp θ_s	733.15	Kelvins	860	°F			
Initial Stack Buoyancy Flux F_o	22.1953	m^4/s^3			$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$		Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)		
No.of Stacks N	1			1.000	Multiple Stack Multiplication Factor ($N^{0.25}$)		
Conditions at End (Top) of Jet Phase:							
Height above Stack z_{jet}	4.445	meters*	14.6	feet*	z_{jet} = 6.25D, meters*=meters above stack top		Sect.3/¶1
Height above Ground $z_{jet}+h_s$	12.165	meters	**39.9**	feet			"
Vertical Velocity V_{jet}	15.220	m/s	49.93	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$		"
Plume Top-Hat Diameter $2a_{jet}$	1.422	meters	4.7	feet	$2a_{jet}$ = 2D	Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase							
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:							
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height		Sect.2/Eq.6
Virtual Source Height z_v	1.591	meters*	5.2	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z_v+h_s	9.311	meters	30.5	feet		where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.6420
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$		Sect.2.1(6)
Product $(Va)_o$	6.950	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$		
Solve for plume-averaged vertical velocity at height	**200.0**	**feet**		60.96	meters above ground ($z'+h_s$)		
Gives the following Height above Stack z'	53.240	meters*	174.7	feet*			
Plume Top-Hat Diameter 2a'	16.528	meters	54.2	feet	$2a'=2*0.16(z'-z_v)$		Sect.2/Eq.6
Vertical Velocity V	**2.360**	**m/s**	7.74	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$		Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**					**Critical VV > Top of Jet (Spillane)**
Find Height above Stack z_{crit}	11.358	meters	37.3	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)		
Height above Ground $z_{crit}+h_s$	19.078	meters	**62.6**	feet	for V=4.3 m/s using the cubic equation $ax^3+bx^2+cx+d=0$, where		
						a=1, c=0, and $b=-(0.12F_o)/(4.3^3 0.16^3)$=	-4.3677
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(4.3^3 0.16^3)$=	-514.94
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet			http://www.1728.org/cubic.htm
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet		gives the real solution x = z-zv =	9.7664
						or z(m/above stack) =	11.358
						z(ft/above ground) =	62.6

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 25.3	***0.00***	**0.356**	**30.44**			
30.0	1.42	0.470	25.56		**Jet Phase Eqs:**	**5 foot Intervals**
35.0	2.95	0.592	20.34		Linearly interpolated from Stack Rel.Ht to Top of Jet	
Top of jet = 39.9	***4.44***	***0.711***	***15.22***		**Spillane Equations:**	
40.0	4.47	0.461	15.08	475.37	$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
50.0	7.52	0.949	7.82	381.11	$a = 0.16(z-z_v)$	**10 foot Intervals**
60.0	10.57	1.436	5.63	349.99	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
Spillane 5.3 m/s Height = 62.6	**11.36**	**1.563**	**5.30**	345.09		
70.0	13.62	1.924	4.61	334.71		
80.0	16.66	2.412	4.03	325.88		
90.0	19.71	2.899	3.65	320.28		**Max<5.30 m/s**
100.0	22.76	3.387	3.39	316.50		
110.0	25.81	3.875	3.18	313.82		
120.0	28.86	4.362	3.02	311.86		
160.0	41.05	6.313	2.61	307.55		**40 foot Intervals**
200.0	53.24	8.264	2.36	305.65		
240.0	65.43	10.215	2.19	304.64		
280.0	77.62	12.165	2.06	304.03		
320.0	89.82	14.116	1.96	303.63		
360.0	102.01	16.067	1.87	303.36		
400.0	114.20	18.017	1.80	303.16		
500.0	144.68	22.894	1.66	302.85		**100 foot Intervals**
600.0	175.16	27.771	1.56	302.67		
700.0	205.64	32.648	1.47	302.56		
800.0	236.12	37.525	1.41	302.49		
900.0	266.60	42.401	1.35	302.44		
1000.0	297.08	47.278	1.30	302.40		
1100.0	327.56	52.155	1.26	302.37		
1200.0	358.04	57.032	1.22	302.35		
1300.0	388.52	61.909	1.19	302.33		
1400.0	419.00	66.785	1.16	302.32		
1500.0	449.48	71.662	1.13	302.31		
1600.0	479.96	76.539	1.11	302.30		
1700.0	510.44	81.416	1.09	302.29		
1800.0	540.92	86.293	1.06	302.28		
1900.0	571.40	91.169	1.05	302.27		

*Summer Max = Monthly Mean of Maximum Daily Temperatures for 1971-2000 (Highest in July)

NOAA Sources: Climatography of the United States No.81 *"Monthly Station Normals of Temperatures, Precipitation, and Heating and Cooling Degree Days, 1971-2000 California"* and Climatography of the United States No. 20 *"Monthly Station Climate Summaries, 1971-2000 California"*

SINGLE/Approximated Plume Average Vertical Velocities for Northtown Chillers - Winter Min*

Based on 9 chillers w/ 6 cells/combined tower. Calc' eff.diam for each tower wiith each cell at 66" ID (368,400 ACFM total for each combined tower).

"*Aviation Safety and Buoyant Plumes*," Peter Best, et. al.

"*The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia*," Dr. K.T. Spillane

Ambient Conditions:					**Constants:**	Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F		0.3048 meters/feet	
Plume Exit Conditions:					Gravity g	9.81 m/s^2	
Stack Height h_s	24.54	meters	80 6/12	feet-inches	λ	1.11	
Individual Chiller Stack Diameter D	4.1063	meters	161.7	inches	λ_o	~1.0	
Stack Velocity V_{exit}	13.13	m/s	43.07	ft/sec	$4Vol/(60\pi D^2)$		
Individual Chiller Volumetric Flow	173.87	cu.m/sec	368,400	ACFM	$\pi V_{exit}D^2/4$		Sect.2/¶1
Stack Potential Temp θ_s	289.26	Kelvins	61.0	°F			
Initial Stack Buoyancy Flux F_o	20.8545	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$		Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)		
Number of Towers n	9			1.732	Multiple Stack Multiplication Factor ($n^{0.25}$)		
Conditions at End (Top) of Jet Phase:							
Height above Stack z_{jet}	25.664	meters*	84.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top		Sect.3/¶1
Height above Ground $z_{jet}+h_s$	50.201	meters	**164.7**	feet			"
Vertical Velocity V_{jet}	6.564	m/s	21.54	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$		"
Plume Top-Hat Diameter $2a_{jet}$	8.213	meters	26.9	feet	$2a_{jet}$ = 2D	Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase							
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:							
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height		Sect.2/Eq.6
Virtual Source Height z_v	0.498	meters*	1.6	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top		Sect.2/Eq.6
Height above Ground z_v+h_s	25.034	meters	82.1	feet		where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9806
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a		Sect.2.1(6)
Product $(Va)_o$	26.432	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$		
Single Chiller Results:							
Solve for plume-averaged vertical velocity at height	**500.0**	**feet**		152.4	meters above ground ($z'+h_s$)		
Gives the following Height above Stack z'	127.864	meters*	419.5	feet*			
Plume Top-Hat Diameter 2a'	40.757	meters	133.7	feet	$2a'=2*0.16(z'-z_v)$		Sect.2/Eq.6
Vertical Velocity V	**1.894**	**m/s**	6.21	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$		Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**				**Critical VV > Top of Jet (Spillane)**	
Find Height above Stack z_{crit}	32.151	meters	105.5	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)		
Height above Ground $z_{crit}+h_s$	56.687	meters	**186.0**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where		
						a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=	-4.10387
Interpolated Height of critical vertical velocity in Jet Phase:						and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=	-27684.01
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet		http://www.1728.org/cubic.htm	
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet		gives the real solution x = z-zv =	31.6532
						or z(m/above stack) =	32.151
						z(ft/above ground) =	186.0

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 80.5	**0.00**	**2.053**	**13.13**			
100.0	5.94	2.529	11.61		**Jet Phase Eqs:**	**20 ft Intervals**
120.0	12.04	3.016	10.05		Linearly interpolated from Stack Rel.Ht to Top of Jet	
140.0	18.14	3.504	8.49		**Spillane Equations:**	
160.0	**24.23**	**3.992**	**6.93**			
160.0	24.23	3.992	6.93		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
Top of Single jet = 164.7	**25.66**	**4.106**	**6.56**		$a = 0.16(z-z_v)$	
180.0	30.33	4.773	5.60	282.49	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
200.0	36.42	5.748	4.73	281.91	**CEC Staff Equation:**	
220.0	42.52	6.723	4.12	281.22	$V_{mp}=n^{0.25}V_{sp}$	
240.0	48.62	7.699	3.68	280.72	**Brigg's Equation:**	
260.0	54.71	8.674	3.34	280.35	$V_{Brigg's} = (2/3) \times 1.6^{(3/2)} \times F_{mp}^{(1/2)} \times u^{(-1/2)} \times z^{(-1/2)}$	
280.0	60.81	9.650	3.07	280.06	where $F_{mp} = nF_{sp}$	
300.0	66.90	10.625	2.86	279.83		
350.0	82.14	13.063	2.47	279.64		
400.0	97.38	15.502	2.21	279.29		**50 ft Intervals**
450.0	112.62	17.940	2.03	279.05	**Max<5.3 m/s**	
500.0	127.86	20.379	1.89	278.88		
550.0	143.10	22.817	1.79	278.76		
600.0	158.34	25.255	1.70	278.67		
650.0	173.58	27.694	1.63	278.59		
700.0	188.82	30.132	1.57	278.53		
800.0	219.30	35.009	1.47	278.49		
900.0	249.78	39.886	1.40	278.42		**100 ft Intervals**
1000.0	280.26	44.763	1.33	278.37		
1100.0	310.74	49.639	1.28	278.33		
1200.0	341.22	54.516	1.24	278.30		
1300.0	371.70	59.393	1.20	278.28		
1400.0	402.18	64.270	1.17	278.26		
1500.0	432.66	69.147	1.14	278.25		
2000.0	585.06	93.531	1.02	278.24		
2500.0	737.46	117.915	0.94	278.20		**500 ft Intervals**
3000.0	889.86	142.299	0.88	278.19		
3500.0	1042.26	166.683	0.84	278.18		
4000.0	1194.66	191.067	0.80	278.17		
4500.0	1347.06	215.451	0.77	278.17		
5000.0	1499.46	239.835	0.74	278.16		

*Winter Min = Monthly Mean of Minimum Daily Temperatures for 1971-2000 (Lowest in December)

NOAA Sources: Climatography of the United

MERGED (along length) Plume Average Vertical Velocities for Northtown Chillers - Winter Min*						
	***"Aviation Safety and Buoyant Plumes,"* Peter Best, et. al.**					
	"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged					
	***Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia,"* Dr. K.T. Spillane**					
Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	278.15	Kelvins	41.0	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	24.54	meters	80 6/12	feet-inches	λ 1.11	
Individual Stack Diameter D	4.1063	meters	161.7	inches	λ_o ~1.0	
Stack Velocity V_{exit}	13.13	m/s	43.07	ft/sec	$4Vol/(60\pi D^2)$	
Individual Volumetric Flow	173.87	cu.m/sec	368,400	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	289.26	Kelvins	61.0	°F		
Initial Stack Buoyancy Flux F_o	20.85	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4 = Vol.Flow(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Total Number of Stacks n	9					
Average Adjacent Stack Separation d	3.57	meters	11.7	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**	
Number of Stacks along Orientation N	9				**plume velocities increased by $N^{0.25}$ at the height where plumes**	
					fully merged (interp. below ht, single merged stack above ht)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	25.664	meters*	84.2	feet*	$z_{jet} = 6.25D$, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	50.201	meters	**164.7**	feet		"
Vertical Velocity V_{jet}	**6.564**	m/s	21.54	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	8.213	meters	26.9	feet	$2a_{jet} = 2D$ Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	0.498	meters*	1.6	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	25.034	meters	82.1	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}=$	0.9806
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	Sect.2.1(6)
Product $(Va)_o$	26.432	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$	
Plume Merging - Based on Single Plume Calculations where:						Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	3.570	meters	11.7	feet	$2a_{touch}=d$, (or $a_{touch}=d/2$)	
Height above Stack z_{touch}	11.654	meters*	38.2	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{touch}+h_s$	36.190	meters	**118.7**	feet		
Vertical Velocity V_{touch}	**14.459**	m/s	47.4	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**28.560**	meters	93.7	feet	$2a_{full}=2d(N-1)/2$, (or $a_{full}=d(N-1)/2$) **FOR 2 STACKS, $2a_{full}=2d$**	
Height above Stack z_{full}	89.748	meters*	294.4	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{full}+h_s$	114.284	meters	**374.9**	feet		
Vertical Velocity V_{full}	2.330	m/s	7.6	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o [(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$	
Product $(V^3a)_{full}$	181	m^4/s^3				
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):						
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height	
Revised Merged Plume **Radius** a_m	24.734	meters	81.1	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs	
Revised Merged Plume Velocity V_m	**4.035**	m/s	13.24	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs	
Revised Virtual Source Height z_{full}	89.748	meters*	294.4	feet*	Height above stack where Total Merging Occurs (shown above)	
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation	
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$	
Multiple Plume Calculations					for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height		**500.0**	**feet**	152.4	meters above ground $(z+h_s)$	
Gives the following Height above Stack z	127.864	meters*	419.5	feet*	REGULAR EQNS	
Plume Top-Hat Radius a	30.832	meters	101.2	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**	
Vertical Velocity V	**3.749**	**m/s**	12.30	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**	
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**	
					V'=single plume values **if $z<z_{touch}$**	
Solve for Height of CASC critical vertical velocity V_{crit}		**5.30**	**m/s**		LESS THAN TOP OF MERGING PHASE-INTERPO	
Find Height above Stack z_{crit}	80.271	meters	263.4	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**	
Height above Ground $z_{crit}+h_s$	104.807	meters	**343.9**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**	

Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:

Height (feet) above ground	**(meters) above stack**	**Plume Radius(m)**	**Vert. Vel(m/s)**	*Single Plume Eqns (see Single Plume spreadsheet)*	
				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
				$a = 0.16(z-z_v)$	
Begin Merging (touch) = 118.7	***11.64***	***1.785***	***14.46***	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
125.0	13.56	#N/A	14.20	*Interpolated Layer Eqns*	**25 ft Intervals**
150.0	21.18	#N/A	13.19	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
175.0	28.80	#N/A	12.17		
200.0	36.42	#N/A	11.15		
225.0	44.04	#N/A	10.14		
250.0	51.66	#N/A	9.12		
275.0	59.28	#N/A	8.10		
300.0	66.90	#N/A	7.08		
325.0	74.52	#N/A	6.07	$V=\{n(V^3a)_{full}/a\}^{1/3}$	**50 ft Intervals**
350.0	82.14	#N/A	5.05	$a=a_m+0.16(z-z_{full})$	
End Merging (full/mp) = 374.9	89.73	24.731	4.04		
400.0	97.38	25.955	3.97		
450.0	112.62	28.394	3.85		
500.0	127.86	30.832	3.75		**100 ft Intervals**
600.0	158.34	35.709	3.57		
700.0	188.82	40.586	3.42		
800.0	219.30	45.463	3.29		
900.0	249.78	50.339	3.18		
1000.0	280.26	55.216	3.09		
1100.0	310.74	60.093	3.00		
1200.0	341.22	64.970	2.92		
1300.0	371.70	69.847	2.85		**500 ft Intervals**
1500.0	432.66	79.600	2.73		
2000.0	585.06	103.984	2.50		
2500.0	737.46	128.368	2.33		
3000.0	889.86	152.752	2.20		
3500.0	1042.26	177.136	2.09		
4000.0	1194.66	201.520	2.01		
4500.0	1347.06	225.904	1.93		

SINGLE/Approximated Plume Average Vertical Velocities for Northtown Chillers - Summer Max*

Based on 9 chillers w/ 6 cells/combined tower. Calc' eff.diam for each tower wiith each cell at 66" ID (368,400 ACFM total for each combined tower).

"Aviation Safety and Buoyant Plumes," **Peter Best, et. al.**

"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Plume from a Gas-Turbine Power Station at Oakey, Queensland, Australia," **Dr. K.T. Spillane**

Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	24.54	meters	80 6/12	feet-inches	λ 1.11	
Individual Chiller Stack Diameter D	4.1063	meters	161.7	inches	λ_o ~1.0	
Stack Velocity V_{exit}	13.13	m/s	43.07	ft/sec	$4Vol/(60\pi D^2)$	
Individual Chiller Volumetric Flow	173.87	cu.m/sec	368,400	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	313.32	Kelvins	104.3	°F		
Initial Stack Buoyancy Flux F_o	19.2457	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2 gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Number of Towers n	9			1.732	Multiple Stack Multiplication Factor ($n^{0.25}$)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	25.664	meters*	84.2	feet*	z_{jet} = 6.25D, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	50.201	meters	**164.7**	feet		"
Vertical Velocity V_{jet}	6.564	m/s	21.54	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	8.213	meters	26.9	feet	$2a_{jet}$ = 2D; Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet Phase						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Plume Top-Hat Radius a	**Solutions in Table Below**				$0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	0.459	meters*	1.5	feet*	$6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	24.995	meters	82.0	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9821
Vertical Velocity V	**Solutions in Table Below**				$\{(Va)_o^3 + 0.12F_o [(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)}$ / a	Sect.2.1(6)
Product $(Va)_o$	26.473	m^2/s			$V_{exit}D/2(\theta_e/\theta_s)^{1/2}$	
Single Chiller Results:						
Solve for plume-averaged vertical velocity at height	**500.0**	**feet**		152.4	meters above ground ($z'+h_s$)	
Gives the following Height above Stack z'	127.864	meters*	419.5	feet*		
Plume Top-Hat Diameter 2a'	40.769	meters	133.8	feet	$2a'=2*0.16(z'-z_v)$	Sect.2/Eq.6
Vertical Velocity V	**1.861**	**m/s**	6.10	ft/sec	$V=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{(1/3)}/(2a'/2)$	Sect.2/Eq.6
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s plume-averaged vertical velocity**			**Critical VV > Top of Jet (Spillane)**	
Find Height above Stack z_{crit}	32.116	meters	105.4	feet	Solve for $x=(z-z_v)$ simultaneously in both eqs. (i.e., Va and a)	
Height above Ground $z_{crit}+h_s$	56.653	meters	**185.9**	feet	for $V=V_{crit}$ using the cubic equation $ax^3+bx^2+cx+d=0$, where	
					a=1, c=0, and $b=-(0.12F_o)/(V_{crit}^3 0.16^3)$=	-3.78727
Interpolated Height of critical vertical velocity in Jet Phase:					and $d=[0.12F_o(6.25D-z_v)^2-(Va)_o^3]/(V_{crit}^3 0.16^3)$=	-28018.28
Find Height above Stack z_{crit}	#N/A	meters	#N/A	feet	http://www.1728.org/cubic.htm	
Height above Ground $z_{crit}+h_s$	#N/A	meters	**#N/A**	feet	gives the real solution x = z-zv =	31.6574
					or z(m/above stack) =	32.116
					z(ft/above ground) =	185.9

Table of Plume Top-Hat Diameters (2a) and Plume-Averaged Vertical Velocities starting at end of jet phase:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	SingleStk VertVel(m/s)	Plume Temp(K)		
Stack.Rel.Ht = 80.5	**0.00**	**2.053**	**13.13**			
100.0	5.94	2.529	11.61		**Jet Phase Eqs:**	**20 ft Intervals**
120.0	12.04	3.016	10.05		Linearly interpolated from Stack Rel.Ht to Top of Jet	
140.0	18.14	3.504	8.49		**Spillane Equations:**	
160.0	**24.23**	**3.992**	**6.93**			
160.0	24.23	3.992	6.93		$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3}$ / a	
Top of Single jet = 164.7	**25.66**	**4.106**	**6.56**		$a = 0.16(z-z_v)$	
180.0	30.33	4.779	5.60	306.56	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^2*\lambda^2)))$	
200.0	36.42	5.754	4.72	305.97	**CEC Staff Equation:**	
220.0	42.52	6.730	4.11	305.29	$V_{mp}=n^{0.25}V_{sp}$	
240.0	48.62	7.705	3.66	304.79	**Brigg's Equation:**	
260.0	54.71	8.680	3.32	304.42	$V_{Brigg's} = (2/3) \times 1.6^{(3/2)} \times F_{mp}^{(1/2)} \times u^{(-1/2)} \times z^{(-1/2)}$	
280.0	60.81	9.656	3.05	304.13	where $F_{mp} = nF_{sp}$	
300.0	66.90	10.631	2.83	303.90		
350.0	82.14	13.070	2.44	303.71		
400.0	97.38	15.508	2.18	303.36		**50 ft Intervals**
450.0	112.62	17.946	2.00	303.13	**Max<5.3 m/s**	
500.0	127.86	20.385	1.86	302.96		
550.0	143.10	22.823	1.75	302.83		
600.0	158.34	25.262	1.67	302.74		
650.0	173.58	27.700	1.60	302.66		
700.0	188.82	30.138	1.54	302.60		
800.0	219.30	35.015	1.44	302.56		
900.0	249.78	39.892	1.36	302.48		**100 ft Intervals**
1000.0	280.26	44.769	1.30	302.43		
1100.0	310.74	49.646	1.25	302.39		
1200.0	341.22	54.522	1.21	302.37		
1300.0	371.70	59.399	1.17	302.34		
1400.0	402.18	64.276	1.14	302.33		
1500.0	432.66	69.153	1.11	302.31		
2000.0	585.06	93.537	1.00	302.30		
2500.0	737.46	117.921	0.92	302.27		**500 ft Intervals**
3000.0	889.86	142.305	0.86	302.25		
3500.0	1042.26	166.689	0.82	302.24		
4000.0	1194.66	191.073	0.78	302.23		
4500.0	1347.06	215.457	0.75	302.23		
5000.0	1499.46	239.841	0.72	302.22		

*Winter Min = Monthly Mean of Minimum Daily Temperatures for 1971-2000 (Lowest in December)

NOAA Sources: Climatography of the United

MERGED (along length) Plume Average Vertical Velocities for Northtown Chillers - Summer Max*

***"Aviation Safety and Buoyant Plumes,"* Peter Best, et. al.**

***"The Evaluation of Maximum Updraft Speeds for Calm Conditions at Various Heights in the Merged Plume from Two Gas-Turbine Power Station at Oakey, Queensland, Australia,"* Dr. K.T. Spillane**

Ambient Conditions:					**Constants:** Assume neutral conditions ($d\theta/dz=0$ or $\theta_a=\theta_e$)	
Ambient Potential Temp θ_a	302.21	Kelvins	84.3	°F	0.3048 meters/feet	
Plume Exit Conditions:					Gravity g 9.81 m/s^2	
Stack Height h_s	24.54	meters	80 6/12	feet-inches	λ 1.11	
Individual Stack Diameter D	4.10632	meters	161.7	inches	λ_o ~1.0	
Stack Velocity V_{exit}	13.13	m/s	43.07	ft/sec	$4Vol/(60\pi D^2)$	
Individual Volumetric Flow	173.87	cu.m/sec	368,400	ACFM	$\pi V_{exit}D^2/4$	Sect.2/¶1
Stack Potential Temp θ_s	313.32	Kelvins	104.3	°F		
Initial Stack Buoyancy Flux F_o	19.25	m^4/s^3	20.0	ΔT(°F)	$gV_{exit}D^2(1-\theta_a/\theta_s)/4$ = Vol.Flow$(g/\pi)(1-\theta_a/\theta_s)$	Sect.2/¶1
Plume Buoyancy Flux F	N/A	m^4/s^3			$\lambda^2gVa^2(1-\theta_a/\theta_p)$ for a,V,θ_p at plume height (see below)	
Total Number of Stacks n	9					
Average Adjacent Stack Separation d	3.57	meters	11.7	feet	**Calcs based on multiple plume treatment in Peter Best Paper:**	
Number of Stacks along Orientation N	9				**plume velocities increased by $N^{0.25}$ at the height where plumes**	
					fully merged (interp. below ht, single merged stack above ht)	
Conditions at End (Top) of Jet Phase:						
Height above Stack z_{jet}	25.664	meters*	84.2	feet*	$z_{jet} = 6.25D$, meters*=meters above stack top	Sect.3/¶1
Height above Ground $z_{jet}+h_s$	50.201	meters	**164.7**	feet		"
Vertical Velocity V_{jet}	**6.564**	m/s	21.54	ft/sec	$V_{jet} = 0.5V_{exit} = V_{exit}/2$	"
Plume Top-Hat Diameter $2a_{jet}$	8.213	meters	26.9	feet	$2a_{jet} = 2D$ Conservation of momentum	"
Spillane Methodology - Analytical Solutions for Calm Conditions for Plume Heights above Jet and Merging Phases						
Single Plume-averaged Vertical Velocity V given by Analytical Solution in Paper where Product Va given by equations below:						
Single Plume Values: Plume Top-Hat Radius a	**Used in Plume Merging Only**				$a = 0.16(z-z_v)$, or linear increase with height	Sect.2/Eq.6
Virtual Source Height z_v	0.459	meters*	1.5	feet*	$z_v = 6.25D[1-(\theta_e/\theta_s)^{1/2}]$, meters*=meters above stack top	Sect.2/Eq.6
Height above Ground z_v+h_s	24.995	meters	82.0	feet	where $(\theta_a/\theta_s)^{1/2} = (\theta_e/\theta_s)^{1/2}$=	0.9821
Single Plume Values: Vertical Velocity V	**Used in Plume Merging Only**				$\{(Va)_o^3 + 0.12F_o[(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	Sect.2.1(6)
Product $(Va)_o$	26.473	m^2/s			$V_{exit}(D/2)(\theta_e/\theta_s)^{1/2}$	
Plume Merging - Based on Single Plume Calculations where:						Sect.3/¶3
Begin Merging Plume Top-Hat Diameter $2a_{touch}$	3.570	meters	11.7	feet	$2a_{touch}=d$, (or $a_{touch}=d/2$)	
Height above Stack z_{touch}	11.615	meters*	38.1	feet*	$z_{touch} = z_v+d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{touch}+h_s$	36.152	meters	**118.6**	feet		
Vertical Velocity V_{touch}	**14.510**	m/s	47.6	ft/sec	$V_{touch} = \{(Va)_o^3 + 0.12F_o[(z-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a$	
Total Merging Plume Top-Hat **Diameter** $2a_{full}$	**28.560**	meters	93.7	feet	$2a_{full}=2d(N-1)/2$, (or $a_{full}=d(N-1)/2$) **FOR 2 STACKS, $2a_{full}=2d$**	
Height above Stack z_{full}	89.709	meters*	294.3	feet*	$z_{full} = z_v+2d/(2*0.16)$, meters*=meters above stack top	
Height above Ground $z_{full}+h_s$	114.245	meters	**374.8**	feet		
Vertical Velocity V_{full}	2.301	m/s	7.6	ft/sec	$V_{full} = \{(Va)_o^3 + 0.12F_o[(z_{full}-z_v)^2 - (6.25D-z_v)^2]\}^{(1/3)} / a_{full}$	
Product $(V^3a)_{full}$	174	m^4/s^3				
Conditions at End (Top) of Merging Phase - Define new values for V_{full} and a_{full} in Merged Plume calculations (based on TOTAL number of stacks):						
Merged Plume Values: Plume Diameter 2a	**Solutions in Table Below**				$2a = 2 \times (a_m + 0.16(z-z_{full}))$, or linear increase with height	
Revised Merged Plume **Radius** a_m	24.734	meters	81.1	feet	where $a_m = n^{0.25}a_{full}$ where Total Merging Occurs	
Revised Merged Plume Velocity V_m	**3.986**	m/s	13.08	ft/sec	and $V_m = n^{0.25}V_{full}$ where Total Merging Occurs	
Revised Virtual Source Height z_{full}	89.709	meters*	294.3	feet*	Height above stack where Total Merging Occurs (shown above)	
Revised Vertical Velocity V	**Solutions in Tables Below**				$V=\{n(V^3a)_{full}/a\}^{1/3}$ for heights above total merging elevation	
					$V=V_{touch}+(V_m-V_{touch})*(z-z_{touch})/(z_{full}-z_{touch})$	
Multiple Plume Calculations					for heights below total merging elevation	
Solve for plume-averaged vertical velocity at height	**500.0**	**feet**	152.4		meters above ground $(z+h_s)$	
Gives the following Height above Stack z	127.864	meters*	419.5	feet*	REGULAR EQNS	
Plume Top-Hat Radius a	30.838	meters	101.2	feet	$a=a_m+0.16(z-z_{full})$ **if $z>z_{full}$**	
Vertical Velocity V	**3.703**	**m/s**	12.15	ft/sec	$V=\{n(V^3a)_{full}/a\}^{1/3}$ **if $z>z_{full}$**	
					$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$ **if $z_{touch}<z<z_{full}$**	
					V'=single plume values **if $z<z_{touch}$**	
Solve for Height of CASC critical vertical velocity V_{crit}	**5.30**	**m/s**			LESS THAN TOP OF MERGING PHASE-INTERPO	
Find Height above Stack z_{crit}	79.956	meters	262.3	feet	$z_{crit} = z_{full} + \{[n(V^3a)_{full}/(V_{crit})^3]-a_m\}/0.16$ **if $V_{crit}<V_m$**	
Height above Ground $z_{crit}+h_s$	104.492	meters	**342.8**	**feet**	$z_{crit}=z_{touch}+(z_{full}-z_{touch})*(V_{crit}-V_{touch})/(V_m-V_{touch})$ **if $V_{crit}>V_m$**	

Table of MERGED Plume-Averaged Vertical Velocities starting at Touching Height:

Height (feet) above ground	(meters) above stack	Plume Radius(m)	Vert. Vel(m/s)		
				Single Plume Eqns (see Single Plume spreadsheet)	
				$V_{plume}=\{(Va)_o^3+0.12F_o[(z-z_v)^2-(6.25D-z_v)^2]\}^{1/3} / a$	
				$a = 0.16(z-z_v)$	
Begin Merging (touch) = 118.6	***11.61***	***1.785***	***14.51***	$\theta_p=\theta_s(1+(1-(\theta_e/\theta_s))*(V_{exit}D^2/(4V_{plume}*a^{2}*\lambda^2)))$	
125.0	13.56	#N/A	14.25	*Interpolated Layer Eqns*	**25 ft Intervals**
150.0	21.18	#N/A	13.22	$V'=V_{touch}+(V_m-V_{touch})*(z'-z_{touch})/(z_{full}-z_{touch})$	
175.0	28.80	#N/A	12.19		
200.0	36.42	#N/A	11.17		
225.0	44.04	#N/A	10.14		
250.0	51.66	#N/A	9.11		
275.0	59.28	#N/A	8.09		
300.0	66.90	#N/A	7.06		
325.0	74.52	#N/A	6.03	$V=\{n(V^3a)_{full}/a\}^{1/3}$	**50 ft Intervals**
350.0	82.14	#N/A	5.01	$a=a_m+0.16(z-z_{full})$	
End Merging (full/mp) = 374.8	89.70	24.733	3.99		
400.0	97.38	25.962	3.92		
450.0	112.62	28.400	3.81		
500.0	127.86	30.838	3.70		**100 ft Intervals**
600.0	158.34	35.715	3.53		
700.0	188.82	40.592	3.38		
800.0	219.30	45.469	3.25		
900.0	249.78	50.346	3.14		
1000.0	280.26	55.222	3.05		
1100.0	310.74	60.099	2.96		
1200.0	341.22	64.976	2.89		
1300.0	371.70	69.853	2.82		**500 ft Intervals**
1500.0	432.66	79.606	2.70		
2000.0	585.06	103.990	2.47		
2500.0	737.46	128.374	2.30		
3000.0	889.86	152.758	2.17		
3500.0	1042.26	177.142	2.07		
4000.0	1194.66	201.526	1.98		
4500.0	1347.06	225.910	1.91		